THE MAMDANI CONTROLLER IN PREDICTION OF THE SURVIVAL LENGTH IN ELDERLY GASTRIC PATIENTS

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Abstract: Strict analytic formulas are the tools derived for determining the formal relationships between a sample of independent variables and a variable which they affect. If we cannot formalize the function tying the independent and dependent variables then we will utilize fuzzy control actions. The algorithm is particularly adaptable to support the problem of prognosticating the survival length for gastric cancer patients. We thus formulate the objective of the current paper as the utilization of fuzzy control action for the purpose of making the survival prognoses.

1 INTRODUCTION

Expert-knowledge designs IF-THEN together with assumptions of fuzzy set theory have given rise to the creation of fuzzy control (Mamdani and Assilian, 1973; Nguyen et al., 2002; Andrei, 2005).

Typical applications of fuzzy control have mostly concerned technical processes but, in the current paper, we intend to prove fuzzy control to make prognoses of the survival length in patients with diagnosis gastric cancer.

In the first trials of survival approximation a survival curve from censored data was introduced (Kaplan and Meier, 1958). The model was used in cancer patient examinations to estimate the length of living (Newland et al., 1994). The Cox regression (Cox, 1972) of life length prediction was developed in such studies as logistic Cox regression (Sargent, 2001). The statistics-based models predicting the survival were compared by Everitt and Rabe-Hesketh (2001) who found such model disadvantages as the lack of normal distribution or missing values among survival times.

The development of computational intelligence brought neural networks as a tool of approximating the life length for cancer patients (Burke et al., 2001; Grumett et al., 2003).

We prove the action of Mamdani controller, which has not been adapted yet to estimation of survival. We count on reliable results to place the controller among other life approximation models.

2 VARIABLE FUZZIFICATION

Fuzzy control model is applied when we cannot formalize the functional connection between independent and dependent variables.

We expect to evaluate the survival length in patients with diagnosis "*gastric cancer*". Variable Z = "*survival length*" is affected by X = "*age*" and Y = "*CRP-value*", selected as the most essential markers of making the prognosis. Since the formula $z = f(x, y)$, $x \in X$, $y \in Y$, $z \in Z$, is not known then we will test the action of fuzzy control.

All variables are divided into levels, which are expressed by lists of terms. The terms are represented by fuzzy sets, restricted by the parametric *s*-functions lying over the variable domains $[x_{\min}, x_{\max}]$, $[y_{\min}, y_{\max}]$ and $[z_{\min}, z_{\max}]$.

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We introduce five levels into X, Y and Z, denoted by X_i, Y_j and Z_k, $i, j, k = 0,...,4$.

A family of fuzzy set membership functions corresponding to X_i will be expressed by the parametric s-class functions $\mu_{X_i}(x)$ split in $left\mu_{X_i}(x)$ and $right\mu_{X_i}(x)$ (Rakus-Andersson, 2007; Rakus-Andersson et al., 2010) where

$$left\mu_{X_i}(x)=\begin{cases} 2\left(\frac{x-((x_{\min}-h_X)+h_X\cdot i)}{h_X}\right)^2 \text{ if} \\ (x_{\min}-h_X)+h_X\cdot i \le x \le \left(x_{\min}-\frac{h_X}{2}\right)+h_X\cdot i, \\ 1-2\left(\frac{x-(x_{\min}+h_X\cdot i)}{h_X}\right)^2 \text{ if} \\ \left(x_{\min}-\frac{h_X}{2}\right)+h_X\cdot i \le x \le (x_{\min})+h_X\cdot i, \end{cases} \tag{1}$$

and

$$right\mu_{X_i}(x)=\begin{cases} 1-2\left(\frac{x-(x_{\min}+h_X\cdot i)}{h_X}\right)^2 \text{ if} \\ (x_{\min})+h_X\cdot i \le x \le \left(x_{\min}+\frac{h_X}{2}\right)+h_X\cdot i, \\ 2\left(\frac{x-((x_{\min}+h_X)+h_X\cdot i)}{h_X}\right)^2 \text{ if} \\ \left(x_{\min}+\frac{h_X}{2}\right)+h_X\cdot i \le x \le (x_{\min}+h_X)+h_X\cdot i. \end{cases} \tag{2}$$

Formulas (1) and (2) are affected by $x_{\min}$ and by the parameter value h_X, which measures the length between the beginnings of two adjacent functions X_i. The h_X quantity is adjusted to the number of X-functions and to the distance between $x_{\min}$ and $x_{\max}$.

The functions of Y_j, $j = 0,\dots,4$, are constructed as similar to (1) and (2) for the accommodated values of parameters h_Y and $Y_{\min}$ to the conditions of Y.

The Z_k's functions $\mu_{Z_k}(z)$, $k = 0,\dots,4$, are split in $left\mu_{Z_k}(z)$, $middle\mu_{Z_k}(z)$ and $right\mu_{Z_k}(z)$ as

$$left\mu_{Z_k}(z)=\begin{cases} 2\left(\frac{z-\left(z_{\min}-\frac{h_Z}{2}+h_Z\cdot k\right)}{\frac{h_Z}{2}}\right)^2 \text{ if} \\ z_{\min}-\frac{h_Z}{2}+h_Z\cdot k \le z \le z_{\min}-\frac{h_Z}{4}+h_Z\cdot k, \\ 1-2\left(\frac{z-(z_{\min}+h_Z\cdot k)}{\frac{h_Z}{2}}\right)^2 \text{ if} \\ z_{\min}-\frac{h_Z}{4}+h_Z\cdot k \le z \le z_{\min}+h_Z\cdot k, \end{cases} \tag{3}$$

$$middle\mu_{Z_k}(z)= \\ 1 \text{ if } z_{\min}+h_Z\cdot k \le z \le z_{\min}+\frac{h_Z}{2}+h_Z\cdot k, \tag{4}$$

and

$$right\mu_{Z_k}(z)=\begin{cases} 1-2\left(\frac{z-\left(z_{\min}+\frac{h_Z}{2}+h_Z\cdot k\right)}{\frac{h_Z}{2}}\right)^2 \text{ if} \\ z_{\min}+\frac{h_Z}{2}+h_Z\cdot k \le z \le z_{\min}+\frac{3h_Z}{4}+h_Z\cdot k, \\ 2\left(\frac{z-(z_{\min}+h_Z+h_Z\cdot k)}{\frac{h_Z}{2}}\right)^2 \text{ if} \\ z_{\min}+\frac{3h_Z}{4}+h_Z\cdot k \le z \le z_{\min}+h_Z+h_Z\cdot k. \end{cases} \tag{5}$$

The parameter h_Z allows designing five functions of fuzzy sets from Z over $[z_{\min}, z_{\max}]$.

Variable X will be restricted over [0, 100]. We thus state $x_{\min}=0$, $h_X=25$ and $i=0,...,4$. For the terms of "*age*" we will get by (1) and (2) the set of membership functions sketched in Fig. 1.

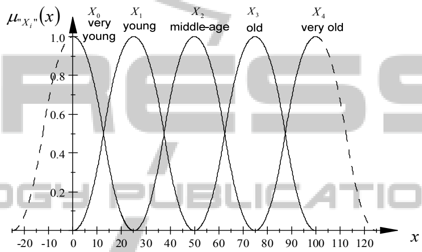

Figure 1: The membership functions for "*age*".

By inserting parameters of $y_{\min}$ and h_Y in (1) and (2), where x is replaced by y, we generate the membership functions for "*CRP-value*" over [0, 50] with $h_Y=15$ and $j = 0,...,4$ to plot them in Fig. 2.

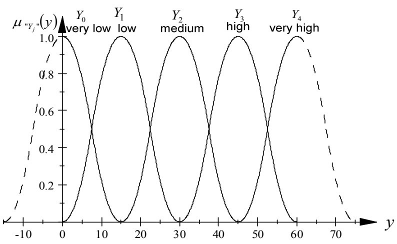

Figure 2: The membership functions for "*CRP-value*".

The output variable Z takes the values in [0, 5]. We determine $h_Z=1$ and set $k=0,...,4$ in (3), (4) and (5) to initialize the functions depicted in Fig. 3.

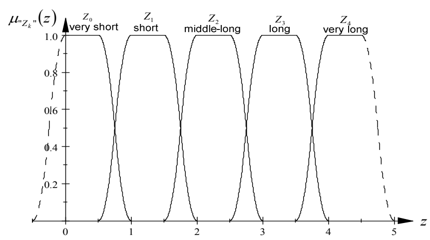

Figure 3: The membership functions for the "*survival length*".

3 THE LOGICAL RULES IF-THEN AND OUTPUT DEFUZIFICATION

After the fuzzification procedure we determine rules, which link the states of two input variables to the state of the output variable. We design Table 1 with entries filled with terms of "*survival length*".

Table 1: Rule base of fuzzy controller estimating "*survival length*".

$X_i \backslash Y_j$	Y_0	Y_1	Y_2	Y_3	Y_4
X_0					
X_1					
X_2	Z_2				
X_3	Z_2	Z_1	Z_1	Z_1	Z_0
X_4	Z_1	Z_1	Z_0	Z_0	Z_0

Some entries in the table are empty, since the essential data was lacking for younger people.

We want to make the survival prognosis for (x, y), $x \in X$, $y \in Y$. Value x can belong to more than one fuzzy set X_i with different membership degrees $\mu_{X_i}(x)$. Element y associated to x is a member of some Y_j with degrees $\mu_{Y_j}(y)$. By means of the IF-THEN statements from Table 1 we determine the contents of rules

$$\text{Rule } R_{(x,y):l}: \text{ If } x \text{ is } X_{i:l} \text{ and } y \text{ is } Y_{j:l}, \text{ then } z \text{ is } Z_{k:l}, \tag{6}$$

where l is the rule number.

To evaluate the influence of the input variables on the output consequences we estimate

$$\alpha_{(x,y):l} = \min\left(\mu_{X_{i:l}}(x), \mu_{Y_{j:l}}(y)\right) \tag{7}$$

for each $X_{i:l}$ and $Y_{j:l}$ concerning the choice of (x,y).

Consequences of all rules $R_{(x,\ y):l}$ become fuzzy sets $R_{(x,y):l}^{conseq}$, which are stated in Z as

$$\mu_{R_{(x,y):l}}{}^{conseq}(z) = \min\left(\alpha_{(x,y):l}, \mu_{Z_k:l}(z)\right). \tag{8}$$

We aggregate the consequence sets $R_{(x,y):l}^{conseq}$ in one common set $conseq_{(x,y)}$ allocated in Z over a continuous interval $[z_0, z_n]$ due to

$$\mu_{conseq_{(x,y)}}(z) = \max_{l}\left(\mu_{R_{(x,y):l}}{}^{conseq}(z)\right). \tag{9}$$

In order to assign a crisp value z to the selected pair (x, y) we defuzzify the consequence fuzzy set (9) in Z. We are furnished with the formula

$$z = f(x, y) = \frac{\int_{z_0}^{z_n} z \cdot \mu^{conseq}(z)dz}{\int_{z_0}^{z_n} \mu^{conseq}(z)dz}. \tag{10}$$

Example

Let $x = 77$ and $y = 16$. Age 77 belongs to fuzzy set X_3 = "*old*". Therefore, for i = 3, h_X = 0.25 and $x_{\min}$ = 0 the membership degree $\mu_{X_3}(77) = 0.9872$.

In X_4 = "*very old*" $\mu_{X_4}(77) = 0.0128$. *CRP*-value y = 16 fits for sets: Y_1 = "*low*" with $\mu_{Y_1}(16) = 0.991$ and Y_2 = "*medium*" with $\mu_{Y_2}(16) = 0.009$.

In accordance with (6) we find the rules

$R_{(77,16):1}$: IF X is X_3 and Y is Y_1 THEN Z is Z_1,

$R_{(77,16):2}$: IF X is X_3 and Y is Y_2 THEN Z is Z_1,

$R_{(77,16):3}$: IF X is X_4 and Y is Y_1 THEN Z is Z_1,

$R_{(77,16):4}$: IF X is X_4 and Y is Y_2 THEN Z is Z_0.

For $\alpha_{(77,16):1} = \min\left(\mu_{X_3:1}(77), \mu_{Y_1:1}(16)\right) = 0.9872$, $\alpha_{(77,16):2} = 0.009$, $\alpha_{(77,16):3} = 0.0128$ and $\alpha_{(77,16):4} = 0.009$, due to (7), we establish consequence sets (8) to determine the final consequence $conseq_{(x,y)}$, which fits for (9) and is shown in Fig. 4.

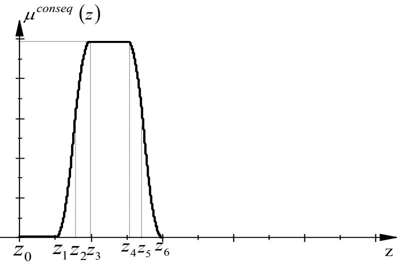

Figure 4: The consequence set $conseq_{(77,16)}$ in Z.

Formula (10) constitutes a basis of an estimation of the survival length expected when assuming "*age*" = 77 and "*CRP-value*" = 16. Over interval $[z_0, z_6] = [0, 2]$, which contains characteristic points z_0 = 0, z_1 = 0.533, z_2 = 0.75, z_3 = 0.96, z_4 = 1.54, z_5 = 1.75 and z_6 = 2, we compute the z-prognosis

$$z = f(77,16) = \frac{\int_0^{0.5335} 0.009 z\,dz + \cdots + \int_{1.75}^{2} 2\left(\frac{z-2}{0.5}\right)^2 z\,dz}{\int_0^{0.5335} 0.009\,dz + \cdots + \int_{1.75}^{2} 2\left(\frac{z-2}{0.5}\right)^2 dz} = 1.05$$

The result converges with the physician's own judgment. For each pair (x,y) we can arrange new actions of the fuzzy control algorithm to estimate the patient's period of surviving.

4 CONCLUSIONS

Fuzzy control system is a powerful method, which mostly is applied to technologies controlling complex processes by means of human experience. In this work we have proved that the expected values of patients' survival lengths can be estimated even if the mathematical formalization between independent and dependent variables is unknown. For each x and each y belonging to continuous spaces X and Y respectively, we can repeat the control algorithm in order to cover the space of pairs over the Cartesian product of X and Y with a continuous surface.

We should emphasize that the Mamdani control system does not need any special assumptions such as distributions of variables, regularity and others, which are necessary to be fulfilled in statistical survival tests (see discussion in Section 1).

The authors' special contribution is the mathematically formalized design of membership functions assisting variable levels.

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