MODELLING A BACKGROUND FOR BACKGROUND SUBTRACTION FROM A SEQUENCE OF IMAGES

Formulation of Probability Distribution of Pixel Positions

Suil Son, Young-Woon Cha and Suk I. Yoo
School of Computer Science and Engineering, Seoul National University, 599 Gwanak-ro, Seoul, Korea

Keywords: Background subtraction, Foreground detection, Probability distribution of pixel positions, Intensity clusters, Kernel density estimation.

Abstract: This paper presents a new background subtraction approach to identifying the various changes of objects in a sequence of images. A background is modelled as the probability distribution of pixel positions given intensity clusters, which is constructed from a given sequence of images. Each pixel position in a new image is then identified with either a background or a foreground, depending on its value from probability distribution of pixel positions representing a background. The presented approach is illustrated using two examples. As compared to traditional intensity-based approaches, this approach is shown to be robust to dynamic textures and various changes of illumination.

1 INTRODUCTION

Detecting a meaningful foreground from a sequence of images, known as background subtraction, has been studied intensively due to its wide area of application such as tracking, identification and surveillance. Two issues in developing background subtraction methods are how to resolve the change of illumination due to noise or light and how to manage dynamic textures such as swaying tree or flow of water.

To manage the change of illumination, most of background subtraction methods have used intensity distributions (Wren, Darrelll and Pentland, 1997, Stauffer and Grimson, 1999, Elgammal, Harwood, and Davis, 2000, Power and Schoonees, 2002, Zivkovic and Heijden, 2006, Dalley, Migdal, and Grimson, 2008). Using intensity distributions, however, does not work very well when there is the large change of illumination in all pixels on the image. To resolve dynamic textures, a mixture of Gaussian (Stauffer and Grimson, 1999, Power and Schoonees, 2002, Zivkovic and Heijden, 2006, Dalley *et al*, 2008) and the kernel density estimation (Elgammal *et al*, 2000, Mittal and Paragios, 2004) have been suggested. To recognize the background having small motion correctly, spatial information of objects is necessary. A window formed with neighbours of a pixel may be used to reflect such spatial information (Elgammal *et al*, 2000, Dalley *et al*, 2008). Although the approach using windows reduces false detection of foreground for dynamic textures, it is not easy to define the exact size of a window in advance. Sheikh and Shah (2005) suggested joint distribution of positions and intensities to reflect the spatial information. Since this joint representation of image pixels reflects the local spatial structure, it works well on motion of background objects. However, it has a difficulty with the curse of dimensionality from its high dimensional data representation.

This paper presents a new approach to relaxing those difficulties of the traditional background subtraction methods: A background is modelled as the probability distribution of pixel positions given intensity clusters. An image in a given sequence is assumed to have M intensity sources. From the image having M intensity sources, M intesity clusters can be formulated. Although it is not easy to figure out the optimal number of M from a given image, specially when the image is complex with various objects, the value of M is assumed to be not larger than six in general due to the range of grey level from 0 to 255.

For each of the M intensity clusters, the distribution of pixel positions is then computed from the sequence of images. The computed distribution of

Son S., Cha Y. and Yoo S..
MODELLING A BACKGROUND FOR BACKGROUND SUBTRACTION FROM A SEQUENCE OF IMAGES - Formulation of Probability Distribution of Pixel Positions.
DOI: 10.5220/0003141105450551
In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence* (ICAART-2011), pages 545-551
ISBN: 978-989-8425-40-9

pixel positions, however, suffers from the discontinuity between pixels due to the limited number of images in the sequence. By smoothing this discontinuity using kernel density estimation, the probability distribution of pixel positions modelling a background is finally constructed. Each pixel position in the new image is then identified with either a foreground or a background, depending on its value from the probability distribution constructed.

In the following section related previous works are visited. In section 3, generation of a probability distribution of pixel positions given intensity clusters from a sequence of images is described. The process to identify each pixel position in a new image with either a background or a foreground is explained in section 4. Finally the presented approach is illustrated and compared to the intensity-based approach in section 5.

2 RELATED WORKS

Most of traditional background subtraction models estimate intensity distributions for each pixel position (Paccardi, 2004).

Stauffer and Grimson (1999) have modelled the value of each pixel as a mixture of M Gaussian distributions in order to represent intensity variations caused by small motion of objects in the background like swaying trees or flow of water. The probability that a pixel position x has an intensity x_t at time t is estimated as

$$P(x_t) = \sum_{j=1}^{M} \frac{w_j}{(2\pi)^{\frac{d}{2}} \, |\Sigma_j^{-1}|^{\frac{1}{2}}} e^{-\frac{1}{2}(x_t-\mu_j)^T \Sigma_j^{-1}(x_t-\mu_j)} \quad (1)$$

where w_j is the weight for each Gaussian distribution, μ_j is the mean and $\Sigma_j=\sigma^2 I$ is the covariance of the jth Gaussian distribution, and M is the number of Gaussians. The M is selected from 3 to 5. This model assumes that the intensity value may result from some candidate sources each of which is modelled as a Gaussian. Therefore, an intensity value has several distributions where it may come from. This model can adapt to small change of intensity or shape in background but in the case where the background has slightly large variations, it fails to achieve correct identification.

Dalley *et al*. (2008) have proposed a model modified from the Stauffer and Grimson(1999)'s one. In this model, a set of mixture components that lie at the local spatial neighbourhood of a pixel are suggested rather a mixture that lies at the same pixel. The probability that a pixel position i has an intensity c_i is estimated as

$$P(c_i) \propto \sum_{j \in Neighbour(i)} w_j N(c_i \mid \mu_j, \Sigma_j) \quad (2)$$

where μ_j is the mean, Σ_j is the covariance of Gaussian distribution N at a pixel position j, and w_j is a mixture weight of a neighbour j. This model considers intensity distributions at neighbours of a pixel simultaneously. Therefore, intensity variations caused by small motion of objects in background can be correctly identified with a background. However, the background subtraction result of this model is dependent on the size of a window applied to include neighbours but the optimal window size is hardly obtained.

Elgammal *et al* (2000) suggested the kernel density estimation to model intensity distribution with multimodality. When there are n samples of intensities, the true distribution of these intensities may be dense where the samples are closely located and may be sparse where the samples are scattered. These characteristics are modelled as a sum of many kernels centred at each sample. Since this model is a data driven approach, the multimodality of distribution for these data is naturally reflected without any assumption of the number of modes. This model for intensity distribution of a pixel can be represented as follows

$$P(x_t) = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{K}(x_t - x_i) \quad (3)$$

where x_t and x_i are intensities of a pixel x at time t and i ,respectively, n is the number of samples, and $\boldsymbol{K}$ is a kernel function. The Gaussian function is usually used for the kernel function $\boldsymbol{K}$. This model can handle situations where the background of scenes contains small motion but it still suffers from large motion and illumination changes. To overcome this difficulty, this model proposed additional policy considering the displacement probability $P_{Neighbour}$.

$$P_{Neighbour}(x_t) = \max_{y \in Neighbour(x)} P(x_t \mid B_y) \quad (4)$$

where B_y is a background sample for a pixel y. This approach significantly reduced false detection of a background but it still has a difficulty in selecting optimal neighbours.

All approaches discussed above are same in considering distribution of intensities for a given pixel position. In this framework, motion of objects in background was modelled as variations of intensities and neighbours of a pixel are introduced to reflect more spatial variations. These approaches may be regarded as an indirect approach to handle spatial variations. In this paper, we model spatial variations directly using distribution of pixel positions given intensity values.

3 MODELLING A BACKGROUND

In this section we first describe how to generate M intensity clusters from a given image and then construct from a sequence of images the probability distribution of pixel positions for each of M intensity clusters modelling a background.

3.1 Generation of *M* Intensity Clusters

When a given image has M intensity sources with background, it is assumed that the image can be characterized with M different intensity clusters. The k-means algorithm (Bishop, 2006) is then used to generate the M intensity clusters:

Let μ_i be a mean of intensity values of those pixels forming the ith cluster C_i where $i=1, \ldots, M$. and let I_{xy} be an intensity value of a pixel at location (x,y) of the image I with height of H and width of W. The k-means algorithm defines the membership value r_{xyi} of the intensity value I_{xy} with respect to the cluster C_i where $x=1,2,\ldots,W$ and $y=1,2,\ldots,H$, to be

$$r_{xyi} = \begin{cases} 1 \;, if\ i = \arg\min_{j} | I_{xy} - \mu_j |^2 \\ 0 \;, otherwise \end{cases} \tag{5}$$

where $j=1,2,\ldots,M$.

The value of μ_i is initially set to a small value and the r_{xyi} is computed by (5). With the computed value of r_{xyi} , the μ_i is then updated by (6). This process is repeated until there is no change in the values of μ_i and r_{xyi}. The value of r_{xyi} indicates whether or not the pixel at (x,y) is associated with the cluster C_i.

$$\mu_i = \frac{\sum_{y=1}^{H}\sum_{x=1}^{W} r_{xyi} I_{xy}}{\sum_{y=1}^{H}\sum_{x=1}^{W} r_{xyi}} \tag{6}$$

For example, the image with two objects, rectangle and circle, is characterized with three intensity clusters as shown in Fig. 1.

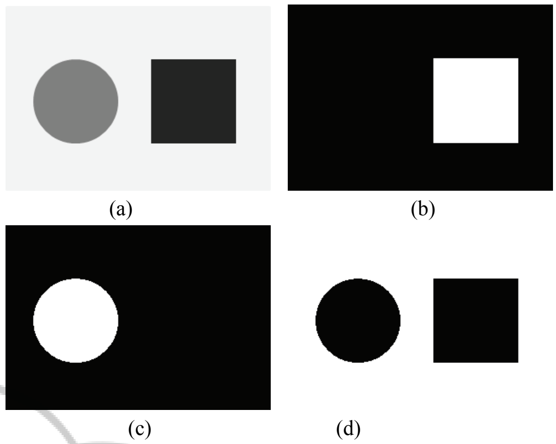


Figure 1: Three intensity clusters, (b), (c), and (d), shown as white areas, from a given image of (a).

3.2 Probability Distribution of Pixel Positions for Intensity Clusters

When M intensity clusters are generated from one image, the same number of clusters can be generated from each of images in a sequence. Once we obtain the same number of clusters from all images in the sequence, we can count the number of occurrence of each pixel position with respect to each of the M clusters. The occurrence of pixel positions for a given cluster may then represent statistical data for the intensity value associated with that cluster.

Let I^l be the lth image, $l=1,2,\ldots,N,$ from a sequence of N images. The histogram $h_N(x,y;C_i)$, defined to be the number of times from the N images that each position (x,y) is located in the cluster C_i, is then

$$h_N(x, y; C_i) = | \{ l : r^l_{xyi} = 1, l = 1, 2, ..., N \} | \tag{7}$$

where r^l_{xyi} is the value of r_{xyi} in the lth image I^l. Since each pixel considered as a background in a new image has its intensity value similar to those of the associated pixels in all of the N images in a sequence, its h value becomes large for one of M clusters. Each pixel considered as a foreground, however, has its intensity value different from those of the associated pixels in all the N images so that its h value becomes small for the associated cluster. A foreground comes from an unusual event and objects formed with those pixels considered as foregrounds may be observed in a few of the N images.

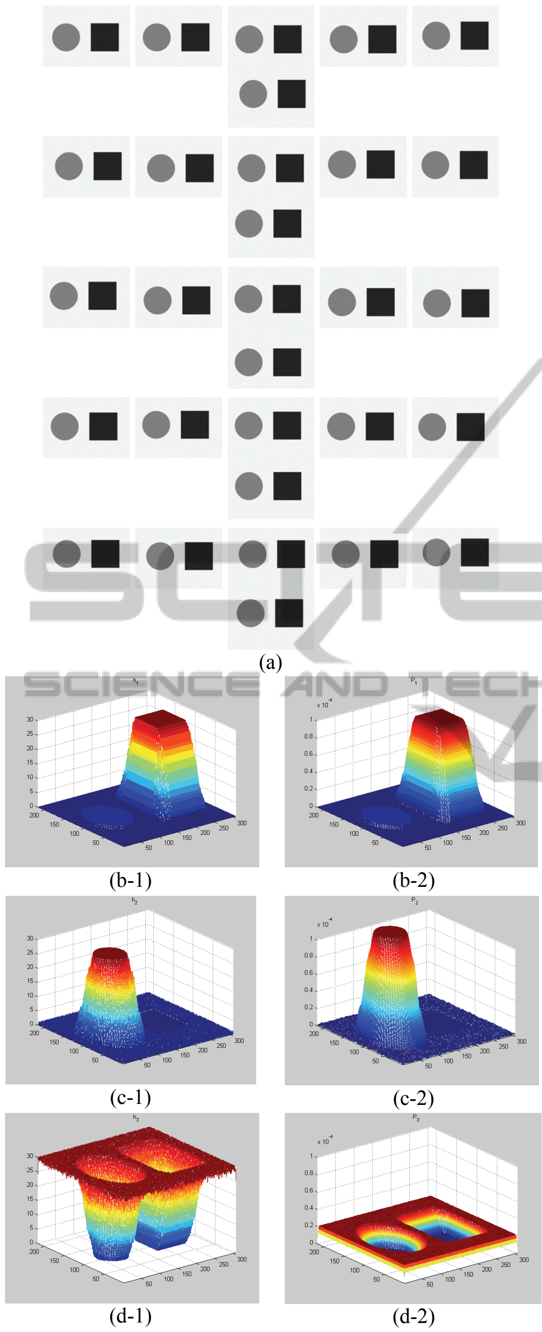


Figure 2: Histograms and Probability distributions of positions for each intensity cluster constructed from for a sequence of 30 images of (a): (b-1) $h_N(x,y;C_1)$, (b-2) $P(x,y|C_1)$, (c-1) $h_N(x,y;C_2)$, (c-2) $P(x,y|C_2)$, (d-1) $h_N(x,y;C_3)$, and (d-2) $P(x,y|C_3)$.

Since the histogram reflects the direct result from a given sequence of images, it suffers from its discontinuity for small motion of objects due to the limited number of images in the sequence. To overcome such difficulty, the kernel density estimation is suggested to smooth the discontinuity.

The kernel density estimation (KDE) is one of well-known nonparametric density estimation methods (Elgammal *et al*, 2000, Bishop 2006). The KDE has many kernels centred at each data point so as to construct a probability distribution of the data. The KDE has the property to relax discontinuity of data and make smooth change between data. Thus we propose a kernel density estimation weighted by the histogram to obtain the smooth and continuous distribution of pixel positions.

When there is a histogram $h_N(x,y;C_i)$ of an intensity cluster C_i, the position distribution for the cluster C_i , $P(x,y|C_i)$, is given by the following kernel density estimation.

$$P(x,y \mid C_i) = \frac{\sum_{p=1}^{H}\sum_{q=1}^{W} h_N(x_q, y_p; C_i) \times \boldsymbol{K}(x - x_q, y - y_p)}{\sum_{p=1}^{H}\sum_{q=1}^{W} h_N(x_q, y_p; C_i)} \tag{8}$$

where $\boldsymbol{K}$ is a two dimensional kernel function. The Gaussian kernel is usually used for the kernel function as follows:

$$\boldsymbol{K}(u,v) = \frac{1}{\left(2\pi b^2\right)^{\frac{1}{2}}} \exp\left\{-\frac{1}{2b^2}\left(u^2 + v^2\right)\right\} \tag{9}$$

where b is a bandwidth of the kernel function.

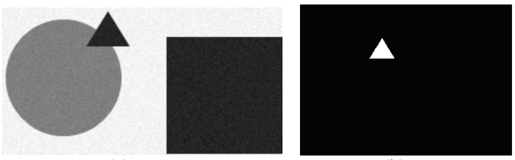
As one example, suppose that a sequence of 30 images shown in Fig. 2 (a) is given where each image has three intensity sources. Three intensity clusters can then be generated. For each of three intensity clusters, the histogram and the associated probability distribution of pixel positions are computed as shown in Fig. 2 (b-1), (b-2), (c-1), (c-2), (d-1), and (d-2). As compared to each of histograms, the associated probability distribution is smoother and normalized.

4 DETECTING A FOREGROUND

Once the pixel position distribution for each of intensity clusters is constructed, the identification of each pixel position in a new image as a background or a foreground is achieved by labelling it as follows:

Let the new image be I^{N+1}. The r_{xyi}^{N+1}, the cluster membership value for I^{N+1}, is computed using the k-means algorithm described in section 2. Each pixel position (x,y) on the I^{N+1} is then labelled with $L(x,y)$ defined to be

$$L(x,y) = \sum_{i=1}^{M} f_T\left(P(x,y \mid C_i)\right) \times r_{xyi}^{N+1} \qquad (10)$$



(a) (b)

Figure 3: (a) A given new image and (b) a detected foreground shown as a white area.

where f_T is a threshold function given by

$$f_T(z) = \begin{cases} 1 & , if\ z < T \\ 0 & , otherwise \end{cases} \qquad (11)$$

with a predefined value T.

Suppose that a given pixel position (x,y) on the new image I^{N+1} is one of them forming the jth intensity cluster. Then the value of r_{xyi}^{N+1} becomes 1 only when i=j. If its value from $P(x,y|C_j)$ is less than T, then it is detected as a foreground. Otherwise, it is a background.

For example, given a sequence of thirty images in Fig. 2 (a), if a new image in Fig. 3 (a) is given, those pixels forming triangle shown in Fig. 3 (b) are detected as a foreground.

5 EXAMPLES

Our approach is illustrated using two examples having dynamic textures and large change of illumination, respectively. It is also compared to the intensity-based approach using KDE (IBA-KDE) (Elgammal *et al*, 2000) and to the intensity-based approach using Gaussian Mixture Model (IBA-GMM) (Dalley *et al*, 2008).

As the first example having dynamic textures, a sequence of 30 images shown in Fig. 4 and four new images shown in Fig. 5 are assumed. From each image in the sequence where four intensity sources are assumed, four intensity clusters are generated. For each of the four intensity clusters, the probability distribution of pixel positions is then computed from 30 images in the sequence.

Figure 4: A sequence of 30 images.

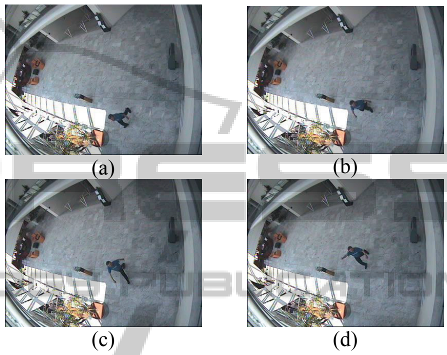


Figure 5: Four new images with different shapes of a walking man.

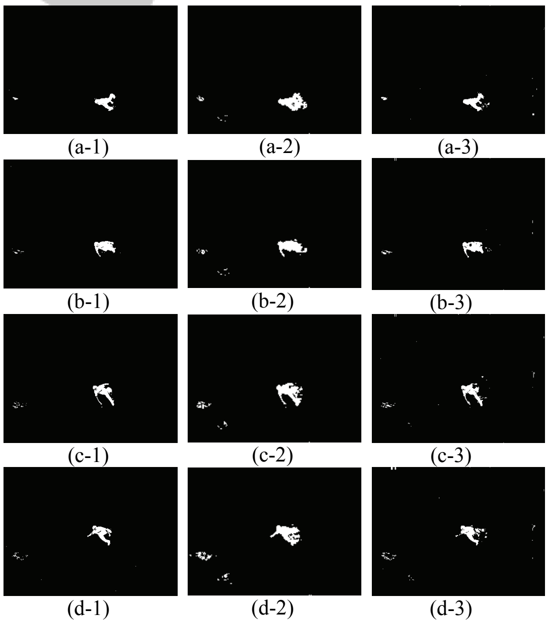


Figure 6: Results from images in Fig. 5 by ours, IBA-KDE, and IBA-GMM.

From the first new image in Fig. 5(a), the result by our approach is shown in Fig. 6(a-1), the result by the IBA-KDE is in Fig. 6(a-2), and the result by the

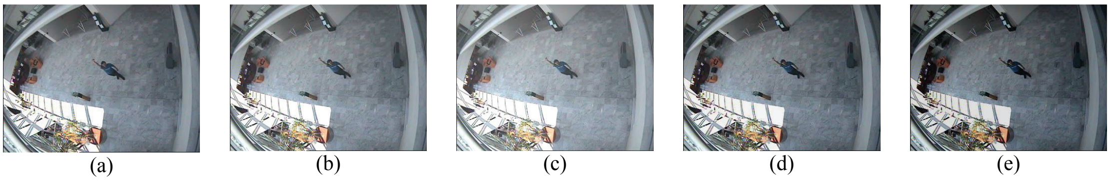


Figure 7: Five new images with different illumination.

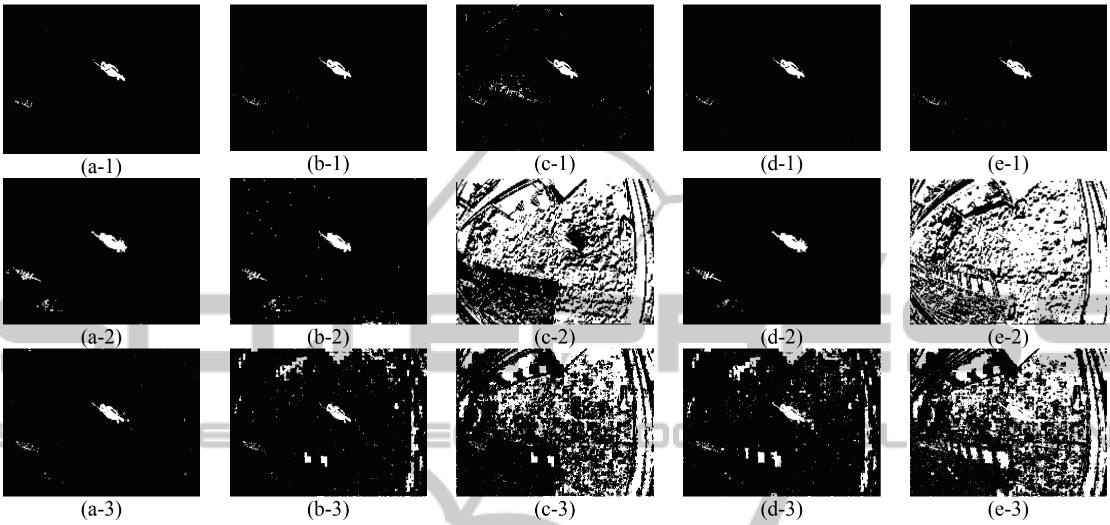


Figure 8: Results from images in Fig. 7 by ours, IBA-KDE, and IBA-GMM.

IBA-GMM is in Fig. 6(a-3). Similarly, from the next three new images in Fig. 5(b), 5(c), 5(d), the result by ours is in Fig. 6(b-1), 6(c-1), 6(d-1), the result by the IBA-KDE in Fig. 6(b-2), 6(c-2), 6(d-2), and the result by the IBA-GMM in Fig. 6(b-3), 6(c-3), 6(d-3). As noticed by comparing three results in Fig. 6, our approach detected the foreground successfully but the IBA-KDE and the IBA-GMM did not.

As the second example having large change of illumination, a sequence of 30 images in Fig. 4 and five new images in Fig. 7 are assumed. All of the five new images represent the same scene with different illumination where the image in (a) is an original image and others in (b), (c), (d), and (e) are modified in their intensities with +10, +30, -10, and -30, respectively. The same probability distribution of pixel positions computed in the first example is then used for detecting a foreground from the five new images.

From Fig. 7(a), the result by ours is in Fig. 8(a-1), the result by the IBA-KDE is in Fig. 8(a-2), and the result by the IBA-GMM is in Fig. 8(a-3). Similarly, from Fig. 7(b), 7(c), 7(d), 7(e), the result by ours is in Fig. 8(b-1), 8(c-1), 8(d-1), 8(e-1), the result by IBA-KDE in Fig. 8(b-2), 8(c-2), 8(d-2), and the result by the IBA-GMM in Fig. 8(b-3), 8(c-3), 8(d-3), 8(e-3). As shown in Fig. 8, our approach detected a foreground successfully from four of the five images except (c) with a little false detection. However, both of the IBA-KDE and the IBA-GMM failed to detect a foreground from each of (c) and (e). Further the IBA-GMM results in false detection from both of (b) and (d).

From these two examples, our approach is shown to be robust to dynamic textures and also large change of illumination as compared to the IBA-KDE and the IBA-GMM.

Finally, the results from using five different values of threshold with four different numbers of clusters are shown in Fig. 9 where as the number of clusters gets larger, the smaller value of threshold becomes more appropriate. Note however that the number of clusters is closely related to the number of intensity sources in the given image.

6 CONCLUSIONS

For modelling a background from a sequence of images, we presented the probability distribution of pixel positions for intensity clusters. To detect a foreground from a given new image, probability of each pixel position is obtained from the probability

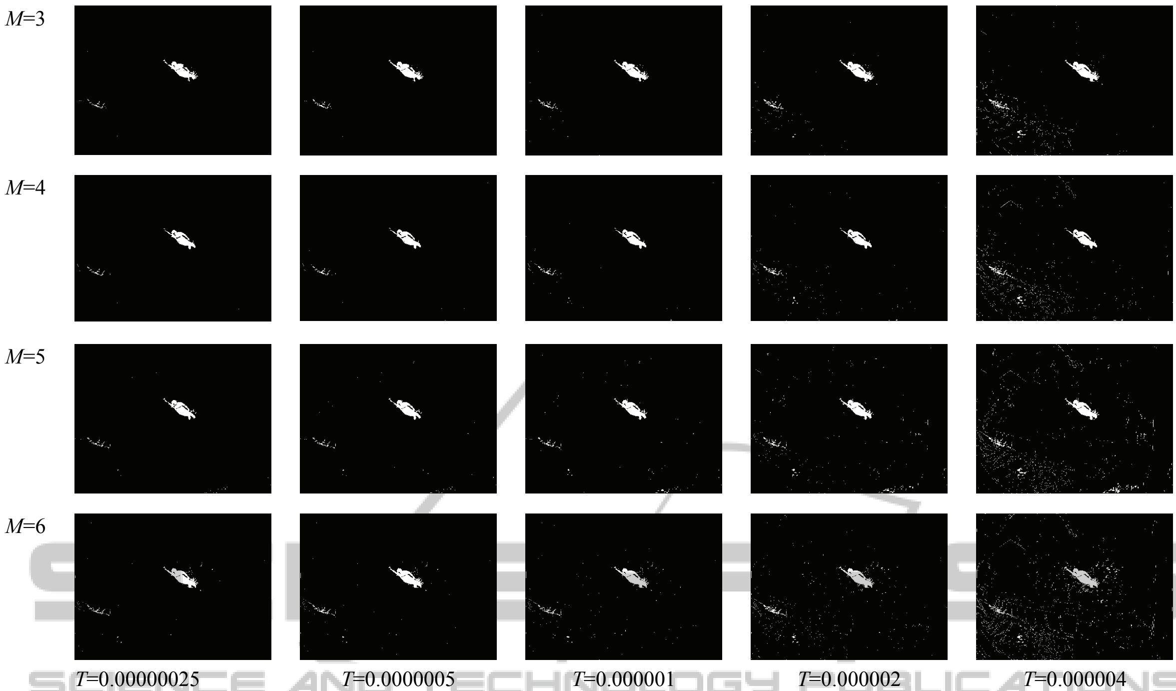


Figure 9: Results from an image in Fig. 7-(a) using different values of threshold with different numbers of clusters.

distribution. If it is less than some predefined threshold, it is detected as a foreground. Otherwise, it is a background. Our approach is illustrated to be not to suffer from dynamic textures and large change of illumination, as compared to the intensity-based approaches. Finally the work to find the general formula to find the optimal number of intensity sources from a given image is left as the future work.

ACKNOWLEDGEMENTS

The ICT at Seoul National University provides research facilities for this study.

REFERENCES

Bishop, C. M., 2006, Pattern recognition and machine learning, *Springer, 1st edition.*

Dalley, G., Migdal, J., and Grimson, W. E. L., 2008, Background subtraction for temporally irregular dynamic textures, *IEEE Workshop on Applications of Computer Vision.*

Elgammal, A., Harwood, D., and Davis L., 2000, Non-parametric model for background subtraction, *European Conference on Computer Vision.*

Mittal, A., and Paragios, N., 2004, Motion-based background subtraction using adaptive kernel density estimation, *IEEE Computer Vision and Pattern Recognition.*

Piccardi, M., 2004, Background subtraction techniques: a review, *IEEE International Conference on Systems, Man and Cybernetics*.

Power, P. W., and Schoonees, J. A., 2002, Understanding background mixture models for foreground segmentation, *Proceedings Image and Vision Computing New Zealand.*

Sheikh, Y., and Shah, M., 2005, Bayesian modelling of dynamic scenes for objects detection, *IEEE Transaction on Pattern Analysis and Machine Intelligence.*

Stauffer, C., Grimson, W. E. L., 1999, Adaptive background mixture models for real-time tracking, *IEEE Computer Vision and Pattern Recognition.*

Wren, C., Azarbayejani, A. Darrel, T., and Pentland, A. P., 1997, Pfinder: real-time tracking of the human body, *IEEE Transaction on Pattern Analysis and Machine Intelligence.*

Zivkovic, Z., and Heijden, F. V. D., 2006, Efficient adaptive density estimation per image pixel for the task of background subtraction, *Pattern Recognition Letters.*