AUTOMATIC STATE SPACE AGGREGATION USING A DENSITY BASED TECHNIQUE

Steven Loscalzo[1,2]
[1]*Department of Computer Science, Binghamton University, Binghamton, NY, U.S.A.*

Robert Wright[2]
[2]*Information Directorate, Air Force Research Lab, Rome, NY, U.S.A.*

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Abstract: Applying reinforcement learning techniques in continuous environments is challenging because there are infinitely many states to visit in order to learn an optimal policy. To make this situation tractable, abstractions are often used to reduce the infinite state space down to a small and finite one. Some of the more powerful and commonplace abstractions, tiling abstractions such as CMAC, work by aggregating many base states into a single abstract state. Unfortunately, significant manual effort is often necessary in order to apply them to non-trivial control problems. Here we develop an *automatic* state space aggregation algorithm, Maximum Density Separation, which can produce a meaningful abstraction with minimal manual effort. This method leverages the density of observations in the space to construct a partition and aggregate states in a dense region to the same abstract state. We show that the abstractions produced by this method on two benchmark reinforcement learning problems can outperform fixed tiling methods in terms of both the convergence rate of a learning algorithm and the number of abstract states needed.

1 INTRODUCTION

For reinforcement learning algorithms, learning in an environment with continuous valued features is a challenging problem since the state space $\mathcal{S}$ will be infinite. The objective of a reinforcement learning (RL) algorithm is to learn an optimal policy π^* which encodes each action to take to get from some start state $s_i \in \mathcal{S}$ to a goal state $s_g \in \mathcal{S}$ while minimizing a utility measure. RL algorithms learn π^* by repeatedly experiencing states on the way to s_g. The probability of reaching the same state twice approaches zero for problems with continuous features, therefore directly applying reinforcement learning algorithms in continuous domain problems is infeasible. State space abstraction is needed to transform the problem in some way to reduce the effective size of $\mathcal{S}$ to make RL applicable.

There are many ways to achieve such an abstraction, and (Li et al., 2006) constructed a five tier categorization scheme for them depending on the "coarseness" of the abstraction and what parts of the optimal policy π^* get carried over from the original space to the abstract one. For three levels of coarseness (model-irrelevance, Q^π-irrelevance, and Q^*-irrelevance) they prove that an optimal policy found in the abstract space $\mathcal{S}'$ will also result in an optimal policy in the ground state space $\mathcal{S}$. The other two categories (a^*-irrelevance an π^*-irrelevance) do not share this same guarantee, though in some ways are more valuable because they can abstract down the size of the state space to a much larger degree than the other three methods.

One of these classes of abstractions, the a^*-irrelevance abstraction, which aggregate two base states together if they share the same optimal action, is the focus of this paper. One of the most popular types of abstraction techniques, tiling, falls into this category. Examples of common tile based abstractions include CMAC (Miller et al., 1990), and U-tile distinction (Mccallum, 1996; Uther and Veloso, 1998). These methods aggregate ranges of a continuous state space to discrete abstract states, resulting in a finite environment for an RL algorithm to learn in. While tile based methods have been shown effective in a number of situations, there are serious drawbacks

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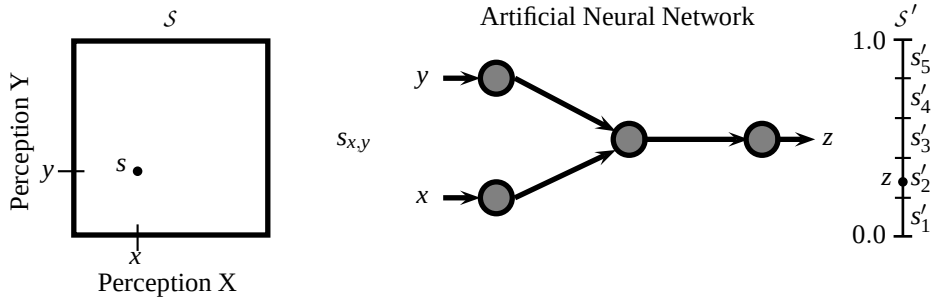


Figure 1: Overview of the RL-SANE algorithm transforming a ground state $s_{x,y}$ in a sample two dimensional state space $\mathcal{S}$ to the abstract state s'_2 in the one dimensional abstract state space $\mathcal{S}'$.

to using them (Sutton, 1996; Whiteson et al., 2007). Engineering a tiling is typically done by hand, and it can be very difficult to find an appropriate tiling for a given problem or to correctly set the parameters in methods that build a tiling during the learning process. Automatic tiling methods (Whiteson et al., 2007), that construct tilings during the learning process, have shown to be very sensitive to parameter choices. Recently, tiling methods have been shown ineffective in the double pole balance setting, a challenging RL standard problem (Gomez et al., 2006).

Here we develop and evaluate an *automatic* aggregation method called Maximum Density Separation that efficiently learns how to abstract a state space but still allows a learner to converge to the optimal policy. This abstraction is constructed on the one dimensional state space produced by the RL-SANE algorithm (Wright and Gemelli, 2009) which allows us to focus on the aggregation technique and not on feature space dimensionality reduction. This method leverages the distribution of observations to choose intuitive abstract states in an effort to improve the speed of learning. By using density patterns found in the observations we hope that useful abstractions can be discovered without significant effort by the user. When used in conjunction with feature space dimensionality reduction techniques, automated state aggregation methods will greatly increase the scalability and applicability of existing RL algorithms.

We show via empirical study on a mountain car and double pole balance benchmark RL problems that the Maximum Density Separation algorithm allows the learning algorithm to significantly improve its rate of convergence to π^* when compared to the base RL-SANE algorithm using a fixed tiling and another more simplistic automated approach based on random variations in the number of fixed tiles in the abstraction. Additionally, we show that our automatic method is capable of creating a more compact abstract space than a traditional fixed tiling approach.

In the next section we give background on the RL-SANE algorithm we use for dimensionality reduction. Next, the Maximum Density Separation algorithm is introduced and explained in detail. This method is then applied to the two problems given in Section 4, and the results of this study are given in the Section 5. Finally, the paper concludes with a summary of our contributions and some future directions.

2 BACKGROUND

The RL-SANE algorithm is a powerful reinforcement learning and state abstraction algorithm (Wright and Gemelli, 2009). It combines a neuroevolution approach to constructing neural networks (Stanley and Miikkulainen, 2002) with a fixed tiling over a one dimensional abstract state space to allow a learner to efficiently solve complex problems by learning the optimal action for each tile in this abstract space. An overview of this process is given in Figure 1.

For any dimensionality of input space $\mathcal{S}$ the artificial neural network (ANN) layer of RL-SANE takes the input measured across m dimensions and reduces it to a single output value $z \in [0,1]$ corresponding to a single abstract state $s'_i \in \mathcal{S}'$. This one dimensional output space still represents infinitely many states, so a tiling is applied to it. The fixed tiling simply splits $\mathcal{S}'$ into a number of equal sized tiles with no consideration given to the exact position of where each split occurs. The tile boundaries are suboptimal since no consideration is given to how observations will gather in the space. However, the ANNs used is this approach are not fixed. They are produced and adapted through the use of the NEAT neuroevolutionary algorithm (Stanley and Miikkulainen, 2002). The NEAT algorithm adapts the ANNs to fit the observed ground states to the structure of the abstract state space lessing the impact of a sub-optimal tiling. This ability of RL-SANE to adapt to the structure of a specified abstract state space improves its ability to discover π^* over other tile coding approaches.

The original RL-SANE algorithm included a user

specified parameter β to determine the number of tiles to lay over $\mathcal{S}'$. Evidence displayed in (Wright and Gemelli, 2009) shows that the algorithm's overall convergence is sensitive to β. In this paper we propose an adaptive tiling approach to overcome RL-SANE's limitations and automatically find a good partitioning. We will show how this approach improves RL-SANE's ability to quickly converge to an optimal policy and eliminates the need to derive an ideal β parameter.

3 MAXIMUM DENSITY SEPARATION

The Maximum Density Separation (MDS) method is a new approach to derive non-arbitrary discretizations of the state space. The rationale for this approach is that there may exist ANNs, used to abstract the state space, in a population that would provide useful abstractions but do not fit a predetermined tiling. MDS designs tilings for each ANN based on observations of the ANN's activation. This method intuitively views dense clusters of observations as belonging to a single state, and abstracts the state space so that these dense clusters are located on separate tiles from one another. The split between tiles occurs at the farthest point between two dense regions of observations. This approach is principled by the idea that nearby states will prefer the same action, however the size of each of these groups may vary so we must use an adaptable solution.

There are two main challenges related to the MDS approach, the first is how to estimate the density of the space or otherwise cluster observations, and the second is deciding the appropriate time to reassess the abstraction. In the context of the first problem, MDS can be thought of as a framework where an appropriate clustering or density estimation technique can be applied at the discretion of the user. In this work, we model the density of the abstract state space using a simple histogram approach as it does not require much additional computation to employ within the learning algorithm. We partition the one dimensional space into 1000 equal width bins in our approach. The number of bins can be taken as a parameter to the method, however, we found that the algorithm is not very sensitive to this value. One possible limitation of this simple technique is if all observed states fall into only a few adjacent bins. This will cause the algorithm to be unable to find a new partitioning because there would seem to be only one peak. To combat this and similar situations other clustering or density estimation techniques may be used. For example, since we are only interested in the most dense areas of the space, a kernel density estimation method like the mean shift algorithm may be employed (Comaniciu and Meer, 2002). While more complicated clustering or density estimation methods can be used, the trade off in terms of possibly more accurate state boundary identification must be balanced against the time cost of performing the analysis, which will depend on the chosen algorithm.

Determining when to repartition the abstract space can have a large impact on the convergence speed of the learning algorithm. If the tiles are repartitioned too frequently the learner may not have time to learn accurate Q-values on the abstract states, causing the algorithm to converge to a less than optimal policy. On the other hand, attempting to learn in a poorly partitioned space can lead to wasted update cycles as the Q-values do not give meaningful direction to the learner. In our implementation of MDS we repartition the space whenever a failure state is reached by the learner since it is possible that a new partitioning might help the learner avoid failing the problem in a subsequent attempt.

An overview of the MDS method is given in Algorithm 1. For a single run of the problem in a given RL algorithm, this method records the frequency of observations across the state space until a failure or goal state is reached. On a failure, the constructed frequency distribution is searched for relative extrema, using a soft-thresholding approach to prevent small fluctuations in the distribution from leading to many spurious extrema. For example, if consecutive bins in the histogram respectively include 9, 8, and 10 observations, peaks are not placed in both the 9 and 10 bins because there was not enough of a decrease between them. This area is instead treated like a plateau in the distribution. Once the relative extrema have been identified, a partition is placed in the space in the center of every two relative maximums. The splits between abstract states are made in this fashion in accordance with the maximum margin principle (Mitchell,

Algorithm 1: Maximum Density Separation (MDS).

required: number of bins for frequency distribution
output: new abstract state mapping
≪embedded within an RL algorithm≫
get next state s' by following π^* from state s
if s' != fail state
  $s := s'$
  increment frequency distribution (s)
else if s' == fail state
  locate relative extrema in frequency distribution
  erase old tiles of $\mathcal{S}'$
  partition $\mathcal{S}'$ in the center of two rel. maximums
end else if

1997), which seeks to minimize the structural variance in a hypothesis. Positioning the splits as far as possible from the dense regions of observations minimizes the risk that in the next run of the problem new observations belonging to one dense region will spill into an adjacent state and mislead the learning there. This process is linear in the number of bins used to measure the frequency distribution, and in practice had only a negligible impact on the running time of each generation of the algorithm, and so is a feasible abstraction algorithm in terms of time complexity.

MDS effectively overcomes the limitations of the fixed tiling: abstract state partitions can be placed anywhere in the space and actively work to fit the outputs of the neural networks. This allows the boundaries to dynamically fit the outputs instead of depending on expensive evolution to fit the neural networks to the abstract states. The MDS method does introduce some other limitations, however. It could be the case that an area of dense observations is not really homogeneous in terms of preferred action, but were coincidentally grouped together by the ANN. In this case, the abstract states might still become successful if the ANN adapts and separates these states into two different clusters in a later evolutionary stage. Another concern is the additional computational burden if a more complicated clustering/density estimation method was used, however our results show that the simple frequency based approach taken here can be effective.

4 EXPERIMENTAL SETUP

Here we see what benefit MDS gives the RL-SANE algorithm in terms of convergence speed and number of abstract states in the solution. In addition to comparing the automatic MDS method against the fixed tiling of RL-SANE, we include another algorithm in the study, a mutation method which allows RL-SANE to mutate the number of abstract states during the evolution of the network. The experiments are carried out on two benchmark RL problems, mountain car and double pole balance.

The mutation method simply introduces another mutation operator into the neural network's evolutionary process which changes the number of tiles used in the abstraction in the next generation. The number of tiles can either increase or decrease up to five values per mutation so that each subsequent abstraction is somewhat similar to the preceding one. The mutation method is not completely automatic; the number of tiles to start with will have an impact on how the algorithm performs. We also present a brief study of this phenomenon in the results.

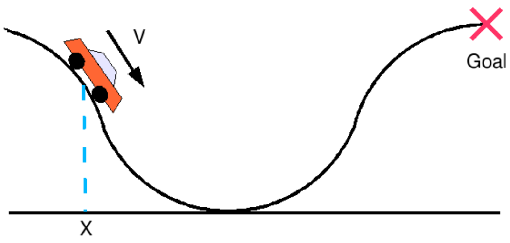
The mountain car problem (Boyan and Moore, 1995) consists of a car trying to escape a valley as illustrated in Figure 2a. The car starts at a random position in the valley and its goal is to drive over the hill to the right of the starting position. Unfortunately, the car's engine is too weak to drive up the hill directly. Instead, the driver must build momentum by driving forward and backward in order to escape the valley.

Only two perceptions are used to define this problem, the position of the car within the valley X, and the velocity of the car V. Time is discretized into small intervals and the learner can choose one of two actions in each time step: drive forward or backward. The only reward that is assigned is -1 for each each action that is taken before the car reaches the goal of escaping the valley. Since RL algorithms seek to maximize the reward, the optimal policy is the one that enables the car to escape the valley as quickly as possible.

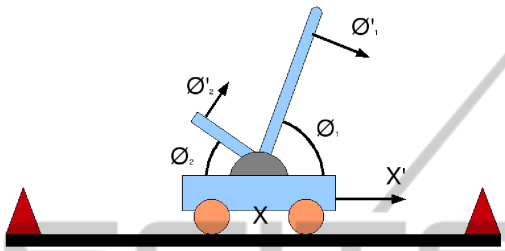
The double inverted pole balancing problem (Gomez and Miikkulainen, 1999) depicted in Figure 2b is a very difficult RL benchmark problem. In this problem, the learner must balance two poles of different length and mass which are attached to a moving cart. The problem is further complicated by maintaining that the cart must stay within a certain small stretch of track. If the learner is able to prevent the poles from falling over after a specified amount of time then the problem is considered solved.

This is a higher dimensional problem than the mountain car problem, with six perceptions being given to the learner: the position of the cart X, the velocity of the cart X', the angle each beam makes with the cart, θ_1 and θ_2, and the angular velocities of the beams, θ'_1 and θ'_2. Once again, time is discretized into small intervals, and during any such interval the learner can choose to push the cart to the left or right or to leave it alone. In our experiment, the learner only receives a -1 reward for dropping either beam or exceeding the bounds of the track. If the learner is able to balance two poles and not exceed the bounds of the track for 10^6 time steps the problem is taken to be solved.

On each of the problems the three methods were evaluated over 25 runs using different random seeds (the same seed values were used for all three methods). For each run, both the mountain car and double pole balance environments used a problem set size of 100 random initial start states. We report the average values across the 25 runs in our results. It should be noted that the mutation and the fixed tiling approaches have a significant dependency on the initial number of abstract states, while the MDS does not.



(a)



(b)

Figure 2: Depictions of the mountain car (a) and double pole balance (b) problem domains.

In the mutation and fixed methods we experimented with setting the number of initial abstract states from $10, 20, \ldots, 100$ and the results show either the average performance of the algorithm over all of these boundaries, or the best performer from the 10, as indicated. For the RL-SANE algorithm with a fixed abstraction, these initial states cannot change during the learning process, while the mutation method is free to alter them over time. The MDS method begins with an arbitrary abstraction over the state space which is quickly replaced by a more competent estimate after the first attempt at learning the problem.

The RL-SANE algorithm was set to use a population of 100 neural networks per generation, with a maximum of 200 generations of learning. Neuroevolution is provided by Another NEAT Java Implementation (ANJI) (James and Tucker, 2004). We used the Sarsa(λ) learning algorithm with learning and neuroevolution parameters set as in (Wright and Gemelli, 2009), and we also limited each learning episode of the mountain car to 2500 time steps to ensure termination. The mutation method was allowed to alter the number of states by up to 5 per generation to provide regularity between generations of neural networks. For MDS the density of the observations in the state space was estimated using a histogram of 1000 evenly spaced bins to collect observations. The exact value of this parameter is unimportant as long as it is significantly larger than the number of expected abstract states in the solution and an episode produces enough observations to partially fill in the space.

Table 1: Average number of final abstract states used $\pm$ standard deviation for MDS and the number of initial states used to derive the best performance for the mutation and fixed approaches.

	Mountain Car	**Double Pole**
MDS	3.36 ± 1.25	13.5 ± 5.93
Mutation	90	10
Fixed	50	10

5 EXPERIMENTAL RESULTS

5.1 Mountain Car

Figure 3 shows the average number of time steps taken to leave the valley over the 25 runs for the three methods. Fewer steps are better. The fixed and mutation curves shown use the best choice of initial number of abstract states, although all values attempted gave similar results for this problem and so are omitted for clarity. The mutation method found the optimal policy fastest with 90 initial states, and the fixed approach did the best on 50 states. The mountain car problem shows all of the methods performing very similarly and all rapidly converging to a policy that takes on average approximately 50 time steps to navigate the car from the valley. The Maximum Density Separation method lags slightly behind the other two methods, which indicates that this problem can be easily learned without complicated abstract state repartitioning.

Analysis of Table 1 gives an additional explanation for the performance of MDS. The MDS method consistently finds that there are only roughly three groups of observations in the space and so only partitions the space into three tiles. It might have taken additional generations to effectively learn the correct placement of these partitions, compared with the larger number of states used by the mutation and fixed methods. Even though the other methods need to learn correct values over more states, there are still relatively few states to learn so they quickly converge. These results do serve to show that automatic repartitioning of the abstract state space does not degrade the convergence of the learning algorithm much in problems where fixed tiling are sufficient.

Table 1 contains the number of abstract states available to the fixed and mutation methods during their best run, as well as the number of states that was determined by the MDS method. We can see that the MDS method on average uses 3 or 4 abstract states with a standard deviation of 1.25, meaning that it was very consistent in the number of states used

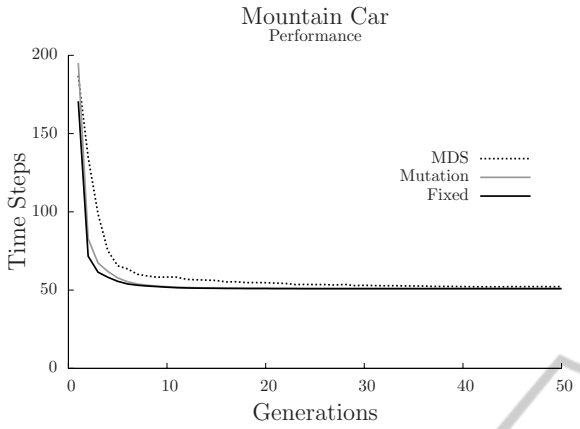


Figure 3: The performance of MDS, mutation and the fixed abstraction methods on the mountain car. The curves for the mutation and fixed methods are the results from the best initial parameter settings.

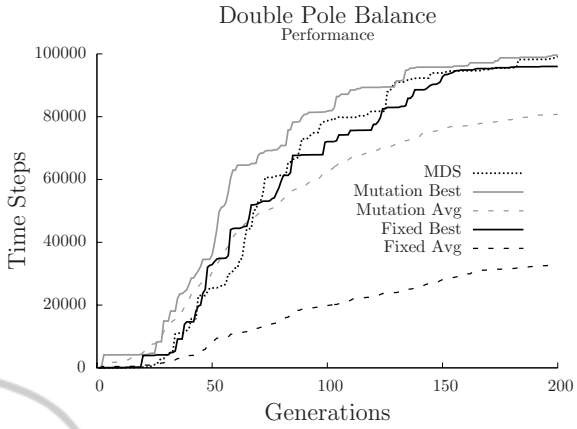


Figure 4: Learning performance evaluation for each of the three methods. For comparison, the mutation and fixed methods include their single best initial parameter setting as well as their average performance across all parameter values.

to learn the problem. The other two methods preferred many more partitions. However, the number of those partitions where observations were placed by the ANNs in the RL-SANE procedure were much smaller. The fixed method used roughly 19.08 ± 9.08 (average$\pm$std) out of the 50 available states, while the mutation method used 30.40 ± 22.67 from 90 possible starting states. From this we can see that it is not only the number of tiles that is important, but the effect they have on where the tiles get placed across the abstract state space. Since each of these two methods use fixed-width partitioning, the number of abstract states will cause the boundaries to fall in different locations, and over-partitioning the space can allow the learner to use more appropriately positioned tiles. This is in contrast to the MDS approach which allows the boundaries to be placed anywhere in the abstract state and does not need to add additional empty states solely to adjust the layout of the useful ones.

Another insight that Table 1 brings is that MDS has by far the most compact and efficient abstraction for the mountain car problem. Reducing an infinite state space to such a compact representation is one of the most important abilities of an abstraction technique. The fact that MDS is able to achieve such a representation suggests that it may allow RL algorithms to scale to more complicated problems better than the other methods shown here.

The disparity in the number of used states between three methods is interesting. Two possible explanations regarding why the fixed and mutation methods had a large number of states are that the large number of initial states induced the learner and ANN to prefer many small groups of observations throughout the one dimensional state space, or that many adjacent states shared the same optimal action preference. In the latter case, these adjacent states could have been merged together resulting in possibly far fewer states, though there is no means to do this in these methods.

5.2 Double Pole Balance

Figure 4 shows the average number of time steps the pole was balanced for each of the three methods, the solid lines for the mutation and fixed methods are the best scores achieved by any initial parameter setting (10 states for both methods). The more time steps, the better the algorithm has learned the problem. The dashed curves show the average performance for the two methods across all tested initial state boundaries. The MDS has no initial parameter selection and so only has the single dotted line in the figure. We can see that MDS and the best settings of the other methods show similar trends, with the score of the two adaptive methods just edging out the best fixed method. Analysis of the average curves (dashed lines) gives more information about the general performance of the mutation and fixed methods as compared to the MDS method. While the MDS method has no choice of initial parameters and still ends up achieving an excellent overall score, different numbers of initial abstract states causes a varied performance in the other two methods. The mutation method is relatively robust with regard to the initial parameter selection as compared to the performance degradation seen by the fixed method if a bad initial abstraction is selected.

Figure 5 explores this phenomenon more completely by showing the performance of the fixed and mutation methods for all 10 initial parameter settings. The individual parameter results are shown in faint

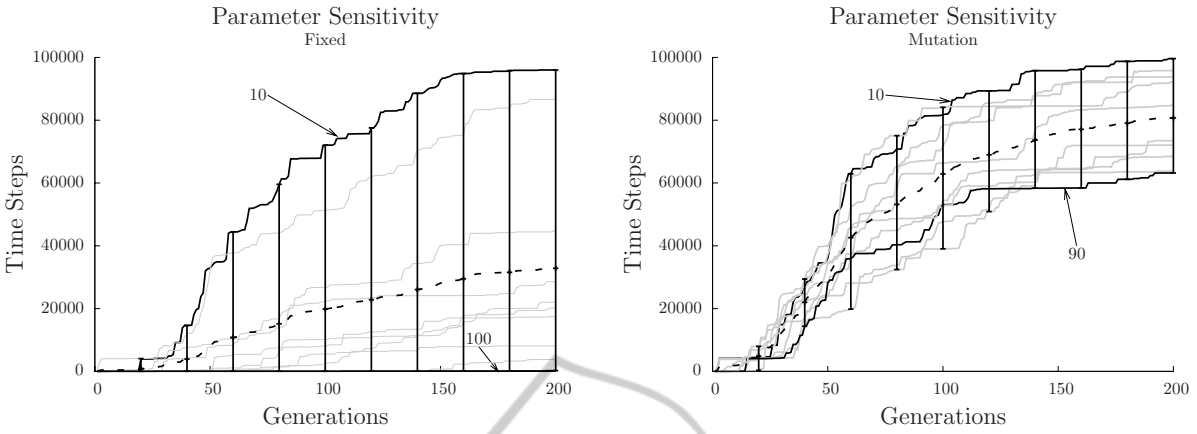


Figure 5: The effect of different numbers of initial abstract states for both the Fixed and Mutation methods on the Double Pole Balance problem. The dashed line is the average of all values; vertical bars show distance between maximum and minimum performance.

gray lines except for the best and worst performers which are solid black lines. The vertical bars span the space between the best and worst performer and highlight the sensitivity of the fixed method. Generally, the smaller parameters perform better than the larger initial values, and the vertical bars show that the fluctuation of performance is much smaller in the mutation method than the fixed method. The reason why the double pole balance problem is much more sensitive to the initial number of abstract states as compared to the mountain car problem has to do with the number of actively used states. In the mountain car problem, as discussed in Section 5.1, even if many states were available only a fraction of them were used. This is in contrast to the double pole balance problem where nearly all of the available states are used. The mutation method allows the number of available states to quickly be reduced down to a number that the learning algorithm can deal with, and thus improve the rate of convergence compared to the fixed method.

All three abstraction methods prefer to use a similar number of abstract states for the double pole balance problem, as indicated by Table 1. The best fixed tiling began with 10 tiles and used all 10 consistently throughout the learning process. The mutation method also achieved its best performance after starting with 10 tiles, however the number of tiles used decreased to 7.56 ± 3.88 by generation 200. The MDS method was also in agreement with the other approaches and by generation 200 was using 13.5 ± 5.93 tiles. Despite the complexity of the problem, many of the states in the ground state space can be successfully aggregated together, as evidenced by the small number of states being used.

6 CONCLUSIONS AND FUTURE DIRECTIONS

We have presented the Maximum Density Separation algorithm which seeks to automatically partition a state space based on dense regions of observations. This method has been shown to improve the learning rate as compared to using a fixed abstraction or naïvely altering the number of states during evolution on the double pole balance problem. MDS did not improve the rate of convergence on the mountain car, but it was competitive while using an abstraction made of far fewer abstract states compared to the others.

This work points to several promising future directions. As illustrated by the mountain car results, intelligent aggregation is not always beneficial. Identifying these situations during the abstraction generation procedure could be one future area of research. Additionally, there is no need to limit MDS to the one dimensional state space used by RL-SANE, and applying this method to the ground state space or in combination with a different dimensionality reduction technique may prove useful. In general, the interplay between state abstraction and dimensionality reduction could be an interesting avenue of future research.

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