A STUDY OF STOCHASTIC RESONANCE AS A MATHEMATICAL MODEL OF ELECTROGASTROGRAM

Matsuura Yasuyuki, Miyao Masaru
Graduate School of Information Science, Nagoya University, Furo-cho, Nagoya, Japan

Yokoyama Kiyoko
Graduate School of Design and Architecture, Nagoya City University, 2-1-10 Kitachikusa, Nagoya, Japan

Takada Hiroki
Graduate School of Engineering, University of Fukui, 3-9-1 Bunkyo, Fukui, Japan

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Abstract: An electrogastrogram (EGG) is a recording of the electric activity of the stomach as measured on the abdominal surface. In this study, our goal is to obtain a mathematical model of an EGG obtained for a subject in the supine position. Initially, we applied the translation error in the Wayland algorithm to the EGG in order to measure the degree of determinism. However, we could not determine whether or not the mathematical model of the EGG could be defined on the basis of a chaotic process. The waveform of the electric potential in the interstitial cells of Cajal is similar to the graphs of the numerical solutions to the Van der Pol equation (VPE). We therefore added the VPE to a periodic function and random white noise was used to represent the intestinal motility and other biosignals, respectively. The EGG and numerical solutions were compared and evaluated on the basis of the translation error and the maximum Lyapunov exponent. The EGG was well described by the stochastic resonance in the stochastic differential equations.

1 INTRODUCTION

Percutaneous electrogastrography is a useful method for examining human gastric electrical activity without invasion. Human gastric electrical activity cannot be measured by any other methods such as magnetic resonance imaging (MRI) or gastro-fiberscopy. An electrogastrogram (EGG) is evaluated by comparing the mean frequency and power values obtained for it to those derived from the spectrum analysis of previous EGG studies. However, the amount of information that can be obtained from such an analysis is limited. Moreover, EGGs are used less often compared to ECGs, EEGs, and other biosignals. However, using a mathematical model for an EGG makes it possible to obtain additional information.

In 1921, Walter C. Alvarez reported performing EGG for the first time in humans (Alvarez, 1922). In EGG, the electrical activity of the stomach is recorded by placing electrodes on the surface of the abdominal wall. In the stomach, a pacemaker placed on the side of the greater curvature generates electrical activity at a rate of 3 cycles per minute (3 cpm); the electrical signal is then transferred to the pyloric side (Couturier et al, 1972).

Gastric electrical potential is generated by the interstitial cells of Cajal (ICCs) (Kenneth and Robert, 2004). ICCs are pacemaker cells that spontaneously depolarize and repolarize at the rate of 3 cpm.

The waveform of the electric potential in ICCs is similar to the graphs of the numerical solutions to the Van der Pol equation. We thus added the Van der Pol equation to a periodic function and random white noise was used to represent intestinal motility and other biosignals.

$$\dot{x} = y - \alpha\, gradf(x) + s(t) + \mu w_1(t) \tag{1.1}$$

$$\dot{y} = -x + \mu w_2(t) \tag{1.2}$$

The function $s(t)=\sin\omega t$ and white noise $w_i(t)$ respectively represent the weak and random

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intestinal movements and other biosignals (i=1,2). The double-well potential, $f(x)$, generates depolarization and repolarization in ICCs.

In this study, the gastrointestinal motility was measured with the aim of obtaining a mathematical model of EGG and speculating factors to describe the diseases resulting from constipation and erosive gastritis.

Some studies have discussed solutions to the forward and inverse problems associated with the dynamics generating the gastric electrical potential. These studies suggest that it is convenient to use current dipoles in an ellipsoid and to use computer simulations to generate a mathematical model for the stomach. However, results available on non-linear analyses of the EGG are insufficient. In order to examine whether or not a mathematical model describes EGG data appropriately, we have proposed a projection of time series on a two-dimensional plane, E_{trans}-λ, estimated by using the Wayland and Rosenstein algorithms (Matsuura et al., 2008).

The Wayland algorithm has been developed in order to evaluate the degree of determinism for dynamics that generate a time series (Wayland et al., 1993). This algorithm can estimate a parameter called translation error E_{trans} to measure the smoothness of flow in an attractor, which is assumed to generate the time-series data.

Chaos processes generate complexity in the attractor, which can be reconstructed from a time series (Takens, 1981). These processes have a sensitive dependence on the initial conditions and can be quantified using the Lyapunov exponent (Sato et al., 1987; Rosenstein et al., 1993). If the Lyapunov exponent has a positive value, the dynamics are considered to be a chaos process. In this study, Rosenstein's algorithm (Sato et al., 1987; Rosenstein et al., 1993) was used to calculate the maximum Lyapunov exponent (MLE), λ.

According to the analysis of the degree of determinism for the time series dynamics, the EGG data obtained 30 min after a subject's postural change were significantly different from the initial EGG data (Matsuura et al, 2008), which were also regarded as a stationary time series in this study. During the latter period, the autonomic nervous system could be represented by a stationary process because it controls the gastric electrical activity, which can be measured by an EGG.

In this study, the gastrointestinal motility was measured with the aim of obtaining a mathematical model of the stationary EGG, and we examined whether numerical solutions to the stochastic resonance (SR) would fit the EGG data.

2 MATERIALS AND METHODS

The subjects were 14 healthy people (7 males and 7 females) aged between 21 and 25 years. A sufficient explanation of the experiment was provided to all the subjects, and a written consent was obtained from them.

2.1 Experimental Procedure

EGGs were obtained at 1 KHz for 150 min for a subject in the supine position by using an A/D converter (AD16-16U (PCI) EH; CONTEC, Japan). The EGGs were amplified using a bio-amplifier (MT11; NEC Medical, Japan) and recorded using a tape recorder (PC216Ax; Sony Precision Technology, Japan).

In this experiment, EGGs are obtained with electrodes arranged for monopolar recordings (Vitrode Bs; Nihon Kohden Inc., Tokyo, Japan). A reference electrode is positioned on the midline of the patient's abdomen near the umbilicus. An active electrode should be placed approximately 10 cm cephalad from the umbilicus and 6 cm to the patient's left. It is the position closest to the pacemaker of gastrointestinal motility.

To remove the noise from the time series of the EGG data $\{y_j | j = 0,1,2\cdots, N-1\}$ obtained at 1 kHz, resampling was performed at 1 Hz. For the analysis, we obtained a resampled time series $\{x_i | i = 0,1,2\cdots,(N/1000)-1\}$ as follows:

$$x_0 = \frac{1}{1000}\sum_{j=0}^{999} y_j,\ x_1 = \frac{1}{1000}\sum_{j=1\times1000}^{1999} y_j,\ \cdots,\ x_i = \frac{1}{1000}\sum_{j=i\times1000}^{i\times1000+999} y_j.$$

The following delay coordinates were used:

$$\{\vec{x}_t = (x_t \quad x_{t+1} \quad \cdots \quad x_{t+(m-1)}\}.$$

Here, m represents the embedding dimension. These delay coordinates could be used to reconstruct a continuous trajectory without intersections in an embedding space having a large m. The embedding delay, τ, is defined as the minimum delay ($\tau \geq 0$) when the auto-correlation coefficient is zero. In this study, we assumed that there was no correlation when The auto-correlation function initially decreased to a value below $1/e$ ($t \geq 0$).

2.2 Calculation Procedure

In this study, we numerically solved Equations (1.1) and (1.2) and verified the SR in the Stochastic differential equations (SDEs). We converted

Equations (1.1) and (1.2) into difference equations and obtained numerical solutions using the Runge–Kutta–Gill formula for the numerical calculations. The initial values were set to (0, 0.5). Pseudorandom numbers were substituted for $w_i(t)$ $(i = 1,2)$. These pseudorandom numbers were generated by using the Mersenne Twister (Matsumoto and Nishimura, 1998). These numerical calculations were performed for N = 24000 time steps. Each time step was 0.05 units.

The values of the numerical solutions were recorded after every 20 time steps, which is equivalent to a signal sampling rate of 1 Hz. For each value of μ, we obtained 20 numerical solutions to Equations (1.1) and (1.2).

1) Using Wayland and Rosenstein's algorithms, estimate the translation errors (E_{trans}) and MLEs (λ) in the attractors generating EGG data, except for 30 min after the postural change. Then, project the stationary EGG onto the E_{trans}–λ plane.
2) Calculate the mean values (m(i)) of E_{trans} and λ for all of the projections obtained in (1). According to statistical theory, 95.5% of the EGGs would project onto the region $\left\{\Re_s^2 \middle| m(E_{trans}) \pm 2\sigma(E_{trans}) \times m(\lambda) \pm 2\sigma(\lambda)\right\}$.
3) Calculate the standard deviations (σ(i)) of E_{trans} and λ for all of the projections obtained in (1).
4) Project the numerical solutions of Equation (1) onto the E_{trans}–λ plane obtained in (1).
5) Count the number of numerical solutions projected onto region $\Re_s^2$ of the E_{trans} –λ plane.
6) Calculate the conformity ratio of the number counted in step (5) to 20, i.e., the number of numerical solutions for each value of μ.

3 RESULTS

3.1 Subjective Evaluation

We analyzed the EGG data. Wayland and Rosenstein's algorithms were applied to the attractors in the case of all 252 (14 subjects × 18 × 10 min-EGGs = 252 EGGs) EGG data items.

The attractors of the EGGs were reconstructed in accordance with Takens' embedding method. The form of the attractors could be evaluated by E_{trans} and λ. The embedding delays and embedding dimensions were distributed from 2 (s) to 4 (s) and from 2 to 7, respectively.

The translation errors were distributed from 0.23 to 0.61. The average ± standard deviation in the E_{trans} was found to be 0.45 ± 0.10.

The MLEs were distributed from 0.67 to 0.81. All of the MLEs were greater than 0. The average ± standard deviation in the MLEs derived from the EGG data was found to be 0.75 ± 0.024.

3.2 Simulation Evaluation

In the 24000 time steps, there was no exception wherein the numerical solutions did not diverge for $\mu = 1,2,\cdots,20$; the value of τ derived from the first component of the numerical solution was not different from that derived from the second component. We compared this numerical solution with the EGG data.

The cross-correlation coefficient between the observed signal, *x(t)*, and the periodic function, *s(t)*, was calculated as a substitute for the SNR used in previous studies in which the occurrence of the SR was investigated. The cross-correlation coefficient between the numerical solutions, $\dot{x}$, and the periodic function, *s(t)*, in Equation (1.1). The cross-correlation coefficient was maximized for a moderate value of noise intensity, 11 < μ ≤ 12. Thus, the SR could be generated using Equations (1.1) and (1.2) with 11 < μ ≤ 12. Numerical solutions were projected onto the E_{trans} –λ plane (Figure 1).

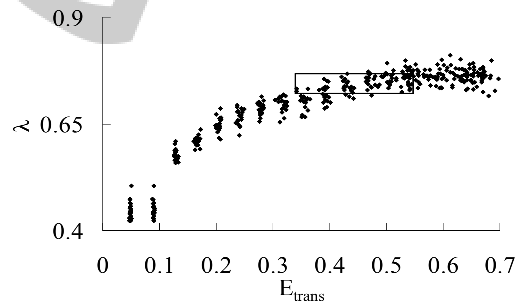

Figure 1: E_{trans}-λ plane (simulation results).

With respect to the EGG data taken 30 min after the postural change, the amount of EGG data projected onto region $\Re_s^2$ was less than the statistical standard. In contrast, 100% of the stationary EGG data was projected on the following region.

$$\{m(E_{trans}) \pm 2\sigma(E_{trans})\} \times \{m(\lambda) \pm 2\sigma(\lambda)\}$$

We quantitatively examined the conformity of the numerical solutions in region $\Re_s^2$ of the E_{trans}-λ plane. The conformity ratio for μ = 11.6 was the highest. Equations (1.1) and (1.2) for μ = 11.6 could be regarded as a mathematical model of the stationary EGG. Therefore, the SR appropriately describes the stationary EGG data.

4 DISCUSSION

In this study, we analyzed EGG time-series data using complex dynamical methods. E_{trans} and λ were calculated from the EGG data.

SDEs were proposed as a mathematical model of an EGG by Matsuura et al. (2008). SR occurs for an appropriate coefficient, μ. Some biosystems are based on the nonlinear phenomenon of SR, in which the detection of small afferent signals can be enhanced by the addition of an appropriate amount of noise (Benzi et al., 1981). We examined whether or not the SR generated using Equations (1) could describe an EGG time series. Stationary EGGs are well described by the SDEs in the case of $\mu = 11.6$, which might represent the SR ($11 < \mu \leq 12$). We herein claim that SR can be regarded as a mathematical model of an EGG. Moreover, distribution of the numerical solutions in the SR fits the distribution of the EGGs, which can be correlated as shown in Figure 2 ($R^2 = 0.972$).

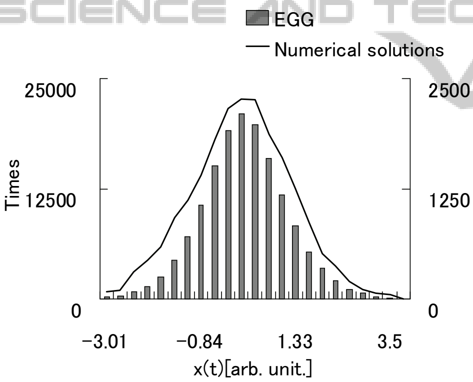

Figure 2: Distributions of EGGs and numerical solutions ($\mu = 11$).

The diseases resulting from constipation and erosive gastritis (an illness in which the inside of the stomach becomes swollen and painful) are accompanied by anomalous autonomic nervous activity. A decline in the electrical activity of the stomach should change the degree of determinism (E_{trans}) and the complexity (λ) in the attractor reconstructed from the EGG data. By using the mathematical model of an EGG, electrogastrography will be of assistance in the diagnosis of diseases of the alimentary canal and autonomic nervous system.

5 CONCLUSIONS

As a mathematical model of an EGG, we added the van der Pol equation to a periodic function and random white noises that represented the intestinal motility and other biosignals, respectively. By projecting the data from a stationary EGG obtained for a subject in the supine position, along with the numerical solutions, onto the E_{trans}-λ plane, we qualitatively evaluated the affinity between them. The SR was statistically the most appropriate with regard to the mathematical model of the stationary EGG. It is necessary to further investigate the reliability of a simplified measurement method by increasing the number of EGGs studied. The next step will also involve the suggestion of a mathematical model for an EGG, derived from data from elderly subjects and meal tolerance tests.

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