# DECISION MAKING BASED ON DUALITY BETWEEN POSITIVE AND NEGATIVE EVALUATIONS

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Abstract: This paper proposes a model of dual hierarchy to study a decision-making structure with positive and negative. It is necessary to evaluate a decision in negative stand point, such as insufficient, hatred, pressure, as a cause of fraud, as well as positive evaluation. In order to treat the positive and negative elements, we propose a model of dual hierarchy process which can evaluate from positive and negative points. Moreover, a technique to judge a consistency of dual evaluation is presented, using a concept of reachability matrix.

## 1 INTRODUCTION

On the decision-making problem under risk environment, it is important subjects how a risk is quantified and modeled. There is a fraud as one of the risks that exist in the company organization etc., and it appears as the serious problem.

AHP(Analytic Hierarchy Process) proposed by Saaty (T.L.Saaty, 1980) is one of the decision-making method which evaluates human's subjective feeling. AHP for risk management (Azuma and Miyagi, 2009) is a research treating the risk in the decision making problem. In the research, degree of satisfaction is regarded as positive utility, and degree of risk is regarded as negative utility. It is based on positive decision-making same as existing methods because negative utility depends on statistical data. However, in fraud prevention assessment, it is necessary to evaluate a decision in negative stand point, such as insufficient, hatred, pressure, as a cause of fraud, as well as positive evaluation.

In this paper, in order to solve this problem, we discuss a development of decision-making model which can be analyze the human's double psychology in fraud prevention.

## 2 DUALITY IN ORDER RELATION

In decision-making process, there are some cases that decision maker can not allow a rational and logical consistent evaluation. As an example, there is the deadlock in a three-cornered tie, and that is the order relation of preferences is uncertain. Such discrepancy can be discovered by calculating consistency index. However, if the order relation is not the deadlock in a three-cornered tie, it is difficult to discover a discrepancy. To discover the discrepancy in decision-maker, it is the one of way to check a fraud action. In the present study, in order to develop a new approach to discover a discrepancy under the human's subconscious, we focus on the duality relation of order as follows:

When the elements of the ordered set $X = x_i$ satisfies the following three rules, the set $X$ has the former theorem $p$, at the same time, it has new theorem $p^*$ that replaced all relations of order.

- $x \leq x$ (reflexivity)
- If $x \leq y$ and $y \leq x$ then $x = y$iasymmetryj
- If $x \leq y$ and $y \leq z$ then $x \leq z$itransitivityj

This concept is the duality theorem (Ataka, 1966). That is, $p^*$ is the duality theorem of $p$. In this paper, we propose the procedure that the discrepancy gener-



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ated in the decision-making process can be shown by utilizing the concept.

# 3 DUAL STRUCTURED DECISION-MAKING MODEL

Existing decision making system is type of selecting superiority. However, it is not easy to deal with negative elements such as ”pressure”, ”recognition of opportunity” and ”self-justification” which may appear in fraud problem. In particular, ”pressure” and ”self-justification” influence psychological factor of decision maker.

Then, we propose inferiority decision procedure on opposite point of superiority in order to treat human’s negative psychology. We believe there is a close relationship between superiority model and inferiority model because they are in the relation of the flip side. In the present study, we discuss both decision-making mechanism and propose the model with dual construction in Fig. 1.

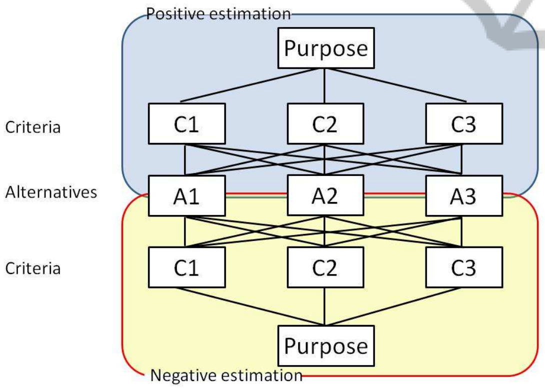


Figure 1: Dual hierarchy process.

Flowchart for proposed model is described as follows:

Step 1) decision-maker is asked to evaluate the decision criteria and alternatives and to make up a pairwise comparison matrix in positive or negative.

Step 2) the consistency index between positive matrix and negative matrix is calculated.

Step 3) If the judgments are consistent, final evaluation is obtained.

## 3.1 Positive Pairwise Comparison Matrix

We define two matrices in order to propose new decision model which has dual construction. One is the superiority pairwise comparison matrix (positive matrix) and another one is the inferiority pairwise comparison matrix (negative matrix).

The positive matrix is described by the ratio scale of criterion, the same as AHP. The ordered set of sequenced item by positive evaluation is defined as $X = x_1, \ldots, x_n$, in an certain decision problem. If the magnitude relation of $x_i$ and $x_j$ is $x_i \geq x_j$ and its ratio is $\omega_i : \omega_j$, then positive matrix $P$ having $\omega_i/\omega_j$ is constructed:

$$P = \begin{bmatrix} \omega_1/\omega_1 & \ldots & \omega_1/\omega_j & \ldots & \omega_1/\omega_n \\ \omega_2/\omega_1 & \ldots & \omega_2/\omega_j & \ldots & \omega_2/\omega_n \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \omega_n/\omega_1 & \ldots & \omega_n/\omega_j & \ldots & \omega_n/\omega_n \end{bmatrix} \quad (1)$$

The relation between matrix $P$ and its eigenvector is given as

$$P\omega = n\omega \quad (2)$$

where

$$\omega^T = [\omega_1, \omega_2, \ldots, \omega_n].$$

## 3.2 Negative Pairwise Comparison Matrix

We consider the opposite problem which has the order relation $x_j \geq x_i$. When the ratio scale of each item is defined as $\omega_j : \omega_i$, the negative matrix $N$ constructed by $x_j \geq x_i$ is defined as

$$N = \begin{bmatrix} \omega_1/\omega_1 & \ldots & \omega_j/\omega_2 & \ldots & \omega_n/\omega_1 \\ \omega_1/\omega_2 & \ldots & \omega_j/\omega_2 & \ldots & \omega_n/\omega_2 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \omega_1/\omega_n & \ldots & \omega_j/\omega_n & \ldots & \omega_n/\omega_n \end{bmatrix}. \quad (3)$$

where $N = P^T$ and $P$ and $N$ is the duality relation for $\geq$. Then, the relation between $N$ and its eigenvector is given as

$$N\omega' = n\omega' \quad (4)$$

where

$$\omega'^T = [1/\omega_1, 1/\omega_2, \ldots, 1/\omega_n].$$

By the above definition, Eq. (4) could be derived from Eq. (2) and we acquire that both matrices is the duality relation by duality theorem: They are in the relation that the order is reversed.

In Step 1, positive matrix and negative matrix are obtained by each evaluation of decision-maker. Select $n-1$ combinations ($n$Fnumber of elements) of element he want in order to reduce the burden on decision-maker’s task to compare. Then, it is not necessary to check each consistency of matrix because 2 combinations is minimum number of combinations to be able to calculate all evaluations.

All order, whether in human thinking, involves proportionality among the parts. Thus, to create positive matrix or negative matrix, we must use ratio scales to capture and synthesize the relations inherent in that order. If, for example, to create positive matrix, decision-maker is comparing each element according to wight we ask: "How much important is the element $i$ than the element $j$ in proportion ?" or "How much inferior is the element $i$ than the element $j$ in proportion ?" If the answer, " $i$ is more important than $j$ in proportion of 8 parts to 2" is gotten, the score of pairwise comparison is $\omega_i/\omega_j = 8/2$. In positive and negative evaluations, it is better to compare pair of elements as differently as possible. Thus, the contradiction between positive question and negative question can be made easy to discover.

## 3.3 Consistency

In step 2, we must judge the consistency for the constructed matrix. There are two kinds of consistency. One is the consistency of each matrix. Another is the consistency of duality matrices. In this paper, Saaty's method (T.L.Saaty, 1980) is adopted for the former and new method is proposed for the latter.

### 3.3.1 Consistency for Each Matrix

Next, consistency of each matrix is checked. The proposed positive matrix and negative matrix are consistent because their values are consisted by $n-1$ of ratio and other values are calculated at the them.

However, if values are obtained by comparing the pair of elements as AHP, we adopt the Saaty's method. Saaty proved that for consistent reciprocal matrix, the largest eigen value is equal to the size of comparison matrix, or $\lambda_{max} = n$. Then a measure of consistency, called Consistency Index is given as deviation or degree of consistency using the following formula

$$CI = \frac{\lambda max - n}{n-1}. \tag{5}$$

### 3.3.2 Consistency in Duality

We propose how to judge the consistency in duality between $P$ and $N$. The consistency in duality means that positive and negative have same order of priority in criteria and alternatives. It is necessary to examine the consistency of duality to discover implicit contradiction of the decision-maker.

We apply the concept of reachability matrix to our procedure. If proposed consistency is satisfied completely, the order relation between $P$ and $N$ is duality. We defined, the relation of priorities in positive and negative matrices is not reversed and it should be considered within the acceptable limits.

Step1) creation of adjacency matrix

Both matrix $P$ and $N$ can be represented by a matrix $M$, called the adjacency matrix, as shown below. There is a row and column for each node; $M[i,j] = 1$ if $(i,j)$ element of matrix is more than 1, if $(i,j)$ element is otherwise $M[i,j] = 0$.

$$\begin{aligned} M_P &= [m_{ij}^P] \\ M_N &= [m_{ij}^N] \\ m_{ij} &= \begin{cases} 1 : \omega_{ij} \geq 1 \\ 0 : \omega_{ij} < 1 or unknown \end{cases} \end{aligned} \tag{6}$$

where $M_P$ is the adjacency matrix of $P$ and $M_N$ is the adjacency matrix of $N$.

Step2) creation of reachability matrix

The next step derives the reachability matrices $R_P$ and $R_N$. It is calculated on gotten the adjacency matrices with Boolean OR operation as follows:

$$R_P = \sum_{k=1}^{m} M_P^k \tag{7}$$

$$R_N = \sum_{k=1}^{m} M_N^k \tag{8}$$

When the number of $k(\leq m)$ is added up, the elements of adjacency matrix $M$ converge in specific $k$. Then, the convergent matrix is reachability matrix. If $R(i,j) = 1$ then it means $i$ element is more important than $j$ element. if $R(i,j) = 0$ then $i$ is less important than $j$, respectively. Thus, $R$ represents the relation of important degree in n-tuple pairwise comparison.

Step3) judgement of the consistency of duality

In the last step, we judge the consistency of duality on $P$ and $N$. When obtaining $R_P$ and $R_N$ in step2 have same elements, it is defined by $\omega = 1/\omega'$ that both matrices are consistent in duality. To judge the consistency, matrix $C$ is defined as

$$R_P \oplus R_N^T = C \tag{9}$$

where operator $\oplus$ represents exclusive OR. As $C = O$(where $O$ is zero matrix)Cwe define the relation between $P$ and $N$ as being consistent. On the other hand, as $C \neq O$, it suggests that there is an inconsistency of order about $i$ and $j$ where 1 in $C$.

# 4 APPLICATION

The following example is taken from the paper presented by H.S.Rian and T.Sekiguchi (RIAN and Takashi, 1995). Suppose that a company choices a excellent person in three persons $A_1$, $A_2$ and $A_3$, on decision-making for person perception. Decision criteria are as follows:

- sense of responsibility ($C_1$)
- inventive idea ($C_2$)
- knowledge ($C_3$)

Decision-maker gives two values ($n = 3$) through a pairwise comparison of above criteria in positive and negative. In positive criteria, for example, to ask a decision-maker "Which is more important a sense of responsibility or an inventive idea for person perception ?". In negative, "Which doesn't need a sense of responsibility or an inventive idea for person perception ?". Then, he gives the ratio as pairwise comparison value. Table 1 and Table 2 are results given by a decision-maker through a pairwise comparison. Values in square are calculated by given comparison value. Both tables consisted of $n-1$ values are satisfied in Saaty's C.I..

Table 1: Positive pairwise comparison by personality.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $C_1$ | 1 | 8/2 | $\boxed{24/4}$ |
| $C_2$ | | 1 | 6/4 |
| $C_3$ | | | 1 |

Table 2: Negative pairwise comparison by personality.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $C_1$ | 1 | 3/7 | 2/8 |
| $C_2$ | | 1 | $\boxed{7/12}$ |
| $C_3$ | | | 1 |

In next step, we check the dual consistency. Two adjacency matrixes are created from Table 1 and Table 2 by Eq. (7).

$$M_P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, M_N = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

Then, reachability matrixes $R_P$ and $R_N$ are obtained as

$$R_P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, R_N = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

Finally, the dual consistency index $C$ can be calculated by Eq. (9).

$$C = O$$

In the calculation, $C$ is zero matrix. It shows that the dual consistency is satisfied under criteria. Moreover, both eigen vectors of maximum eigen values are weight vector on criteria, givens as

$$W_P^T = (0.706, 0.176, 0.118)$$
$$W_N^T = (0.136, 0.318, 0.546).$$

By the above result, the order relation is $C_1 \geq C_2 \geq C_3$ in positive and negative. It shows that they are duality in order relation.

# 5 CONCLUSIONS

This paper suggested a new decision-making model to analyze the human's double psychology in fraud prevention.

We proposed the dual hierarchy, having positive matrix and negative matrix to evaluate two-sided question. We discussed about the relation that both matrixes are duality. Further, the technique to judge a dual consistency of both matrixes, provides consistency index based on a concept of reachability matrix through the order relation of both matrixes. As a result, we are able to check any inconsistency between positive and negative mentals.

In our future work, we plan to develop a method for fraud detection with our decision-making model.

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