ULTRASONIC OFDM PULSE DETECTION FOR TIME OF FLIGHT MEASUREMENT OVER WHITE GAUSSIAN NOISE CHANNEL

Daniel F. Albuquerque, José M. N. Vieira, Carlos A. C. Bastos and Paulo J. S. G. Ferreira
Signal Processing Lab – IEETA/DETI, University of Aveiro, 3810-193 Aveiro, Portugal

Keywords: Pulse Detection, OFDM, Ultrasonic Location System.

Abstract: In this paper we evaluate the probability of detection and the probability of false alarm for an OFDM pulse over AWGN channel. This type of pulse is useful for indoor location systems using ultrasounds due to the ability to accurately measure the time of flight by pulse detection while transmitting some data. Moreover, we can avoid the RF auxiliary channel by using an OFDM-based sync. The probability of OFDM pulse detection over white Gaussian noise will be presented as function of the probability of false alarm when a threshold technique is employed at the receiver. Furthermore, the pulse detection probability will be compared with a chirp pulse.

1 INTRODUCTION

Indoor location is an active area of research in the signal processing community with a large potential from the point of view of applications (Sayed et al., 2005; Liu et al., 2007). To perform indoor location, there are 2 main types of solutions: Ultrasonic (US) and Radio Frequency (RF) based systems. RF power strength based systems are inexpensive but require the profiling of the entire location scenario to get a RF fingerprint resulting in an accuracy from 1 to 5 meters approximately (Stuntebeck et al., 2008; Bahl and Padmanabhan, 2000). On the other hand, the ultrasound technology is the best suited to achieve the accuracy required on indoor environments, that can be less than 1 cm in some cases (Gonzalez and Bleakley, 2009; Prieto et al., 2007). Our location system, LocUS, aim to be entirely based on ultrasound signals. Most of the known ultrasonic location systems use an auxiliary RF channel for synchronization. An RF pulse is used as a time reference for measuring the propagation delay between the source and the receiver (Hazas and Hopper, 2006). Although this auxiliary RF channel allows very simple clock synchronization and delay measurement solutions, it also gives away two important advantages that US-based systems bear in reference to RF-based ones: the immunity to RF interference, and the ability to safely operate in the presence of critical electronic instrumentation such as medical or life-support systems. Therefore, one way to avoid the use of an auxiliary RF signal to measure the time-of-flight (TOF) is to synchronize the clocks of the nodes (Skeie et al., 2001). To achieve this, the nodes should be able to send to each other the clock information using the ultrasonic channel. Due to the reflection of the ultrasonic signals on the walls the acoustic communication channel presents a strong multipath effect causing inter-symbolic interference.

LocUS solved this problem by using OFDM (Orthogonal Frequency Division Multiplexing) to perform data transmission. This technique has already been used in ultrasonic underwater communications (Mason et al., 2007; Nakashima et al., 2006). OFDM is a very flexible modulation technique, robust to multipath and that simplifies the channel equalization. It is also very sensitive to synchronization, which may be an advantage when the application requires the measurement of the TOF (Levanon and Mozeson, 2004). As we intend to send just one OFDM data pulse with the carriers phase modulated, to measure the phases we need to send another pulse to measure the phases difference. That way, the first pulse has a double purpose, it should have a high energy to be effectively detected by a matched filter and it will be used as a phase reference by the second pulse to decode the transmitted data.

In this paper it will be presented an analytical expression for the probability of OFDM pulses detection in the presence of white Gaussian noise. For that propose it will be presented, in section 2, the

F. Albuquerque D., M. N. Vieira J., A. C. Bastos C. and J. S. G. Ferreira P..
ULTRASONIC OFDM PULSE DETECTION FOR TIME OF FLIGHT MEASUREMENT OVER WHITE GAUSSIAN NOISE CHANNEL.
DOI: 10.5220/0003375103450350
In *Proceedings of the 1st International Conference on Pervasive and Embedded Computing and Communication Systems* (PECCS-2011), pages 345-350
ISBN: 978-989-8425-48-5

asynchronous data transmission with OFDM pulses. In section 3 it is presented the probability of detection and the the probability of false alarm for OFDM pulses over a white Gaussian noise channel when a threshold technique is used in the receiver. Some simulation results will be presented in section 4. At the end it will be presented a brief conclusion.

2 ASYNCHRONOUS DATA TRANSMISSION WITH OFDM

An architecture for asynchronous data transmission using OFDM pulses is proposed. We use two concatenated OFDM pulses, one for time synchronization (e.g. time-of-flight measurement) and another for some data information transmission (e.g. source identification). The group of the synchronization pulse and the data information pulse is called frame.

2.1 Frame Prototype

Figure 1 presents the proposed frame prototype, as mention before, there are two different main pulses in the frame: The *OFDM Sync.* and the *OFDM Data*. The first pulse is detected by the receiver by matched filter and used to synchronization. It will also be used to demodulate the second pulse, the *OFDM Data*, by a differential demodulation scheme (Haykin, 2001). This method was chosen manly because the OFDM pulse is robust to environments with multipath but it does not produce a very high resolution in time synchronization, as will be seen later on. Therefore, the differential demodulation will be robust to this small jitter.

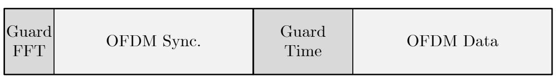

Figure 1: Asynchronous data transmission with OFDM, frame prototype.

The *Guard FFT*, presented in Figure 1 is a cyclic extension to protect the demodulation process due to the time synchronization jitter. The *Guard Time* is a cyclic extension of the OFDM data pulse to avoid inter-symbolic interference caused by the room impulse response (Schulze and Luders, 2005).

3 OFDM PULSE DETECTION

Two different techniques to perform pulse detection can be used, the conventional time domain MF (Matched Filter) or the FFT (Fast Fourier Transform) followed by a small scalar product. Figure 2 presents the FFT technique, where the incoming signal, x (pulse to detect plus noise) enters in a *Buffer Delay* that has the same size of the pulse s. Therefore, the information in the buffer is converted to the frequency domain. Moreover, due to the OFDM properties $S(k)$ (Fourier transform of s) is only different from zero in the information carriers so the system only needs to perform the dot product in that carriers.

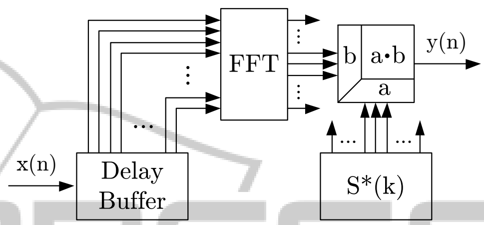

Figure 2: Pulse detection with FFT.

In the following sections is considered that the FFT and IFFT are normalize to $1/\sqrt{N}$:

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{nk}{N}} \tag{1}$$

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi\frac{nk}{N}}. \tag{2}$$

Therefore, the output $y_{FFT}(n)$ of FFT detector can be written as:

$$y_{FFT}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S^*(k) \sum_{m=0}^{N-1} x(m+n-N+1) W^{-mk}. \tag{3}$$

where $W = e^{j\frac{2\pi}{N}}$ and $()^*$ is the conjugate.

On the other hand, the output of the MF detector $y_{MF}(n)$ is given by (Levanon and Mozeson, 2004):

$$\begin{aligned} y_{MF}(n) &= \sum_{m=0}^{N-1} s^*(N-1-m) x(n-m) \\ &= \sum_{m=0}^{N-1} x(m+n-N+1) s^*(m) \end{aligned} \tag{4}$$

where $s(m)$ is the inverse Fourier transform of $S(k)$:

$$y_{MF}(n) = \sum_{m=0}^{N-1} x(m+n-N+1) \left(\frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S(k) W^{mk} \right)^* \tag{5}$$

reorganizing the terms in the equation:

$$y_{MF}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S^*(k) \sum_{m=0}^{N-1} x(m+n-N+1) W^{-mk}. \tag{6}$$

As can be seen, equation (6) is equal to equation (3).

In the FFT technique the scalar product has the size of the number of carriers. On the other hand, the MF technique needs to have a filter with the size of the pulse. So the choice between one of these techniques depends on the pulse size and the number of carriers. If the pulse is small and there is a large number of carriers, the MF is better than the FFT technique, otherwise, if the pulse is big and there are only a few carriers the FFT is better than the MF technique.

In order to simplify equation (6) and without loss of generalization in the next section we will consider the detector output with a time advance of $(N-1)$

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S^*(k) \sum_{m=0}^{N-1} x(m+n) W^{-mk}. \quad (7)$$

3.1 Probability of Detection

In order the detect the presence of a pulse the output $y(n)$ must be compared to a given threshold. Therefore, we consider that a pulse is detected if the output $y(n)$ is greater than the given threshold. Therefore, the threshold must be chosen in a way that a very weak pulse must be detected and the noise does not produce false alarms. In this way, it is very important to keep the probability of false alarm, which is the probability of detecting the pulse when in the input there is only noise, very small and the probability of detection very high. To evaluate these probabilities we must consider two competing hypotheses:

$$\begin{cases} \mathcal{H}_0 & : \quad x(n) = w(n) \\ \mathcal{H}_1 & : \quad x(n) = w(n) + As(n) \end{cases} \quad (8)$$

which is equivalent to:

$$\begin{cases} \mathcal{H}_0 & : \quad x(n) = w(n) + As(n) \quad , A = 0 \\ \mathcal{H}_1 & : \quad x(n) = w(n) + As(n) \quad , A \neq 0 \end{cases} \quad (9)$$

where $\mathcal{H}_0$ is called the *null hypothesis*, $\mathcal{H}_1$ the *alternative hypothesis* and $w(n)$ is white Gaussian noise. The system choose the hypothesis by the magnitude of the signal $y(n)$. Therefore, if the magnitude of $y(n)$ is greater than a given threshold γ the system decides $\mathcal{H}_1$ otherwise the system decides $\mathcal{H}_0$. This procedure is shown in Figure 3.

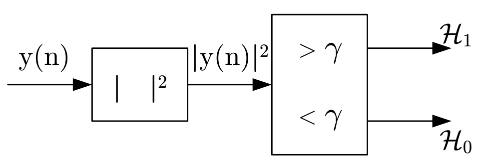

Figure 3: Detection decision block.

The output of the first block, after the comparators, is given by:

$$|y(n)|^2 = \frac{1}{N} \left| \sum_{k=0}^{N-1} S^*(k) \sum_{m=0}^{N-1} x(m+n) W^{-mk} \right|^2 \quad (10)$$

Using (9):

$$|y(n)|^2 = \frac{1}{N} \left| \sum_{k=0}^{N-1} S^*(k) \left(\sum_{m=0}^{N-1} w(m+n) W^{-mk} + \right.\right. \left.\left. + A \sum_{m=0}^{N-1} s(m+n) W^{-mk} \right) \right|^2 \quad (11)$$

The signal $S(k)$ is only different from zero in a set of carriers of the OFDM pulse, $k \in (k_0, k_1, ..., k_{N_p-1})$ (where N_p is the number of OFDM pulse carriers), they will be real due to the differential demodulation and without loss of generalization we can consider that $|S(k)| = 1$ for $k \in (k_0, k_1, ..., k_{N_p-1})$. So we can simplify the equation 11 to:

$$|y(n)|^2 = \frac{1}{N} \left| \sum_{k=k_0}^{k_{N_p-1}} S(k) \left(U\left(\frac{k}{N}\right) + jV\left(\frac{k}{N}\right) \right) + A \sum_{k=k_0}^{k_{N_p-1}} S(k) \sum_{m=0}^{N-1} s(m+n) W^{-mk} \right|^2 \quad (12)$$

where $U(k/N)$ and $V(k/N)$ for $k = 1, 2, ..., N/2 - 1$ are random independent and Gaussian variables (Steven, 1993). Therefore, if $w(m)$ is white noise with zero mean and variance σ^2, the $U(k/N)$ and $V(k/N)$ also has zero mean and variance $N\sigma^2/2$. Normalizing U and V to have unit variance, $|y(n)|^2$ becomes:

$$|y(n)|^2 = \frac{N_p \sigma^2}{2} \left| \frac{U(k/N)}{\sqrt{N\sigma^2/2}} + j \frac{V(k/N)}{\sqrt{N\sigma^2/2}} + \mu(n) \right|^2 \quad (13)$$

where $\mu(n)$ can be seen as the mean of the sum of this two variables, moreover, it is dependent of the pulse and it is only different from zero when the pulse is present, $\mu(n)$can be written as:

$$\mu(n) = \frac{A}{\sigma} \sum_{k=k_0}^{k_{N_p-1}} \sum_{l=k_0}^{k_{N_p-1}} \frac{S(k) S(l) W^{l(n-n_i)} P(k-l, n)}{N\sqrt{N_p/2}} \quad (14)$$

where n_i is the first sample of the pulse and $P(k,n)$ is the Fourier transform of the boxcar function:

$$P(k,n) = W^{-\frac{(N-1+n_i-n)k}{2}} \psi(k,n) \quad (15)$$

where, for $k \neq 0$ $\psi(k,n)$ is defined as:

$$\psi(k,n) = \begin{cases} \frac{\sin\left(\frac{(N+n_i-n)\pi k}{N}\right)}{\sin\left(\frac{\pi k}{N}\right)} & n_i \leq n < n_i + N \\ \frac{\sin\left(\frac{(N+n-n_i)\pi k}{N}\right)}{\sin\left(\frac{\pi k}{N}\right)} & n_i - N < n < n_i \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

Rewriting this equation, $\psi(k,n)$ becomes:

$$\psi(k,n) = \frac{\sin\left(\frac{(n-n_i)\pi k}{N}\right)}{\sin\left(\frac{\pi k}{N}\right)}(-1)^k \upsilon(n) \quad (17)$$

were $\upsilon(n)$ is:

$$\upsilon(n) = u(n-n_i-1+N) - 2u(n-n_i) + u(n-ni-N) \quad (18)$$

and $u(n)$ is a unitary step (it is zero for $n < 0$ and one otherwise). For $k = 0$, $\psi(k,n)$ is defined as:

$$\psi(0,n) = N\upsilon(n)^2 + (n-n_i)\upsilon(n) \quad (19)$$

So the equation 14 can be simplified to:

$$\mu(n) = \frac{A}{\sigma} \sum_{k=k_0}^{k_{N_p-1}} \sum_{l=k_0}^{k_{N_p-1}} \frac{S(k)S(l)W^{\frac{(k+l)(n-n_i)}{2}} \psi(k-l,n)}{N\sqrt{N_p/2}} \quad (20)$$

Separating $\mu(n)$ in real and complex part it can be written as:

$$\mu(n) = \mu_U(n) + j\mu_V(n) = \sqrt{\eta}\left[\mu'_U(n) + j\mu'_V(n)\right] \quad (21)$$

where η is:

$$\eta = \frac{A^2}{\sigma^2}. \quad (22)$$

As S is real, $\mu'_U(n)$ and $\mu'_V(n)$ can be written separately:

$$\mu'_U(n) = \sum_{k=k_0}^{k_{N_p-1}} \sum_{l=k_0}^{k_{N_p-1}} \frac{S(k)S(l)\psi_U(k,l;n)}{N\sqrt{N_p/2}} \quad (23)$$

$$\mu'_V(n) = \sum_{k=k_0}^{k_{N_p-1}} \sum_{l=k_0}^{k_{N_p-1}} \frac{S(k)S(l)\psi_V(k,l;n)}{N\sqrt{N_p/2}} \quad (24)$$

where $\psi_U(k,l;n)$ and $\psi_V(k,l;n)$ for $k \neq l$ are defined as:

$$\psi_U(k,l;n) = \frac{\sin\left(\frac{2\pi l(n-n_i)}{N}\right) - \sin\left(\frac{2\pi k(n-n_i)}{N}\right)}{2\sin\left(\frac{\pi(k-l)}{N}\right)}\upsilon(n) \quad (25)$$

$$\psi_V(k,l;n) = \frac{\cos\left(\frac{2\pi l(n-n_i)}{N}\right) - \cos\left(\frac{2\pi k(n-n_i)}{N}\right)}{2\sin\left(\frac{\pi(k-l)}{N}\right)}\upsilon(n) \quad (26)$$

for $k = l$ $\psi_U(k,l;n)$ and $\psi_V(k,l;n)$ are defined as:

$$\psi_U(l,l;n) = \cos\left(\frac{2\pi l(n-n_i)}{N}\right)\psi(0,n) \quad (27)$$

$$\psi_V(l,l;n) = -\sin\left(\frac{2\pi l(n-n_i)}{N}\right)\psi(0,n) \quad (28)$$

Rewriting the equation 13, $|y(n)|^2$ becames:

$$|y(n)|^2 = \frac{N_p\sigma^2}{2}\left[\left(\frac{U(k/N)}{\sqrt{N\sigma^2/2}} + \mu_U(n)\right)^2 + \right. \\ \left. + \left(\frac{V(k/N)}{\sqrt{N\sigma^2/2}} + \mu_V(n)\right)^2\right] \quad (29)$$

The two quadratic terms in equation 29 can be seen as a Chi-Squared distribution with two Gaussian white independents variables ($\chi'^2_2(\lambda(n))$) (Steven, 1993), one with $\mu_U(n)$ mean and another with $\mu_V(n)$ mean. Where $\lambda(n)$ is given by (Steven, 1993):

$$\lambda(n) = \mu_U(n)^2 + \mu_V(n)^2 = \eta\lambda(n)' \quad (30)$$

where $\lambda(n)'$ is given by:

$$\lambda(n)' = \mu'_U(n)^2 + \mu'_V(n)^2 \quad (31)$$

From equations 23, 24, 30 and 31 we can get two important conclusions: First, the value of N does not change the magnitude of λ', second, λ is proportional to η consequently λ is proportional to the signal to noise ratio.

For the particular case when $n = n_i$, (first sample of the pulse), λ' takes the value $2N_p$ so λ becomes:

$$\lambda(n_i) = \eta 2N_p \quad (32)$$

The Signal to Noise Ratio (SNR) is equal to $\eta N_p/N$, therefore, λ can also be written in function of the SNR and the size of the pulse:

$$\lambda(n_i) = 2N \times \text{SNR} \quad (33)$$

With these results it would be easy to compute the *probability of false alarm* (P_{fa}) and the *probability of detection* ($P_d(n)$) for any instant n.

The *probability of false alarm* can be seen as the probability of choose $\mathcal{H}_1$ when the system is in the presence of $\mathcal{H}_0$ i.e. the system believes to be in the presence of the pulse but it only has noise in the input ($\mu_U(n)$ and $\mu_V(n)$ are equal to zero). The *probability of false alarm* can be defined as:

$$P_{fa} = Pr\left\{|y(n)|^2 > \gamma; \mathcal{H}_0\right\} \quad (34)$$

Replacing $|y(n)|^2$ we obtain:

$$P_{fa} = Pr\left\{\left(\frac{U(k/N)}{\sqrt{N\sigma/2}}\right)^2 + \left(\frac{V(k/N)}{\sqrt{N\sigma/2}}\right)^2 > \right. \\ \left. > \frac{2\gamma}{N_p\sigma^2}; \mathcal{H}_0\right\} \quad (35)$$

Therefore the *probability of false alarm* is:

$$\begin{aligned} P_{fa} &= Q_{\chi'^2_2(0)}\left(\frac{2\gamma}{N_p\sigma^2}\right) = Q_{\chi^2_2}\left(\frac{2\gamma}{N_p\sigma^2}\right) \\ &= e^{-\frac{\gamma}{N_p\sigma^2}} \end{aligned} \tag{36}$$

where χ^2_2 represent a Chi-Squared distribution with two Gaussian white independents variables with unit variance and zero mean (Steven, 1993).

The *probability of detection* can be seen as the probability of choosing $\mathcal{H}_1$ when the system is in the presence of the pulse i.e. the system considers that a pulse was received and and one is present. The *probability of detection* can be defined as:

$$P_d(n) = Pr\left\{|y(n)|^2 > \gamma; \mathcal{H}_1\right\} \tag{37}$$

Doing the same for P_d that it was done for P_{fa}

$$P_d(n) = Q_{\chi'^2_2(\lambda(n))}\left(\frac{2\gamma}{N_p\sigma^2}\right). \tag{38}$$

Using (36) P_d can be written as function of P_{fa}

$$P_d(n) = Q_{\chi'^2_2(\lambda(n))}\left(\ln({P_{fa}}^{-2})\right) \tag{39}$$

or

$$P_d(n) = Q_{\chi'^2_2(\eta\lambda'(n))}\left(\ln({P_{fa}}^{-2})\right). \tag{40}$$

4 RESULTS

In order to validate the theoretical detection probability a test for a probability of false alarm of 10^{-16} was performed. In this test, it was considered the following practical situation:

- An OFDM pulse with 17 carriers sampled at 100 kHz with the following amplitudes

$$\begin{aligned} S(k) = \quad & \{1,1,\text{-}1,\text{-}1,\text{-}1,\text{-}1,1,1,\text{-}1,1,1,\ldots \\ & \ldots\text{-}1,1,\text{-}1,1,\text{-}1,1\},\ k = k_0 \ldots k_{N_p-1}; \end{aligned}$$

- the lowest carrier frequency was set to 40 KHz ($k_0 = 400$);
- the pulse has 1000 samples;
- the signal to noise ratio was set to 0 dB;
- for each sample 1000 simulations were performed;

Figures 4 and 5 present the probability of detection for a probability of false alarm of 1×10^{-16} and 1×10^{-8} respectively. The horizontal axis represents the incoming instant sample of the pulse with n_i as the instant when the last sample of the pulse was received.

Figure 4 shows that there is a great probability to detect the pulse before the system receive the whole pulse. Therefore, to improve the detection capabilities we can use a very simple algorithm: When the system detects a pulse it can wait for a period of time equal to the length of the pulse and choose the maximum value at the match filter output.

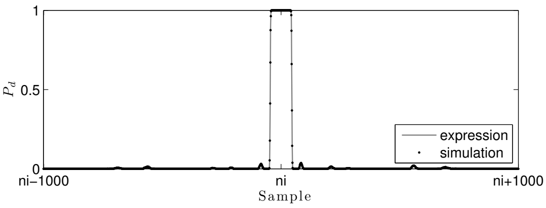

Figure 4: Probability of detection for a probability of false alarm of the 1×10^{-16}.

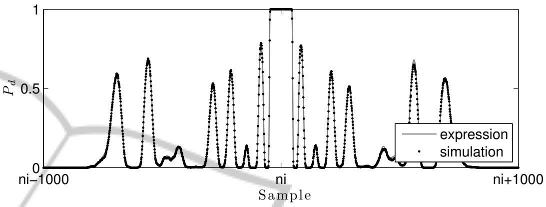

Figure 5: Probability of detection for a probability of false alarm of the 1×10^{-8}.

4.1 Comparison with other Usual Pulse

It was chosen a linear chirp also known as a linear frequency-modulated pulse (Levanon and Mozeson, 2004) to compare with the proposed pulse. A linear chirp is a signal where its instantaneous frequency change linearly with time. Moreover, this type of signals are usually used in radar and sonar systems (Levanon and Mozeson, 2004).

For this test we chosen a chirp pulse with similar characteristics (bandwidth, length, maximum amplitude) to the OFDM pulse. The autocorrelation functions of the OFDM and chirp pulses are shown in Figure 6.

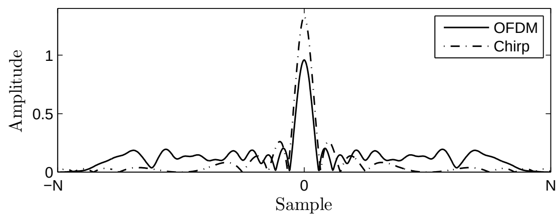

Figure 6: Comparison of the autocorrelation functions of a chirp pulse and an OFDM pulse.

From Figure 6 it is possible to say that the Chirp pulse is better for detection over noise environments than the proposed OFDM pulse. Manly because it produces a bigger amplitude in the matched filter output. To prove this statement we evaluated the probability of detection as a function of the probability of false alarm for 0 dB ratio of signal amplitude to noise standard deviation. The results are presented in Figure 7.

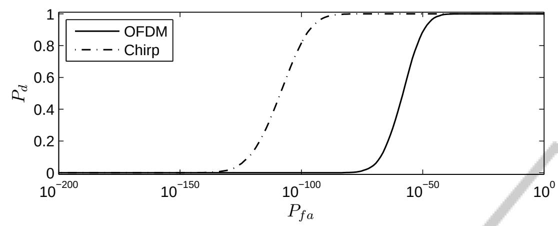

Figure 7: Probability of detection in function of probability of false alarm.

5 CONCLUSIONS

In this paper we evaluated the probability of detection of an OFDM pulse over white Gaussian noise channels for a given probability of false alarm. Therefore, the resultant expression was validated with computer simulations. The results demonstrated that the probability of detection of an OFDM pulse is lower than a chirp with similar characteristics. This lower performance comes from the reduced energy of the OFDM compared to the chirp, this energy difference is due to the amplitude concerns. However, this worst performance does not compromise the potential for using OFDM pulses for asynchronous communication. Simulation results have sown that if the probability of false alarm is set to 10^{-50} the pulse has a 90% probability of detection. Moreover, the proposed OFDM pulse will be used not only for TOF measurements but also for some data communication.

REFERENCES

Bahl, P. and Padmanabhan, V. N. (2000). RADAR: an in-building RF-based user location and tracking system. In *Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies*, volume 2, pages 775–784 vol.2.

Gonzalez, J. R. and Bleakley, C. J. (2009). High-Precision Robust Broadband Ultrasonic Location and Orientation Estimation. *IEEE Journal of Selected Topics in Signal Processing*, 3(5):832–844.

Haykin, S. (2001). *Communication Systems*. John Wiley & Sons, 4th edition.

Hazas, M. and Hopper, A. (2006). Broadband ultrasonic location systems for improved indoor positioning. *IEEE Transactions on Mobile Computing*, 5(5):536–547.

Levanon, N. and Mozeson, E. (2004). *Radar Signals*. JOHN WILEY & SONS.

Liu, J., Chu, M., and Reich, J. E. (2007). Multitarget Tracking in Distributed Sensor Networks. *Signal Processing Magazine, IEEE*, 24(3):36–46.

Mason, S., Anstett, R., Anicette, N., and Zhou, S. (2007). A Broadband Underwater Acoustic Modem Implementation Using Coherent OFDM. In *The National Conference On Undergraduate Research*, California.

Nakashima, Y., Matsuoka, H., and Yoshimura, T. (2006). Evaluation and Demonstration of Acoustic OFDM. In *Fortieth Asilomar Conference on Signals, Systems and Computers*, pages 1747–1751.

Prieto, J. C., Jimenez, A. R., and Guevara, J. I. (2007). Subcentimeter-accuracy localization through broadband acoustic transducers. In *IEEE International Symposium on Intelligent Signal Processing*, pages 1–6.

Sayed, A. H., Tarighat, A., and Khajehnouri, N. (2005). Network-based wireless location: challenges faced in developing techniques for accurate wireless location information. *Signal Processing Magazine, IEEE*, 22(4):24–40.

Schulze, H. and Luders, C. (2005). *Theory and Applications of OFDM and CDMA*. John Wiley & Sons, first edition.

Skeie, T., Johannessen, S., and Holmeide, O. (2001). Highly accurate time synchronization over switched Ethernet. In *8th IEEE International Conference on Emerging Technologies and Factory Automation*, pages 195–204 vol.1.

Steven, M. K. (1993). *Fundamentals of statistical signal processing: estimation theory*. Prentice-Hall, Inc.

Stuntebeck, E. P., Patel, S. N., Robertson, T., Reynolds, M. S., and Abowd, G. D. (2008). Wideband powerline positioning for indoor localization. In *Proceedings of the 10th international conference on Ubiquitous computing*, pages 94–103, Seoul, Korea. ACM.