

MULTIPLE TARGETS DETECTION AND LOCALIZATION BASED ON BLIND ESTIMATION IN WIRELESS SENSOR NETWORK

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Abstract: Observations of sensors are modeled as mixed signals in multiple targets scenario. Each element of mixing matrix represents the power decay of a pair of target and sensor, and each column preserves the waveform formed by the corresponding target respectively. Making use of blind estimation algorithms, we get the estimation of mixing matrix. Target locations are then estimated using the least squares method.

1 INTRODUCTION

Research on single target localization and tracking approaches in wireless sensor network has been carried out for a decade, and effective algorithms have been proposed (Savarese et al., 2001; Taff, 1997). Some existing approaches aiming at multiple targets apply sensor arrays (Nehorai et al., 1994), which are different from adhoc sensor networks as the latter are with unstable topological structure of sensors. Research corresponding to multiple targets scenario has just emerged in recent years and some methods have been proposed, most of which are under the framework of maximum likelihood and expectation-maximization like methods (Xiao et al., 2005; Krasny et al., 2001).

In this paper, by taking into consideration of the statistical properties of targets, we use the independent component analysis to estimate the number of target, and make use of blind separation algorithms to solve the mixing matrix which describes the overlap of the multiple targets in each sensor's measurement. A target localization algorithm based on the least squares methods is then obtained.

In section II, the system model, mixing model and assumption of sources are presented. In section III, a source detection method is given to estimate the target number. In section IV, algorithms of sources separation and estimation of mixing matrix are introduced. Finally, the target locations are estimated in section V.

2 SYSTEM MODEL

2.1 Source Model

Whatever signals are transmitted by certain form of wave, e.g. acoustic, radio, earth wave, etc, signals produced by sources are carried by waves from sources to receivers. Due to physical essences, signals are described as stochastic processes with statistical properties. Whether the waves are generated by sources or reflected by sources, some inherited properties of sources are loaded on carriers inevitably.

Source signal can be modeled as

$$s(t) = \sum_{k=-\infty}^{\infty} s(k)g_T(t-nT_s) \quad (1)$$

where $s(k)$ is the source signal. It can be modelled as a zero mean stationary process with a non-singular covariance matrix. $g_T(t)$ is unit amplitude rectangular pulse of width T_s .

2.2 Fading Model

For wireless radio, sonar, or earth waves, signals generally suffer from two major sorts of fading, one is caused by space condition, e.g. loss of distance, multi-path and the other caused by relative motion between transmitter and receiver.

Signals suffers from different sorts of path fading, here we only take into consideration of

signal power decays related to path length in free space. Let $f(d)$ denote the path fading coefficient, the received signal by a sensor can be expressed as

$$r(t) = f(d)s(t) \quad (2)$$

where d is the distance between source and sensor. One of the generally used models is

$$f(d) = \frac{1}{(d+1)^2}$$

After all, how to model the path fading does not affect the source detection and separation essentially.

2.3 Mixing Model

Suppose there are m sources and n sensors in the sensor field, generating signal $s_j(k)$, $j = 1, \dots, m$ and observation records $x_i(k)$, $i = 1, \dots, n$, $n > m$, respectively. Let $\mathbf{s}(t) = [s_1(t), \dots, s_m(t)]^T$ and $\mathbf{x}(t) = [x_1(t), \dots, x_n(t)]^T$ denote the source and sensor observation vector, respectively. The observation vector $\mathbf{x}(t)$ can be expressed as the sum of linear combination of signals and noises as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (3)$$

where $\mathbf{A} = (a_{i,j}) \in \mathbb{C}^{n \times m}$ is the mixing matrix between sources and sensors. $\mathbf{n}(t) = (n_i) \in \mathbb{C}^n$ is an additive Gaussian noise with zero mean, covariance matrix $\sigma_n^2 \mathbf{I}$, and \mathbf{I} is the identity matrix. Note that power decays and Doppler shift are independent to each other,

$$a_{i,j}(t) = f(d_{i,j}) \quad (4)$$

where, $d_{i,j}$ is the distance between the j th source and the i th sensor.

Assumptions

(A1) \mathbf{A} has full column rank, i.e. $\text{rank}(\mathbf{A}) = m$.

(A2) $\{s_1(t), \dots, s_m(t)\}$ are uncorrelated.

(A3) There exists a $\tau > 0$ such that $E(s_i(t)s_i^*(t+\tau)) \neq 0, i = 1, \dots, m$.

(A4) Sampling rate satisfies $1/\Delta t > f_D$.

(A5) Source location is approximated to be unchanged in T_s , and observation window $N\Delta t < T_s$, where N is the sample number in observation window.

3 SOURCES DETECTION

Under the above assumptions, the covariance matrix of $\mathbf{x}(t)$ can be given by

$$\begin{aligned} \mathbf{R}_x(0) &= E\{\mathbf{x}(t)\mathbf{x}^H(t)\} \\ &= \mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H + \sigma_n^2\mathbf{I} \end{aligned} \quad (5)$$

where H denotes conjugate transpose, $\mathbf{R}_s(0) = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the nonsingular covariance matrix of $\mathbf{s}(t)$. For assumption (A2), $\{s_1(t), \dots, s_m(t)\}$ are uncorrelated, $\mathbf{R}_s(0)$ are diagonal. $\mathbf{R}_x(0)$ is full rank, so it can be diagonalized.

$$\mathbf{R}_x(0) = \mathbf{U} \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n) \mathbf{U}^H \quad (6)$$

where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n]$ is an $n \times n$ matrix. $\mathbf{u}_i, i = 1, \dots, n$ are eigenvectors of $\mathbf{R}_x(0)$. $\lambda_i, i = 1, \dots, n$ are eigenvalues of $\mathbf{R}_x(0)$, $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m > \lambda_{m+1} = \dots = \lambda_n = \sigma_n^2$, and $\lambda_i = \alpha_i + \sigma_n^2$, $\alpha_i, i = 1, \dots, m$ are eigenvalues of $\mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H$.

Source number m can be determined from the multiplicity of the smallest eigenvalue of $\mathbf{R}_x(0)$. According to MDL information theoretic criteria (Wax et al., 1989), value of m is estimated as \hat{m} , where \hat{m} is chosen such that

$$MDL(\hat{m}) = \min\{MDL(0), \dots, MDL(n-1)\} \quad (7)$$

with

$$MDL(k) = -\log L_k^{(n-k)N} + v(k, n)$$

where

$$L_k = \frac{\prod_{i=k+1}^n \lambda_i^{1/(n-k)}}{1/(n-k) \sum_{i=k+1}^n \lambda_i}$$

$$v(k, n) = \frac{1}{2}k(2n-k)\log N$$

The noise variance σ_n^2 can be estimated as

$$\hat{\sigma}_n^2 = \frac{1}{n-m} \sum_{i=m+1}^n \lambda_i \quad (8)$$

and the eigenvalues of $\mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H$ can be estimated as

$$\alpha_i = \lambda_i - \sigma_n^2 \quad (9)$$

4 SOURCES SEPARATION AND ESTIMATION OF MIXING MATRIX

From (5)

$$\begin{aligned} \mathbf{R}_x^H(0)\mathbf{R}_x(0) &= (\mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H + \sigma_n^2\mathbf{I})^H (\mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H + \sigma_n^2\mathbf{I}) \\ &= \mathbf{A}\mathbf{R}_s^H(0)\mathbf{A}^H\mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H + \sigma_n^2\mathbf{I} \\ &= \mathbf{R}_x(0)\mathbf{R}_x^H(0) \end{aligned} \quad (10)$$

Hence, $\mathbf{R}_x(0)$ is a normal matrix. According to matrix theory, there must exist an unitary matrix, which can diagonalize $\mathbf{R}_x(0)$, so \mathbf{U} is also an unitary matrix that satisfies $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Denote $\mathbf{R}_A(0) = \mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H$. $\mathbf{R}_A(0)$ can be diagonalized as

$$\mathbf{R}_A(0) = \mathbf{U}_s \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_m) \mathbf{U}_s^H \quad (11)$$

where $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_m]$.

Define $\mathbf{T} \equiv \text{diag}(\alpha_1^{-1/2}, \alpha_2^{-1/2}, \dots, \alpha_m^{-1/2}) \mathbf{U}_s^H$. Multiply equation (5) by \mathbf{T} from the left, then it can be expressed as

$$\mathbf{y}(t) = \mathbf{B}\mathbf{s}(t) + \mathbf{w}(t) \quad (12)$$

where

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{T}\mathbf{x}(t) \\ \mathbf{B} &= \mathbf{T}\mathbf{A} \\ \mathbf{w}(t) &= \mathbf{T}\mathbf{n}(t) \end{aligned}$$

For $\mathbf{B}^H\mathbf{B} = \mathbf{I}$, \mathbf{B} is an unitary matrix.

$$\begin{aligned} \mathbf{R}_y(\tau) &= E(\mathbf{y}(t)\mathbf{y}^H(t+\tau)) \\ &= E((\mathbf{B}\mathbf{s}(t) + \mathbf{w}(t))(\mathbf{B}\mathbf{s}(t+\tau) + \mathbf{w}(t+\tau))^H) \\ &= E(\mathbf{B}\mathbf{s}(t)\mathbf{s}^H(t+\tau)\mathbf{B}^H) \\ &= \mathbf{B}E(\mathbf{s}(t)\mathbf{s}^H(t+\tau))\mathbf{B}^H \\ &= \mathbf{B}\mathbf{R}_s(\tau)\mathbf{B}^H \end{aligned}$$

For Assumptions (A3), $\mathbf{R}_s(\tau)$ is diagonal. $\mathbf{R}_y(\tau)$ can be obtained from the statistics of observation of $\mathbf{x}(t)$. For $\mathbf{R}_y(\tau)\mathbf{R}_y^H(\tau) = \mathbf{R}_y^H(\tau)\mathbf{R}_y(\tau)$, $\mathbf{R}_y(\tau)$ is a normal matrix and must have an eigen-decomposition of the following form

$$\mathbf{R}_y(\tau) = \mathbf{V}\mathbf{R}_s(\tau)\mathbf{V}^H \quad (13)$$

where \mathbf{V} is an unitary matrix. Although waveform of each column is preserved by unitary transformation, the inherited indeterminacies of blind estimation associated with the magnitude of

sources and the order in which sources are arranged are inevitable. \mathbf{B} and \mathbf{V} are related by

$$\mathbf{B} = \mathbf{V}\mathbf{E}\mathbf{J} \quad (14)$$

where \mathbf{E} is a nonsingular diagonal matrix and \mathbf{J} is a permutation matrix. Substitute $\mathbf{B} = \mathbf{T}\mathbf{A}$, the mixing matrix can be estimated as

$$\hat{\mathbf{A}} = \mathbf{T}^+\mathbf{V}\mathbf{E}\mathbf{J} \quad (15)$$

where superscript $+$ denotes the Moore-Penrose pseudoinverse.

5 LOCALIZATION OF SOURCES

In (15), each element of \mathbf{E} multiplies an arbitrary coefficient to each column of $\hat{\mathbf{A}}$, respectively. \mathbf{J} exchanges the columns of $\hat{\mathbf{A}}$. Without loss of generality, we assume $\mathbf{E} = \mathbf{I}$ and $\mathbf{J} = \mathbf{I}$, that will not affect the estimation result as shown in the following. $\hat{\mathbf{A}}$ can be rewritten as

$$\hat{\mathbf{A}} = \mathbf{T}^+\mathbf{V} \quad (16)$$

Assume that $\hat{\mathbf{a}}_j, j=1, \dots, m$ is the i th column of $\hat{\mathbf{A}}$. As in (6) $\hat{\mathbf{a}}_i = [\hat{a}_{1,j}, \hat{a}_{2,j}, \dots, \hat{a}_{m,j}]$ represents the proportion between fading coefficients of the i th source to each sensor, where the magnitude of $\hat{\mathbf{a}}_j$ represents the proportion between path fading coefficients $p(d_{i,j}), i=1, \dots, n$, and the angle of $\hat{\mathbf{a}}_j$ represents the proportion between Doppler shifts $g(f_{i,j}), i=1, \dots, n$.

A. Location Estimation

$$|\hat{a}_{1,j}| : |\hat{a}_{2,j}| : \dots : |\hat{a}_{m,j}| = p(d_{1,j}) : p(d_{2,j}) : \dots : p(d_{n,j}) \quad (17)$$

where $|\cdot|$ denotes magnitude of a complex number. Introduce a reference coefficient ρ , (18) can be expressed as

$$p(d_{i,j}) = \rho |\hat{a}_{i,j}| \quad i=1, 2, \dots, n \quad (18)$$

Then $d_{i,j}$ can be expressed by the inverse function of $\hat{a}_{i,j}$

$$d_{i,j} = p^{(-1)}(\rho |\hat{a}_{i,j}|) \quad (19)$$

Substitute $d_{i,j}$ by coordinate of source, we can get

$$(\hat{x}_j - \bar{x}_i)^2 + (\hat{y}_j - \bar{y}_i)^2 = (p^{(-1)}(\rho|\hat{a}_{i,j}|))^2 \quad (20)$$

where $[\hat{x}_j, \hat{y}_j]$, $j=1,2,\dots,m$ are the coordinates of the j th source, and $[\bar{x}_i, \bar{y}_i]$, $i=1,2,\dots,n$ are the coordinates of the i th sensor.

By substituting $i=1,2,\dots,n$ into (21), and subtracting each other, we have that

$$\begin{aligned} & \hat{x}_j(\bar{x}_{i+1} - \bar{x}_i) + \hat{y}_j(\bar{y}_{i+1} - \bar{y}_i) \\ &= \frac{1}{2} \left(\bar{x}_{i+1}^2 - \bar{x}_i^2 + \bar{y}_{i+1}^2 - \bar{y}_i^2 + (p^{(-1)}(\rho|\hat{a}_{i+1,j}|))^2 - (p^{(-1)}(\rho|\hat{a}_{i,j}|))^2 \right) \\ & i=1,2,\dots,n-1 \end{aligned}$$

The above equations can be expressed as

$$\mathbf{C}_j [\hat{x}_j, \hat{y}_j]^T = \mathbf{e}_j \quad j=1,2,\dots,m \quad (21)$$

$$\text{where, } \mathbf{C}_j = \begin{bmatrix} \bar{x}_2 - \bar{x}_1, \bar{y}_2 - \bar{y}_1 \\ \vdots \\ \bar{x}_n - \bar{x}_{n-1}, \bar{y}_n - \bar{y}_{n-1} \end{bmatrix}$$

$$\mathbf{e}_j = \frac{1}{2} \begin{bmatrix} \bar{x}_{i+1}^2 - \bar{x}_i^2 + \bar{y}_{i+1}^2 - \bar{y}_i^2 + (p^{(-1)}(\rho|\hat{a}_{2,j}|))^2 - (p^{(-1)}(\rho|\hat{a}_{1,j}|))^2 \\ \vdots \\ \bar{x}_n^2 - \bar{x}_{n-1}^2 + \bar{y}_n^2 - \bar{y}_{n-1}^2 + (p^{(-1)}(\rho|\hat{a}_{n,j}|))^2 - (p^{(-1)}(\rho|\hat{a}_{n-1,j}|))^2 \end{bmatrix}$$

The least squares estimate of source position can be given as

$$[\hat{x}_j, \hat{y}_j]^T = \mathbf{C}_j^+ \mathbf{e}_j \quad j=1,2,\dots,m \quad (22)$$

Introduce (22) to (20), ρ can be solved. Return ρ to (22), the source position is obtained.

6 CONCLUSIONS

Path fading is introduced to model the multiple target network. Based on blind estimation, a range free multiple target localization algorithm was presented. This development is especially applicable in fast, time-varying environments, where multiple targets maneuver quickly and randomly.

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