IRREGULAR PLACEMENT PROBLEM
Solved with a 2-Level Algorithm and Collision Free Region

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Abstract: The two-dimensional irregular open dimension packing problem is a combinatorial optimization problem that searches a layout for a given set of irregular items within a rectangular container so that no item overlaps with other items or protrudes from the container, where each irregular item is not necessarily convex. The container has a fixed width, while its length can change so that all items are placed in it. The objective is to find a layout of the set of polygons that minimizes the length of the container. The proposed algorithm constructively creates layouts from an ordered list of items and a placement heuristic. The placement determines the collision free region (represents the set of translations to create a feasible layout) for the item to be placed. It is shown that the collision free region must be calculated using non-regularized Boolean operations, as contours of no-fit polygons should be ignored. The proposed algorithm to solve the placement problem has two levels, in the internal level the container has fixed dimensions, and the external level reduces or increases the variable dimension. The placement heuristic searches for degenerated vertices and edges as they represent local maximum compaction. Computational comparisons on benchmark problems show that the proposed algorithm generated highly competitive solutions. Moreover, our algorithm updated some best known results.

1 INTRODUCTION

This paper is related to the classic problem of finding the most efficient layout for a set of irregular items out of a given container with the objective of minimizing the wasted material. There are numerous applications for this problem, including wood, textile, sheet metal, plastics, and glass industries. Wäscher et al. (2007) proposed a typology that classifies the packing problems based on size, objective, assortment of items and container type. According to this typology, this work investigates the two-dimensional irregular open dimension problem (ODP), in which the items can be represented by irregular polygons and the container is rectangular with an variable dimension. The objective is to find the smallest container that fits all items inside in such way that no item overlaps.

Although several strategies to search and represent the solutions exist in the literatures, they can be divided in two main categories: the first represents the solution as an ordered list of items and applies placement rules to construct a feasible layout, i. e., layouts where no item overlaps or protrudes from the container (Burke et al., 2007). The second considers a feasible layout on the container and attempts to move the items inside the container, overlapped items are permitted, and a penalty method is used to reach feasible layouts (Egeblad et al., 2007). The problem is NP-complete (Fowler et al., 1981) and, consequently, the searching strategies usually adopt heuristics. In this work, a simulated annealing (SA), which was originally proposed as an combinatorial optimization algorithm, is used in combination with a placement heuristic.

This work is structured as follows. Section 2 explains three basic concepts: no-fit polygon (NFP), inner-fit polygon (IFP) and collision free region (CFR). Section 3 explains that the correct and robust determination of CFRs requires non-regularized Boolean operations, as contours of NFPs should be ignored . Section 4 presents the proposed algorithm with two levels, in the internal level the container has fixed dimensions, and the external level reduces or increases the variable dimension. The placement heuristic searches for exactly fitting positions, as they usually represent local maximum compaction. Section 5 presents some results and the conclusions are in section 6.

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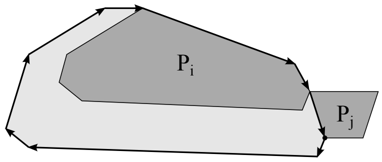


Figure 1: No-fit polygon (contour with oriented edges) induced by the item P_i to item P_j. The reference point is the bottom-left vertex of the item P_j.

2 BASIC CONCEPTS

When irregular items are involved an extra dimension of complexity is generated by the geometry. More precisely, there is a need to determine whether two items touch, overlap or are separated. There exists a number of solutions to this problem ranging from simple to complex, these include raster method, direct trigonometry and NFP. In this work, the NFP is used to evaluate all the feasible positions for the placement of a new item.

The NFP alone has limited utility in cutting and packing problems. Some recent works are using the CFR to determine feasible layouts in containers with fixed dimensions. Martins and Tsuzuki (2009, 2010) proposed a constructive heuristic where the CFR represents feasible placements for a moveable item considering the already placed items and the container. This section describes how the CFR is determined using NFPs and IFPs.

2.1 No-Fit Polygon

The NFP represents translations applied to items and is mathematically represented by a set of vectors. It represents the set of forbidden translations that, when applied to a moveable item, intersects with a fixed item. The NFP is induced by the fixed item to the movable item. An example of a no-fit polygon is represented in Fig. 1. If the reference point is placed at the interior of the NFP then the moveable item collides with the fixed item. On the other hand, if the reference point is placed at the boundary of the NFP then the moveable and fixed items touch. For an item P, let $i(P)$ be its interior, ∂P be its boundary and $c(P)$ be its complement.

Definition 2.1. *Consider the fixed item P_i and the moveable item P_j. The NFP induced by item P_i to item P_j, denoted by $\Upsilon(P_i,P_j)$, is the set of translation vectors applied to P_j that leads it to a collision with P_i. Thus,*

$$\begin{aligned}\Upsilon(P_i,P_j) &= i(P_i)\ominus i(P_j) \\ &= \left\{\vec{\mathbf{v}}\,|\,\exists \boldsymbol{a}\in i(P_j), \boldsymbol{a}+\vec{\mathbf{v}}\in i(P_i)\right\}.\end{aligned} \tag{1}$$

Numerous algorithms to determine the NFP were proposed. Mahadevan (1984) developed an algorithm based on a sliding scheme. However, more efficient algorithms based on Minkowski sums exist (Burke et al., 2007; Bennell and Song, 2008). It is possible to obtain the NFPs using Minkowski sums and the opposed polygon.

Definition 2.2. *The Minkowski sum of two polygons P_i and P_j, noted $P_i \oplus P_j$, is defined as the set of points $\left\{\boldsymbol{O}+\vec{\mathbf{v}}+\vec{\mathbf{w}}\,|\,\boldsymbol{O}+\vec{\mathbf{v}}\in P_i, \boldsymbol{O}+\vec{\mathbf{w}}\in P_j\right\}$.*

Definition 2.3. *The opposed polygon for a given polygon P_j, denoted by $-P_j$, is defined as the set of points $-P_j = \left\{\boldsymbol{O}-\vec{\mathbf{w}}\,|\,\boldsymbol{O}+\vec{\mathbf{w}}\in P_j\right\}$.*

From the above definitions, one can see that

$$i(P_i)\ominus i(P_j) = i(P_i)\oplus(-i(P_j)) \tag{2}$$

meaning that the NFP is produced by the Minkowski sum of the fixed item with the opposed item to be placed. This algorithm is very efficient for convex polygons. The result of a Minkowski sum of two convex polygons is a convex polygon built from the original polygons edges ordered counterclockwise. Non-convex polygons can be decomposed into convex polygons, as the applied transformations (rotation and translation) do not affect such decomposition.

2.2 Inner-Fit Polygon

The IFP is a concept derived from the NFP and it represents the set of translations that places an item inside a container (Dowsland et al., 2002). The IFP can be obtained by sliding the item along the internal contour of the container. (see Fig. 2).

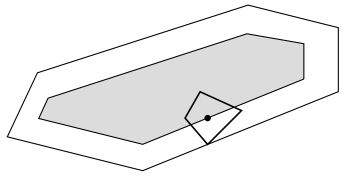


Figure 2: Inner-fit polygon (gray polygon) induced by container to item.

Definition 2.4. *The IFP induced by container $\mathcal{C}$ to item P_j, denoted by $\Lambda(\mathcal{C},P_j)$, is the set of translation vectors applied to P_j that leads it to be inside the container. Thus,*

$$\begin{aligned}\Lambda(\mathcal{C},P_j) &= c(c(\mathcal{C})\oplus(-i(P_j))) \\ &= \left\{\vec{\mathbf{v}}\,|\,\forall \boldsymbol{a}\in i(P_j), \boldsymbol{a}+\vec{\mathbf{v}}\in\mathcal{C}\right\}.\end{aligned} \tag{3}$$

2.3 Collision Free Region

Consider a container $\mathcal{C}$ and a set of placed items $\mathcal{P}=\{P_1,\cdots,P_n\}$, with no collision and totally inside the container. A new item P_m, $m > n$ will be placed in the container, keeping the layout feasible, i.e. no items collide or protrude from the container. The CFR represents such a set of translations for item P_m (see Fig. 3).

Definition 2.5. *The CFR is the set of all translations, that, when applied to a specific item, place this item in the interior of the container without colliding with the already placed items.*

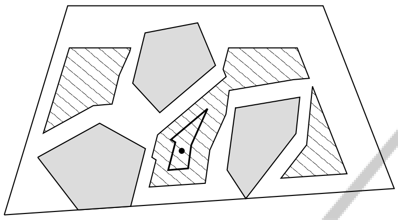


Figure 3: The CFR is filled with a hatch pattern. The item to be placed is not filled and the already placed items are filled with gray.

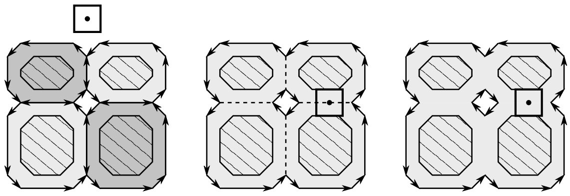
When there are no placed items, the CFR is the IFP. The first step to determine the CFR is to obtain the corresponding IFP. The next step is to subtract the NFPs induced by the placed items. The CFR, denoted $\Pi(\mathcal{C},\mathcal{P},P_m)$, can be determined using the expression:

$$\Pi(\mathcal{C},\mathcal{P},P_m) = \Lambda(\mathcal{C},P_m) - \bigcup_{P_i\in\mathcal{P}} i(P_i) \ominus i(P_m). \quad (4)$$

3 NON-REGULARIZED BOOLEAN OPERATIONS

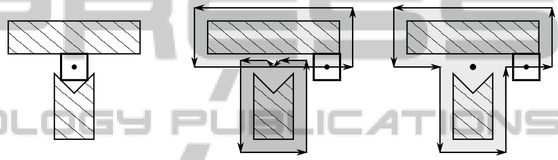
As shown in the previous section, the determination of the CFR requires the use of Boolean operations over polygons. However, those Boolean operations cannot be regularized as they assume that a polygon contains its boundary. The CFR is determined through Boolean operations applied to NFPs and IFPs. As NFPs represent forbidden translations, their boundaries represent allowable translations. The NFP's boundaries represent translations that place the moveable item touching the fixed item, which is not forbidden. Then, for the determination of the CFR, it is necessary to use non-regularized Boolean operations.

Fig. 4 shows an example where four items are already placed and the moveable item can be placed between the already placed items. The reference point of the moveable item can be placed at the common boundaries of the original NFPs shown in Fig. 4.(a). The union of all NFPs must result in a polygon with internal edges (degenerated edges) as shown in Fig. 4.(b). However, when regularized Boolean operations are used, the final result is shown in Fig. 4.(c) without the internal edges. Fig. 5 shows a similar situation where the the union of all original NFPs must result in a polygon with one internal vertex (degenerated vertex).



(a) (b) (c)

Figure 4: (a) On top, the moveable item with a central reference point. On the bottom, four fixed items (filled with a hatch pattern) and the four induced NFPs represented by its oriented contours. (b) CFR, with its four degenerated edges shown (dashed lines). (c) Result of the regularized union of all the NFPs, which does not have degenerated edges.

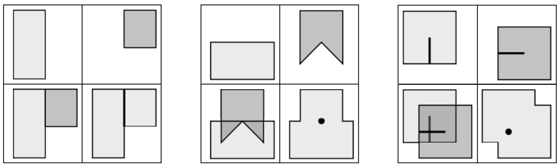


(a) (b) (c)

Figure 5: (a) Desired placement of a rectangular item with a central reference point. Fixed items are represented by hatched polygons. (b) The two NFPs represented by its oriented contours. (c) CFR, with its degenerated vertex shown (internal vertex).

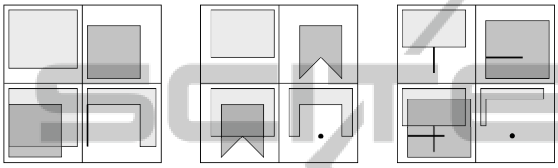
The result of non-regularized Boolean operation may have edges or vertices that do not belong to the boundary. They are called degenerated edges and degenerated vertices. When not specified, the edges or vertices are part of the contour. When moveable items are placed on degenerated vertices or edges usually implies in a more compact layout. This affirmation is discussed in subsection 4.1.

In this work, new non-regularized Boolean operations were developed. Generally, regularized Boolean operations are implemented in three steps (Vatti, 1992): edge intersection determination, edge and contour labeling and edge and contour collection. The new algorithm developed in this work has the same three steps, and the main differences are in edge and contour labeling and edge and contour collection. They are associated with the creation and collection of degenerated edges and vertices. As equation (4) uses exclusively unions and subtractions, the intersection was not implemented. Degenerated edges and vertices are created in situations where they exist in the results but not in the input polygons, as can be seen on Fig. 6 (union) and Fig. 7 (subtraction). The collection of degenerated edges and vertices is a decision to maintain or eliminate them. The developed algorithm consists of a robust implementation, high performance and works with finite precision (Sato et al., 2010).



(a) (b) (c)

Figure 6: Three cases of degenerated elements generation for non-regularized Boolean union operation. Consider $A \cup B$ operation. Top left: polygon A. Top right: polygon B. Bottom left: superposition. Bottom right: result.



(a) (b) (c)

Figure 7: Three cases of degenerated elements generation for non-regularized Boolean subtraction operation. Consider $A - B$ operation. Top left: polygon A. Top right: polygon B. Bottom left: superposition. Bottom right: result.

4 PROPOSED APPROACH

The proposed algorithm searches for layouts for a given set of irregular items out of a given rectangular container with an open dimension such that this dimension is minimized. Each irregular item can be rotated by a finite set of angles. The irregular items can be any polygon, convex or non-convex and may contain holes.

To solve this problem, a two level algorithm was developed (see Fig. 8). The internal level considers the container with fixed dimension. SA is used to control the sequence, orientation and position of each item. The objective function is the wasted space of the container. The global convergence condition happens when, for a given temperature, the wasted space of the container is exactly the same in all tries and they are equal to the best already found wasted value. The local convergence condition is a predefined maximum number of iterations. When considering a container width fixed dimensions, the wasted space can only assume a set of discrete values (Martins and Tsuzuki, 2009, 2010).

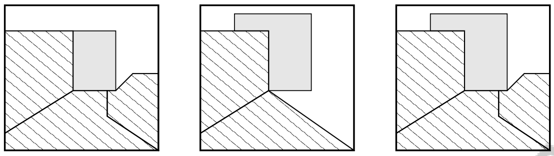
The external level controls the value of the open dimension and the initial temperature of the SA. Once the internal level ends, a feasible layout with all irregular items placed is found, the external level shrinks the container and resets the SA. When no solution with all items placed is found, the open dimension of the container is increased and the internal level is restarted. For the external level, two parameters p_{dec} and p_{inc} are used to control, respectively, the shrinkage and expansion of the container.

$\mathbf{x} \leftarrow$ <Initial random solution>
$T_0 \leftarrow$ <Initial temperature>
$L \leftarrow$ <Initial container dimension>
while <*Not finished*> **do**
  while <*Global stop condition not satisfied*> **do**
    $T_i \leftarrow T_i * \alpha$; $i \leftarrow i + 1$
    while <*Local stop condition not satisfied*> **do**
      $val \leftarrow random(0,1)$
      if $val < 0.5$ **then**
        $\mathbf{x}^* \leftarrow$<Modify placement sequence>
      else
        <Select the movable item>
        $\mathbf{x}^* \leftarrow$<Select a vertex using adopted placement heuristic>
      $\Delta E = F(\mathbf{x}^*) - F(\mathbf{x})$
      if $\Delta E < 0$ **then**
        $\mathbf{x} \leftarrow \mathbf{x}^*$
      else
        if $random(0,1) < e^{-\Delta E/kT}$ **then**
        $\mathbf{x} \leftarrow \mathbf{x}^*$
      if <*All items are placed inside the container*> **then**
        $L \leftarrow (1 - p_{dec})L$
        <Container dimension>$\leftarrow L$
        <Global and local stop condition satisfied>
  if <*At least one of the items is not placed inside the container*> **then**
    $L \leftarrow (1 + p_{inc})L$
    <Container dimension>$\leftarrow L$
    $i \leftarrow 0$; $T_0 \leftarrow$ <Initial temperature>

Figure 8: Proposed Algorithm.

4.1 Placement Heuristic

In this work, a placement heuristic that gives priority to exactly fitting position is used. When an exactly fitting placement exists then the moveable item touches the fixed item in at least two different locations. The placement of a new item in a exactly fitting position often represents a local compaction. The main motivation of this work is that degenerated edges and vertices represent exactly fitting positions. In the proposed algorithm, placement in these situations have higher priority. Another placement heuristic of interest is at convex vertices from the CFR. Fig. 9 shows examples of moveable items placed at convex vertices.



(a) (b) (c)

Figure 9: Placement on a concave vertex of the CFR. Fixed items are represented by polygons filed with hatch pattern and the movable item by a gray filled polygon.

The placement always occurs at vertices from the CFR. Previous works showed that this placement helps the algorithm finding a solution in smaller time without compromising the quality of the final solution. However, there are special cases where the algorithm is not capable to find the optimal solution just by placing new items exclusively at CFR's vertices.

The adopted placement heuristics have the following order of priority: degenerated vertex, degenerated edge, convex contour vertex. If the CFR has more than one degenerated vertex, the algorithm chooses one randomly. If no degenerated vertex are found, then the placement should occur in one degenerated edge's vertex, randomly chosen. If no degenerated elements exists, one convex vertex from the boundary randomly chosen is selected, with no priority order. The proposed algorithm is shown in Fig. 8.

5 RESULTS

The proposed algorithm was tested with 13 benchmark data sets, found on ESICUP's (EURO Special Interest Group on Cutting and Packing) website[1]. These sets are irregular strip packing problems with the objective of minimizing the length of the container with a fixed width. The objective was to find the minimum length for the container such that all items fit inside the container and they do not overlap. The irregular items have up to four possible orientations (0°, 90°, 180° or 270°). All tests were executed on a i7 860 processor with 4GB RAM.

The adopted external level parameters p_{dec} and p_{inc} were 1% e $0,3$% respectively. Table 1 shows the minimum length and the density of the most compact layout found by the algorithm. Fig. 10 show the optimal layouts found by the proposed algorithm. The Albano, Jakobs and Marques layouts found are the best results published in the literature. As for the Jakobs1 set, it has the same density as the one published by (Egeblad et al., 2007). The algorithm was also capable of achieving 100% density for the problems Dighe1 and Dighe2.

Table 1: Results from Benchmark Data Sets. Minimum length and density (%) are specified.. I: (Imamichi et al., 2009). B: (Bennell and Song, 2008). E: (Egeblad et al., 2007) G: (Gomes and Oliveira, 2006). The data sets with * are the best results in the literature.

Case	Proposed	Best
Alb	**9848.72 (88.39)***	9905.94 (88.16) (I)
Da	57.82 (87.71)	**57.63 (87.87)*** (B)
D1	**1000 (100)***	**1000.00 (100)*** (BG)
D2	**1000 (100)***	**1000.00 (100)*** (BG)
Fu	30.99 (91.96)	**30.97 (92.03)*** (E)
J1	**11.00 (89.07)***	**11.00 (89.07)*** (E)
J2	**22.75 (84.83)***	23.39 (82.51) (I)
Mao	1753.20 (84.07)	**1731.26 (85.15)*** (E)
Mar	**76.85 (90.01)***	77.04 (89.82) (E)
S0	59.03 (67.59)	**58.30 (68.44)*** (E)
S1	55.51 (71.88)	**54.04 (73.84)*** (EI)
S2	25.93 (83.30)	**25.64 (84.25)*** (I)
Sh	61.65 (87.59)	**60.18 (89.69)*** (B)
Tr	241.83 (90.07)	**241.23 (90.46)*** (E)

6 CONCLUSIONS

Using the proposed non regularized Boolean operations algorithm, it was possible to determine degenerated edges and vertices. A new placement heuristic was proposed considering the importance of placement at degenerated elements. A two-level algorithm was proposed to solve the open dimension problem. The results from the benchmark data sets from literature showed to be very competitive, finding in some cases the best result in the literature.

The proposed algorithm has two parameters: the ratio by which the container is shortened, and the ratio by which the container is increased. The tests were conducted using constant ratios. However, we think that it is possible to control the ratios through an algorithm. This was left as future work.

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[1]http://paginas.fe.up.pt/~esicup/tiki-index.php.

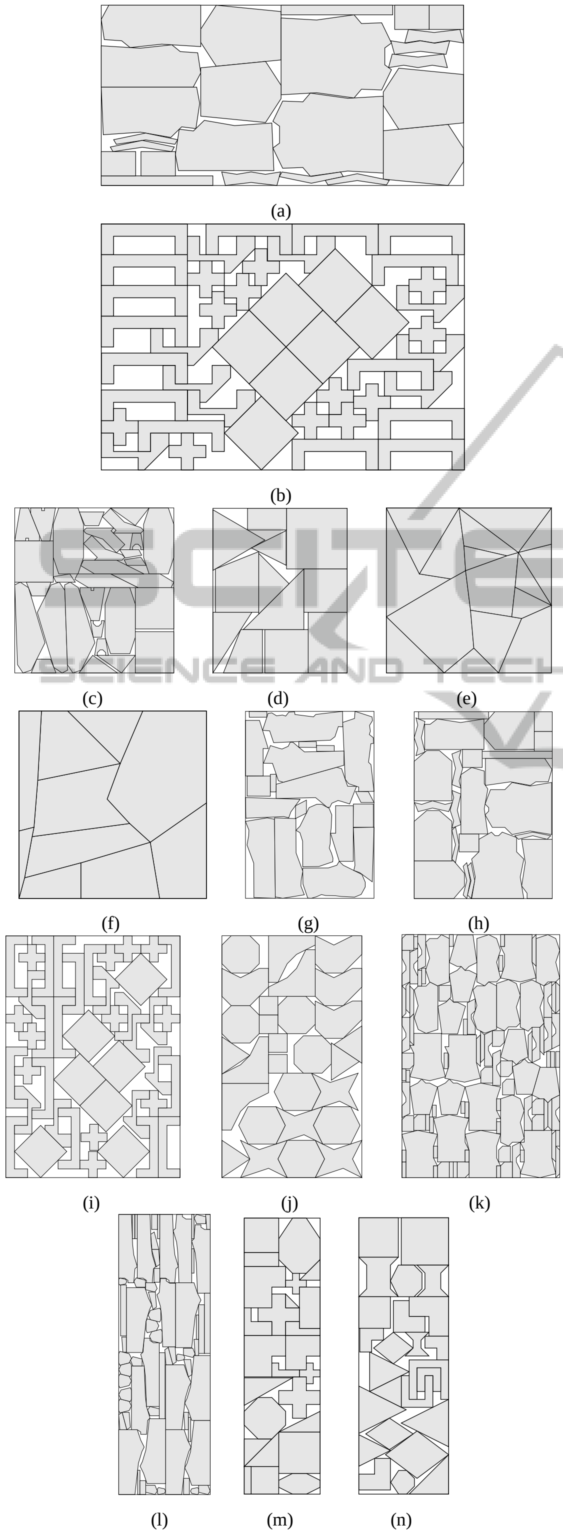

Figure 10: The best solutions obtained by the proposed algorithm. (a) albano. (b) shapes0. (c) dagli. (d) fu. (e) dighe1. (f) dighe2. (g) mao. (h) marques. (i) shapes1. (j) shapes2. (k) shirts. (l) trousers. (m) jakobs1. (n) jakobs2.

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REFERENCES

Bennell, J. A. and Song, X. (2008). A comprehensive and robust procedure for obtaining the no-fit polygon using minkowski sums. *Eur J Oper Res*, 35:267–281.

Burke, E. K., Hellier, R. S. R., Kendall, G., and Whitwell, G. (2007). Complete and robust no–fit polygon generation for the irregular stock cutting problem. *Eur J Oper Res*, 179:27–49.

Dowsland, K. A., Vaid, S., and Dowsland, B. W. (2002). An algorithm for polygon placement using a bottom–left strategy. *Eur J Oper Res*, 141:371–381.

Egeblad, J., Nielsen, B. K., and Odgaard, A. (2007). Fast neighborhood search for two- and three–dimensional nesting problems. *Eur J Oper Res*, 183:1249–1266.

Fowler, R. J., Paterson, M., and Tanimoto, S. L. (1981). Optimal packing and covering in the plane are np-complete. *Inform Process Lett*, 12(3):133–137.

Gomes, A. M. and Oliveira, J. F. (2006). Solving irregular strip packing problems by hybridising simulated annealing and linear programming. *Eur J Oper Res*, 171:811–829.

Imamichi, T., Yagiura, M., and Nagamochi, H. (2009). An iterated local search algorithm based on nonlinear programming for the irregular strip packing problem. *Discrete Optim*, 6:345–361.

Mahadevan, A. (1984). *Optimization in computer-aided pattern packing (marking, envelopes)*. PhD thesis, North Carolina State University.

Martins, T. C. and Tsuzuki, M. S. G. (2009). Placement over containers with fixed dimensions solved with adaptive neighborhood simulated annealing. *B Pol Acad Sci Techn Sci*, 57:273–280.

Martins, T. C. and Tsuzuki, M. S. G. (2010). Simulated annealing applied to the irregular rotational placement of shapes over containers with fixed dimensions. *Expert Syst Appl*, 37:1955–1972.

Sato, A. K., Takimoto, R. Y., Martins, T. C., and Tsuzuki, M. S. G. (2010). Translational placement using simulated annealing and collision free region with parallel processing. *Proc 9th IEEE/IAS INDUSCON*.

Vatti, B. R. (1992). A generic solution to polygon clipping. *CACM*, 35:56–63.

Wäscher, G., Haussner, H., and Schumann, H. (2007). An improved typology of cutting and packing problems. *Eur J Oper Res*, 183:1109–1130.