STOCHASTIC MODELLING IN HEALTHCARE SYSTEMS

Srinivas R. Chakravarthy
Department of Industrial and Manufacturing Engineering, Kettering University, MI-48504, Flint, U.S.A.

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Abstract: We are at a point in time where healthcare (in USA) is getting more attention from law makers, government agencies, doctors, hospitals, pharmaceutical companies, and population at large. The costs for healthcare have been steadily growing. The healthcare system offers challenging and interesting opportunities for operations researchers from both theoretical and practical points of view. This paper is an attempt to use simulation as a tool to study a healthcare system at a macroscopic level.

1 INTRODUCTION

With baby-boomers growing at a faster rate and the required workforce (to support the existing older people and the boomers) dwindling in size, the United States healthcare system (HCS) is receiving attention at every level. The country is facing multi-dimensional problems with regards to HCS. On one hand the worry is to make sure that everyone living in this country has an affordable health insurance. For way too long this has been largely ignored in spite of the constant exposure of this problem. Obviously, one of the main reasons is the cost associated with making everyone insured. Hence, insured people as well as the governmental (both local and federal) agencies have been putting up the bill on the uninsured. While some are uninsured due to their own choice, majority of them cannot afford to pay for their insurance. With the current economic condition the problem is even more exacerbated. On the other hand, HCS has so much waste (Thomson Reuters, 2009, Washington Post, 2009) that a small percentage of the savings will pay for the costs associated with the uninsured patients. In fact, if done properly the overall costs can be significantly brought down. For example, identifying the areas of wastage, underutilized resources, and needing significant improvement, will help this cause.

According to a white paper published by Thomson Reuters, 2009, the U.S. HCS wastes between $600 billion and $850 billion annually. This is about one-third of the nation's healthcare bill. This report identifies a number of categories (in broader terms) where wastage occurs (see Figure 1 below). About 40% of the wastage is estimated under the "unnecessary care" category.

This is defined as "Unwarranted treatment, such as the over-use of antibiotics and the use of diagnostic lab tests to protect against malpractice exposure, accounts for $250 billion to $325 billion in annual healthcare spending."

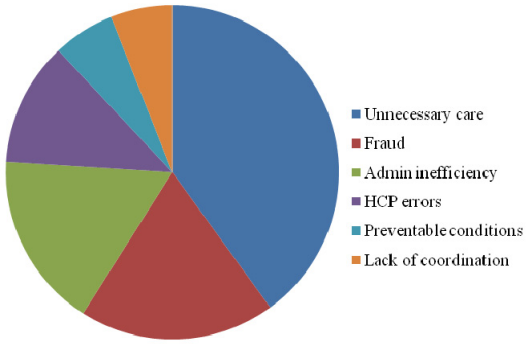

Figure 1: Percentage of waste in US HCS.

One of the items mentioned in the 40% category is the "use of diagnostic lab tests". Diagnostic labs cover a wide range of labs such as blood test, X-rays, MRI, and Cardio. In an ideal world a patient requiring any type of lab test should be able to get it without having to wait for excessively long period of time. But as we all know this is not the case for a variety of reasons. These (not necessarily in any order) are: (a) lack of resources; (b) improper allocation of resources; (c) scheduling of patients; and (d) queuing delays due to unexpected arrivals.

R. Chakravarthy S..
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We are in an era where the competition is so strong. As technology grows exponentially, both the patients and the doctors would like to use the technology to cure the patients efficiently and in a short period of time. Hence, a HCS would like to attract and enrol more patients and the doctors to its organization. One of the ways of doing this is to provide "quality" service at an affordable price and still make profit. Note that a typical HCS consists of many (major) groups such as (a) administrative; (b) doctors who serve the patients insured through the system; (c) hospitals that serve the patients covered by the system and (d) laboratories that serve the hospitals, doctors, and patients. It should be noted that there may be interactions between these groups. For example, some hospitals have their own laboratories which serve the patients admitted to the hospital either as in-patients or out-patients. Most doctors visit hospitals to take care of not only their own patients but also other patients who have been admitted. We will not model that aspect in this paper.

The study of such systems in which most of the underlying variables are random falls in the area of stochastic modelling and one can avail the tools therein to study healthcare models. However, tracking analytically the system performance measures such as the mean waiting times and the utilization factors is almost infeasible due to inherent complexities and the significant interactions that are present among various segments of a healthcare system. An alternative approach to analytical modelling is through simulation. While simulation in healthcare has evolved at a slow pace (as compared to other non-healthcare systems such as manufacturing and telecommunications) over the last three decades or so (see e.g., Jun, et al., 1999, Baldwin, et al., 2004), it is recognized as an important tool in solving problems arising in healthcare systems. Most of the published papers dealing with simulation in healthcare systems focus on sub-systems such as emergency room, outpatient clinics, etc. (Eldabi, et al., 2010, Gunal and Pidd, 2010). For latest developments in simulation and its application to many fields including healthcare systems we refer the reader to journals such as Simulation Modelling Practice and Theory, Simulation in Healthcare, Journal of Simulation, and Proceedings of Winter Simulation Conference.

Thus, the objective of this paper is to provide insights into how stochastic modelling can be applied to a typical HCS at a macroscopic level so as to help the management with aggregate planning. It is our intent here to focus on the use of simulation to identify bottlenecks that cause excessive delays in patients receiving service, and areas of under utilization of the resources, by looking at a HCS at a macroscopic level. More specific ones (within this type of HCS) requiring microscopic level modelling will be addressed elsewhere.

2 MODEL DESCRIPTION

As indicated earlier a HCS is plagued with wastage, underutilized resources, and excessive delays experienced by patients. Whether one is dealing with handling patients to go through diagnostic labs or to process paperwork before (and after) the patients go through diagnostics or the doctors to notify the patients of the results, or the patients need to be admitted in the hospitals, or the patients requiring operations need to wait for beds, equipments, personnel, delays are inevitable due to available finite resources and the way they are allocated among competing service providers. These delays are further compounded by inherent randomness. For example, the arrivals of the patients to service providers are not deterministic. Different classes of patients arrive and they have to be attended based on their priorities. These are also random. The service times are usually random and some patients may have to go through the same service more than once for reasons that cannot be anticipated. Thus, a natural approach to solving such problems is the use of stochastic modelling. While analytical modelling is important, there are instances such as the current study where one has to rely on simulation due to complex nature of the model.

Patients are the central focus in any HCS and so we start with assuming that patients arrive to a HCS according to a Markovian arrival process (MAP) with representation (D_0, D_1) of order m. Note that the transitions corresponding to no arrivals are governed by D_0 and the transitions corresponding to arrivals are governed by D_1. The underlying continuous-time Markov chain (CTMC) has the generator given by $Q = D_0+D_1$. This representation of MAP is a special case of batch Markovian arrival process (BMAP). This BMAP was originally introduced by Neuts (1979) as a versatile Markovian point process in 1979. MAP, a very versatile point process used extensively in stochastic modelling, includes several well-known point processes such as Poisson, Erlang, and hyperexponential. For full details on MAP and its applications to stochastic models we refer to (Lucantoni, 1991, Chakravarthy, 2001, Chakravarthy, 2010). The fundamental rate (the rate of arrivals per unit of time), λ, is given by

$\lambda = \pi D_1 \mathrm{e}$, where π is the steady state probability vector of the generator Q governing the underlying CTMC satisfying $\pi Q = 0$, $\pi \boldsymbol{e} = 1$ and where $\boldsymbol{e}$ is a column vector of 1's with dimension m.

While one can model the arrivals of different priority type patients to follow independent MAPs, we choose here to model the arrivals to be dependent on each other generated by a common MAP with an associated probability vector. However, it is easy to modify our model to accommodate any variation to the current one. Also, the idea of using MAP to model patient arrivals is to incorporate inherent correlation present in the inter-arrival times of patients.

Note that the patients in any HCS require different types of services. Thus, the patients are classified based on their service requirements that range from a simple administrative query to a more serious one requiring key resources such as doctors, labs, etc. We assume that a HCS under consideration has N groups and that with probability p_i, $1 \leq i \leq N$, an arriving patient belongs to group i and let $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$. We will, henceforth, refer to them as patients of type i. Type i patients have to go through K_i stages of servicing. This is again typical of a HCS. For example, a patient admitted into a hospital has to go through registration, triage, examination room, etc. Also, patients who call administrative people for any query related to billing, office visits, and other activities go through various stages before hanging up the phone.

We assume that the service times of patients of type i, $1 \leq i \leq N$, need to go through K_i stages and in each stage the time required to process the patients is of phase type. (A phase type distribution (PH-distribution) is obtained as the time until absorption in a finite state continuous time Markov chain with n transient states and one absorbing states. Thus, a PH distribution is represented by $(\boldsymbol{\beta}, S)$ of order n. PH-distributions include well-known distributions such as exponential, (generalized) Erlang, and hyperexponentials as very special cases (Neuts, 1995).

These stages represent the patients going through admission process, filling necessary paperwork, triage, etc. It is possible for some patients to seek direct or indirect services from another group after getting serviced in the group they entered. For example, patients getting into hospitals may have to seek administrative help for follow up paperwork or examination/billing details. Some may opt to ask at a later point in time (in which case we can treat them as new arrivals to the system) or as part of their current visit. We model this scenario by specifying the routing mechanism. We also put a restriction that a patient may not seek services from more than two groups (including the one that was entered). This is not only to mimic the most practical situations but also to avoid patients cycling through many groups more than once. This one requires more book-keeping. A pictorial description of this model is displayed in Figure 2.

Thus, the processing time of a priority i patient in stage j, $1 \leq j \leq K_i$, $1 \leq i \leq N$, is assumed to be of phase type with representation $(\boldsymbol{\beta}(i,j), S(i, j))$ of order n_{ij}.

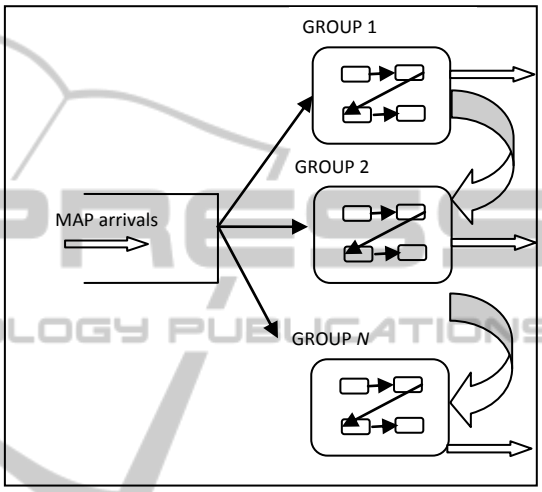

Figure 2: A typical HCS.

By keeping track of the phase of the arrival process, the number of type i patients in the system, and the phase of the services in various stages, one can study the model under consideration using Markov chain theory and algorithmic methods (Neuts, 1989, 1995). However, the state space for the model grows exponentially and the book-keeping is very involved. Furthermore, the computations of the distributions of the waiting times in the system of patients are very complicated to describe analytically. Thus, we will use simulation to study our model. We have chosen ARENA to simulate the model under study.

2.1 Simulation with ARENA

In this section we will outline how ARENA is used to simulate the HCS under study. The following assumptions are made in developing the model in ARENA.

(a) The number of groups and the number of stages within each group are as follows:

$N = 4$, $K_1 = 2$, $K_2 = 2$, $K_3 = 3$, and $K_4 = 5$.

(b) Type 1 patients go through both their stages and then leave the system after getting the services.

(c) Type 2 patients go through both stages before leaving the system.
(d) Type 3 patients go through one of the three sequences: (i) Stage 1 to Stage 2 to Stage 3; (ii) Stage 1 to Stage 2; (iii) Stage 1 to Stage 3; according to a probability vector, say, $\boldsymbol{p_2} = (p_{21}, p_{22}, p_{23})$ before leaving the system.
(e) Type 4 patients go through one of the five sequences: (i) Stage 1 to Stage 2 to Stage 3 to Stage 4 to Stage 5; (ii) Stage 1 to Stage 2 to Stage 5; (iii) Stage 1 to Stage 5; (iv) Stage 1 to Stage 2 to Stage 4 to Stage 5; and (v) Stage 1 to Stage 2 to Stage 3 to Stage 5; according to a probability vector, say, $\boldsymbol{p_3} = (p_{31}, p_{32}, p_{33}, p_{34}, p_{35})$ before leaving the system.
(f) Patients of types 2, 3, and 4 create additional work for servers in Stage 2 of Group 1 when they leave the system. These have a lower priority as compared to Type 1 patients.

3 ILLUSTRATIVE EXAMPLE

For our illustrative example, we consider five different arrival processes and different service time distributions. The five arrival processes with parameter matrices D_0 and D_1 are as follows. The base time units are taken to be minutes.

EXPA: Exponential: $D_0 = -1$ and $D_1 = 1$.

ERLA: Erlang of order 5

$$D_0 = \begin{bmatrix} -5 & 5 & 0 & 0 & 0 \\ 0 & -5 & 5 & 0 & 0 \\ 0 & 0 & -5 & 5 & 0 \\ 0 & 0 & 0 & -5 & 5 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

HEXA: Hyperexponential: This is the mixture of two exponential with mixing probabilities 0.9 and 0.1, and with parameters 1.9 and 0.19. Here

$$D_0 = \begin{bmatrix} -1.9 & 0 \\ 0 & -0.19 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -1.710 & 0.190 \\ 0.171 & 0.019 \end{bmatrix}.$$

MNCA: MAP with negatively correlated arrivals: Here we take D_0 and D_1 to be

$$D_0 = \begin{bmatrix} -1.00222 & 1.00222 & 0 \\ 0 & -1.00222 & 0 \\ 0 & 0 & -225.75 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.01002 & 0 & 0.9922 \\ 223.4925 & 0 & 2.2575 \end{bmatrix}.$$

MPCA: MAP with positively correlated arrivals: Here we take D_0 and D_1 to be

$$D_0 = \begin{bmatrix} -1.00222 & 1.00222 & 0 \\ 0 & -1.00222 & 0 \\ 0 & 0 & -225.75 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.9922 & 0 & 0.01002 \\ 2.2575 & 0 & 223.4925 \end{bmatrix}.$$

All these five MAP processes are normalized during simulation so as to have an arrival rate of 12/minute. However, these are qualitatively different in that they have different variance and correlation structure. The first three arrival processes, namely ERLA, EXPA, and HEXA, correspond to renewal processes and so the correlation is 0. The arrival process labeled MNCA has correlated arrivals with correlation between two successive inter-arrival times given by -0.4889 and the arrivals corresponding to the processes labeled MPCA has a positive correlation with values 0.4889. The ratio of the standard deviations of the inter-arrival times of these five arrival processes with respect to ERLA are, respectively, 1, 2.2361, 5.0194, 3.1518, and 3.1518.

For services in various stages in different groups we pick among the following three special cases of PH-distributions. These are displayed in general notations and in the examples we will point out the specific values used for these parameters.

EXPS: Exponential: $\boldsymbol{\beta} = (1)$, $S = (-\xi)$.

ERLS(μ, m): Erlang of order m

$$\beta = (1, 0, \ldots, 0) \quad S = \begin{bmatrix} -\mu & \mu & & \\ & -\mu & \mu & \\ & & \ddots & \\ & & & -\mu \end{bmatrix}.$$

HEXS($\boldsymbol{\beta}$,$\boldsymbol{\mu}$): This is the mixture of two exponential with mixing probabilities $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_m)$ and with parameters $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_m)$. These parameters will be chosen so as to arrive at a desired mean.

$$\beta = (\beta_1, \ldots, \beta_m) \quad S = \begin{bmatrix} -\mu_1 & & & \\ & -\mu_2 & & \\ & & \ddots & \\ & & & -\mu_m \end{bmatrix}.$$

All three PH-distributions will be normalized by modifying the parameters so as to have a specific mean. However, these are qualitatively different in that they have different variance structure. Note that the coefficient of variation of ERLS, EXPS, and HEXS are, respectively, less than 1, equal to 1 and greater than 1.

In Table 1 we list the values of the parameters of the model under study used in simulation. In the following we denote Stage j in Group i by G_iS_j, for $1 \le j \le K_i$, $1 \le i \le N$.

Before we specify other parameters of the model, we display in Figure 3 the bar diagram of some key statistics (related to our model) taken from various

sources such as state health facts (SHF), US Census (Census), and population by state that are available on the public domain. We first group the 50 states and the District of Columbia of USA into 5 regions as: (a) Northeast consisting of 13 states; (b) Southeast with 12 states; (c) Midwest with 12 states; (d) Southwest with 4 states; and (e) West with 10 states. The number of HMOs (HMO) and the number of hospitals (HOS) are in actual units; the units for doctors (DOC) are the rate per 100,000 residents; the population (POP) is in units of 100,000s; the number of patients (PAT) served by Federally-funded Federally qualified health centers are in units of 100,000s, and the number of healthcare employees (HCE) are in 100,000s.

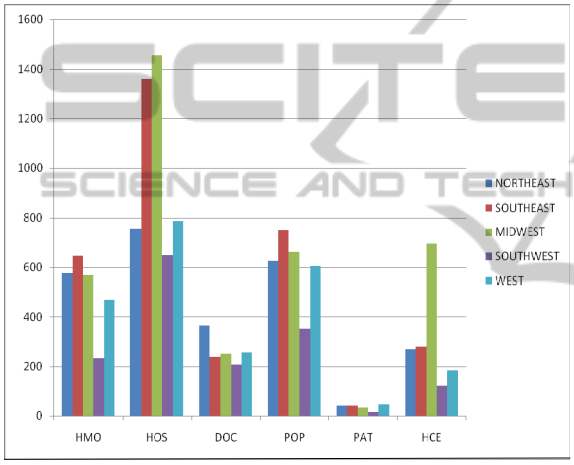

Figure 3: Key statistics related to a HCS.

It should be noted that such statistics pertaining to specific HMOs or hospitals or doctors or any other category may not only be proprietary in nature but also difficult to obtain. So, we try our best to reasonably estimate the parameters for our simulation model. Also this is the first step that we take in dealing with modelling a healthcare system at the macroscopic level (mainly for aggregate planning) and hence there is room for considerable improvement in the future.

In the following let c_{ij}, $1 \leq j \leq K_i$, $1 \leq i \leq N$, denote the number of service providers such as doctors or healthcare administrative personnel, etc., available to serve type i patients in Stage j. Based on the statistics seen above coupled with additional statistics on one of the local HMOs we fix our other parameters as follows. All the time units are in minutes. The simulation was run for 365 days on a 24-hr basis. In Tables 2 through 4 we display the (a) utilization of resources; (b) mean and (c) coefficient of variation (CV) of the waiting time in the system.

Table 1: Parameter values.

Parameter	Values
N	4
(K_1, K_2, K_3, K_4)	(2, 2, 3, 5)
λ	5/minute
(c_{11}, c_{12})	(20, 40)
(c_{21}, c_{22})	(500, 250)
(c_{31}, c_{32}, c_{33})	(40, 30, 30)
$(c_{41}, c_{42}, c_{43}, c_{44}, c_{45})$	(50, 75, 125, 150, 200)
$\boldsymbol{p}$	(0.1, 0.6, 0.2, 0.1)
$\boldsymbol{p_2}$	(0.30, 0.35, 0.35)
$\boldsymbol{p_3}$	(0.1, 0.2, 0.2, 0.2, 0.3)
Service time at G_1S_1	ERLS(0.2, 5)
Service time at G_1S_2	HEXS ((0.6,0.3,0.1),(0.68, 0.068, 0.0068)) for type 1 patients; ERLS(1/0.3,10) for additional work
Service time at G_2S_1	ERLS(1/3, 5)
Service time at G_2S_2	HEXS ((0.85,0.1,0.05),(0.3425, 0.03425, 0.003425)
Service time at G_3S_1	ERLS(1/3, 5)
Service time at G_3S_2	ERLS(2.5, 5)
Service time at G_3S_3	ERLS(1, 5)
Service time at G_4S_1	ERLS(0.25, 5)
Service time at G_4S_2, G_4S_3, G_4S_4, and G_4S_5	HEXS ((0.85,0.1,0.05),(0.17125 0.017125, 0.0017125)
MAP	ERLA, EXPA, HEXA, MNCA, MPCA

Looking at these tables we notice that all arrival processes have pretty much the same utilization in all sectors. The utilization is not high for any of the sectors. In fact, the largest value is 0.590. With regards to the mean waiting time in the system, we find that the five arrival processes appear to have similar values for all types of patients. However, with respect to the additional paperwork (created by types 2, 3, and 4 patients) the mean time taken is much higher for the positively correlated arrivals. In fact, the mean is almost three times as large as the other arrival processes.

The CV of the waiting time in the system of patients shows a different pattern as compared to the mean waiting time for the five arrival processes

considered. For example, the smallest value for CV seems to occur for type 3 patients with positively correlated arrivals. This measure, for type 3 patients, is larger than 1 in all cases indicating that standard deviation of the waiting time in the system to be much larger than the mean.

Table 2: Utilization of the resources.

MAP	ERLA	EXPA	HEXA	MNCA	MPCA
G_1S_1	0.251	0.250	0.249	0.250	0.251
G_1S_2	0.590	0.583	0.584	0.587	0.588
G_2S_1	0.090	0.090	0.090	0.090	0.090
G_2S_2	0.240	0.240	0.239	0.240	0.238
G_3S_1	0.374	0.375	0.375	0.375	0.374
G_3S_2	0.067	0.067	0.067	0.067	0.067
G_3S_3	0.166	0.166	0.166	0.167	0.166
G_4S_1	0.200	0.200	0.200	0.200	0.200
G_4S_2	0.271	0.269	0.264	0.263	0.270
G_4S_3	0.161	0.160	0.157	0.158	0.159
G_4S_4	0.135	0.131	0.134	0.133	0.133
G_4S_5	0.100	0.099	0.099	0.100	0.101

Table 3: Mean waiting time in the system.

MAP	ERLA	EXPA	HEXA	MNCA	MPCA
Type 1	30.23	29.64	29.83	29.91	31.43
Type 2	34.99	35.01	34.94	34.98	34.83
Type 31	22.02	22.01	22.00	22.03	24.77
Type 32	22.00	22.03	22.01	22.00	24.78
Type 33	21.98	22.01	22.02	21.99	24.81
Type 41	185.84	175.79	180.16	181.02	180.31
Type 42	181.60	180.57	179.38	180.24	179.83
Type 43	177.86	179.46	178.96	178.08	183.42
Type 44	180.56	179.82	177.87	177.34	180.10
Type 45	181.50	179.43	178.97	177.81	179.42
Paperwork	3.00	3.00	3.00	3.00	8.80

Table 4: CV of the waiting time in the system.

MAP	ERLA	EXPA	HEXA	MNCA	MPCA
Type 1	0.471	0.470	0.472	0.470	0.490
Type 2	0.385	0.382	0.383	0.383	0.385
Type 31	3.086	3.095	3.078	3.078	2.463
Type 32	3.084	3.090	3.087	3.094	2.479
Type 33	3.086	3.075	3.083	3.088	2.470
Type 41	0.498	0.501	0.488	0.499	0.491
Type 42	0.490	0.486	0.494	0.495	0.493
Type 43	0.497	0.497	0.498	0.491	0.498
Type 44	0.496	0.490	0.505	0.494	0.495
Type 45	0.494	0.500	0.495	0.492	0.498
Paperwork	0.316	0.316	0.316	0.316	1.631

In the case of additional paperwork, we notice that CV is about 5 times larger for the positively correlated arrivals as compared to all the other arrivals (which all have roughly the same value). This illustrates that one cannot solely depend on the means. In practice, the management normally uses the means to allocate appropriate resources and this example points out the danger in doing so.

Finally, we display the fitted distributions of the waiting times of different patients in various stages in Table 5. In most applications the waiting time distribution will be skewed to the right since some patients have to wait unusually longer than the others. Therefore, we notice that most of the fitted distributions are either gamma, lognormal, or beta, which are very common in situations that exhibit a large variation. In the case of all but positively correlated arrival processes, we observe that the best fit for the time spent by the additional paperwork is same as the processing time (Erlang of order 10 with parameter 10/3). This indicates that the additional paperwork is processed soon after its arrival. However, for the positively correlated this is not the case and there appears to exhibit a large variation requiring a beta distribution. Thus, in practice one should integrate fully the type of distribution used for the arrivals rather than just a few descriptive measures such as mean, standard deviation, and correlation.

4 CONCLUSIONS

In this paper we used ARENA simulation software to study a healthcare system at a macroscopic level and identified a few underutilized resources as well as areas for improvement (with regards to delay in waiting for services). We used a versatile point process to model the arrivals of patients and phase type distributions for the services of the patients in various stages of a HCS. Different types of patients require different sequencing to get services and are routed accordingly. It should be pointed out that the intent of this paper is not to simulate any specific unit of a HCS but to highlight the need (especially for aggregate planning) for modelling at a macroscopic level through an example. Thus, in this first attempt the results are only approximate and should be taken and interpreted carefully. There are several variants and improvements to the current model and will be addressed elsewhere.

Table 5: Fitted distributions of the waiting time in the system (using ARENA notation).

	ERLA	EXPA	HEXA	MNCA	MPCA
Type 1	2 + LOGN(22.4, 29.4)	2 + LOGN(22, 28.5)	1 + LOGN(22.9, 26.9)	1 + LOGN(22.9, 26.9)	1 + LOGN(24.7, 28.7)
Type 2	3 + LOGN(23.9, 25.5)	2 + LOGN(24.8, 24.4)	2 + LOGN(24.8, 24.4)	2 + LOGN(24.8, 24.4)	2 + LOGN(24.7, 24.3)
Type 31	5 + GAMM(3.06, 5.57)	5 + GAMM(3.05, 5.58)	5 + GAMM(3.07, 5.54)	5 + GAMM(3.07, 5.55)	5 + GAMM(4.59, 4.31)
Type 32	5 + GAMM(3.07, 5.55)	5 + GAMM(3.06, 5.56)	4 + GAMM(2.83, 6.36)	4 + GAMM(2.82, 6.39)	5 + GAMM(4.54, 4.35)
Type 33	5 + GAMM(3.06, 5.56)	5 + GAMM(3.08, 5.53)	5 + GAMM(3.06, 5.56)	5 + GAMM(3.05, 5.57)	5 + LOGN(19.9, 10.6)
Type 41	8 + LOGN(140, 244)	8 + LOGN(132, 221)	8 + LOGN(133, 227)	8 + LOGN(136, 233)	7 + LOGN(135, 225)
Type 42	8 + LOGN(135, 232)	8 + LOGN(133, 229)	7 + LOGN(134, 223)	7 + LOGN(135, 226)	8 + LOGN(134, 228)
Type 43	7 + LOGN(134, 222)	8 + LOGN(135, 228)	7 + LOGN(135, 224)	7 + LOGN(133, 220)	8 + LOGN(137, 238)
Type 44	7 + LOGN(135, 226)	8 + LOGN(133, 226)	8 + LOGN(134, 227	8 + LOGN(133, 224)	7 + LOGN(135, 224)
Type 45	7 + LOGN(136, 228)	6 + LOGN(136, 221)	6 + LOGN(135, 219)	6 + LOGN(134, 217)	8 + LOGN(134, 228)
Paperwork	ERLA(0.3, 10)	ERLA(0.3, 10)	ERLA(0.3, 10)	ERLA(0.3, 10)	152 * BETA(0.296, 4.82)

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