

# SECURITY INVESTMENT ANALYSIS ON GAMING THEORY WITH MEASUREMENTS OF COST AND DECISION BEHAVIOR

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Abstract: The enterprises face with problems such as coordination with the security administration, market competition, decision-making of production, security investment and so on. With the cost of raw material has been going up in recent years, in order to reduce the cost and maximize the profit, the enterprises are gaming in the security investment, which results in safety accidents. Based on the gaming model with the measurements of cost and decision behavior, this paper presents a security investment analysis for decision maker to enhance the security supervision and improve the production and security status.

## 1 INTRODUCTION

From the incomplete statistics, the average preventive investment in developed countries accounted for 3.3% of GDP. It is estimated that the security engineering, security facilities, and outstanding security loans amounted to hundreds of billions, not including the other expenditures. On the other hand, the overall losses annually in recent years occurred in all types of security incidents is more than trillion dollars of direct losses, plus inestimable indirect losses. Through the analysis of security consciousness, security input, security legal system and on-the-spot government, more and more countries pay attention to the importance of security administration and inputs. The emphasis is placed on as strengthening enterprise's security management, employing principle of risk concentration to arrange invested funds for security rationally, using risk-transfer to lower accident rates, using financial methods reasonably to reduce losses of accident (Dixit and Pindyck, 1994; Goeree and Holt, 2005).

Why do not companies want to invest in security? Firstly, Let us explain this phenomenon from the principle of minimum security cost and maximum profit (Kort et al., 1999):

### (i) Minimum Security Cost Principle

Considering security investment consists of accident losses and security cost:

$$B(S)=L(S)+C(S) \quad (1)$$

Where, S represents a variable for the security production, L(S) expresses the loss function of security; C(S) expresses the cost function of security.

The optimal case is when B (S) is minimum. To achieve this objective, the optimal S can be derived through seeking  $dB (S) / dS = 0$ .

### (ii) The Maximum Profit Principle

Security investment return E (S) can be expressed as:

$$E(S)=F(S) - C(S) \quad (2)$$

Where, F (S) is a security function which is equal to the profit appreciated and loss saved of security inputs. The optimal case is when E (S) the maximum. It can be derived through setting  $dE (S) / dS = 0$ .

No matter whether it is the minimum cost principle or maximum profit principle, the analysis comes out from the economics perspective of a business, and does not take into account the utility functions of market competition, inter-firm interaction and other constraints. This paper introduces game theory into the analysis of enterprise security investment from the perspective of technology cost and the decision-making behavior.

## 2 THE MODEL OF GAMING

### 2.1 Utility Function

In order to study the security investment gaming between enterprises, the constructed model involves two entities ( $i, j = 1, 2$ ) which has its own utility function  $u_i$  under the security investment of  $a_i$  ( $a_i \geq 0$ ), and assumed that

- the enterprise  $i$  invested in security at the level of  $a_i=0$  and  $a_i > 0$ ;
- the probability  $P$  of a security accident is very small. Otherwise, it is not attractive for a rational investor to invest and the government would supervise such kind of risky project. The objective of security investment is to control the probability of security accident so that the accident probability decreases to a small level of  $P$ ;
- only the price competition come out of profit consideration is taken into account for the market competition;
- an infinite loss of security accident would result in a bankruptcy.
- The Security investment utility function can be constructed as follows (Guang-mao et al., 2005):

$$u_i = x_i(a_i, a_j) + y_i(a_i) - v_i(a_i) \quad (3)$$

Where,  $x_i(a_i, a_j)$  expresses the overall profit utility value changed for enterprise  $i$  from external competition of security investment with other enterprises;  $y_i(a_i)$  is the utility value for enterprise  $i$  in the security investment;  $v_i(a_i)$  is the cost of security investment, and  $v_i(a_i) > 0$ .

Taking enterprise 1 as example, and let  $x_1(a_1, a_2) = w_1 a_1 a_2$ . Where  $w_1 = \partial^2 x_1 / \partial a_1 \partial a_2$  reflects the contribution of  $x_1$  to the overall utility value and the relative impact of competition in security investment between enterprises, its sign is the same as  $(a_2 - a_1)$ . Under the competition condition that a security investment would increase the cost for enterprise  $i$ ,  $a_1$  has a negative impact and  $a_2$  has a positive impact on the utility value.

$$y_i(a_i) = (1 - p')u^c - p'u^s, \quad (4)$$

$$= [1 - (p - \gamma_i a_i)]u^c - (p - \gamma_i a_i)u^s, \quad \gamma_i \geq 0$$

Where,  $\gamma_i$  expresses the learning ability of security investment for enterprise  $i$ ;  $(p - \gamma_i a_i)$  indicates the probability change of accidents for a security investment,  $(p - \gamma_i a_i) \geq 0$ ;  $u^c$  is the profit for enterprise without security accident,  $u^s$  is the profit loss caused

by a security accident, and  $u^s \rightarrow +\infty$ . For a very small number  $p$ ,  $(p - \gamma_i a_i) \rightarrow 0$ . It is hard to determine the value of  $pu^s$  and  $(p - \gamma_i a_i)u^s$  which are meaningless in the utility function. Consequently, the term  $(p - \gamma_i a_i)u^s$  can be truncated, and the impact of security investment is focused on the term of  $[1 - (p - \gamma_i a_i)]u^c$ .

$$v_i(a_i) = -b_i a_i + \frac{1}{2} c_i a_i^2, \quad c_i \geq 0, \quad (5)$$

Where  $b_i$  is security investment intention for enterprise  $i$ ,  $c_i$  is the significance of security investment  $a_i$ . Under a security investment, the utility value for enterprise 1 and enterprise 2 is:

$$u_i(a_1, a_2) = w_i a_1 a_2 + [1 - (p - \gamma_i a_i)] u^c + b_i a_i - \frac{1}{2} c_i a_i^2, \quad (i=1, 2) \quad (6)$$

If enterprise  $i$  does not have a security investment, then  $a_i$  can be substituted by 0, and

$$x_i(a_i, a_j) = z_1 a_1 + z_2 a_2 \quad (7)$$

Where,  $z_1$  and  $z_2$  are constant. If the impact on enterprise 1 is negative, then the impact on enterprise 2 would be positive, accordingly,  $z_1 < 0$  and  $z_2 > 0$ .

### 2.2 Utility Matrix

Suppose the gaming utility matrix for two business entities is shown in Figure 1, from equation (6),  $u_1(a_1, a_2)$  and  $u_2(a_1, a_2)$  can be calculated (Wei-ying, 1996):

$$u_1(0, 0) = u_2(0, 0) = (1 - p)u^c$$

$$u_1(a_1, 0) = z_1 a_1 + [1 - (p - \gamma_1 a_1)]u^c + b_1 a_1 - \frac{1}{2} c_1 a_1^2$$

$$u_1(0, a_2) = z_2 a_2 + (1 - p)u^c$$

$$u_2(0, a_2) = z_2 a_2 + [1 - (p - \gamma_2 a_2)]u^c + b_2 a_2 - \frac{1}{2} c_2 a_2^2$$

$$u_2(a_1, 0) = z_1 a_1 + (1 - p)u^c$$

		Entity 2	
		0	$a_2$
Entity 1	0	$u_1(a_1, a_2), u_2(0, 0)$	$u_1(0, a_2), u_2(0, a_2)$
	$a_1$	$u_1(a_1, 0), u_2(a_1, 0)$	$u_1(a_1, a_2), u_2(a_1, a_2)$

Figure 1: Two entity's security invest utility matrix.

If  $0 < a_1 < 2(z_1+b_1+\gamma_1u^c)/c_1$ , then  $u_1(a_1,0) > u_1(0,0)$ . It means enterprise 1 continues to invest in security even though enterprise stops the security investment.

### 3 GAMING ANALYSIS

#### 3.1 Graphical Analysis of Gaming Zones

By means of a game graphics with gaming zones, the gaming can be divided into two situations (Jun-jie et al., 2006; Li and Sun, 2005):

##### Situation 1:

Corresponding to  $u_1(0,0)=u_1(a_1,0)$  and  $u_2(0,0) = u_2(0,a_2)$ , there are four gaming zones (I, II, III, IV) divided by the cross lines of  $a_1=2(z_1+b_1+\gamma_1u^c)/c_1$  and  $a_2= 2(z_2+b_2+\gamma_2u^c)/c_2$ , which is shown in Figure 2.

In zone I,  $u_1(0,0) < u_1(a_1,0)$  and  $u_2(0,0) < u_2(0,a_2)$ , the security investment choices for the two enterprises are  $(a_1,a_2)$ . Similarly, in zone II, III, IV the security investment choices are  $(0,a_2)$ ,  $(a_1,0)$  and  $(0,0)$ .

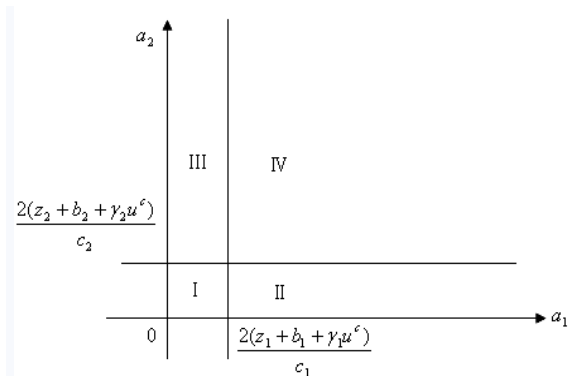


Figure 2: Graphics for gaming situation 1.

##### Situation 2:

Corresponding to  $u_1(a_1,a_2)=u_1(0,a_2)$  and  $u_2(a_1,a_2) = u_2(a_1,0)$ , the four gaming zones (I, II, III, IV) are divided by the cross lines of

$$a_1 = (\gamma_2 a_2 u^c + b_2 a_2 - 0.5 c_2 a_2^2) / (-w_2 a_2 + z_2) \text{ and } a_2 = (\gamma_1 a_1 u^c + b_1 a_1 - 0.5 c_1 a_1^2) / (-w_1 a_1 + z_1),$$

which is shown in Figure 3. Through an iterative calculation, the gaming zones can be obtained and shown briefly as Figure 3. Any change of the parameters can only affect the size of the zones but not the relationship between zones. The security investment choices for gaming zone I, II, III, IV are  $(0,0)$ ,  $(0,a_2)$ ,  $(a_1,0)$ ,  $(a_1,a_2)$ , respectively.

Combining Figure 2 with Figure 3 into one coordinate frame, there are nine gaming zones available in Figure 4.

It should be noted that the relative location of gaming zones in Figure 4 might be changed based on the value of  $(z_1+b_1+\gamma_1u^c)/c_1$ ,  $(z_2+b_2+\gamma_2u^c)/c_2$ ,  $(\gamma_2 a_2 u^c + b_2 a_2 - 0.5 c_2 a_2^2) / (-w_2 a_2 + z_2)$ , and  $(\gamma_1 a_1 u^c + b_1 a_1 - 0.5 c_1 a_1^2) / (-w_1 a_1 + z_1)$ . It can also be observed from Figure 4 that the gaming results are always the same for zone IV, V, and VII, which are  $(0,0)$ ,  $(0, a_2)$ ,  $(a_1, 0)$ , respectively. The gaming results for the other zones might vary from different gaming situations.

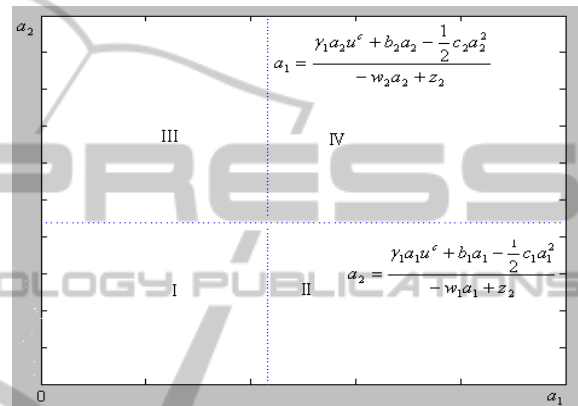


Figure 3: Graphics for gaming situation 2.

#### 3.2 Gaming Analysis for Security Investment

Another observation from the gaming model and gaming zone analysis is that the security investment is very much dependent upon the parameters. However, the actual security investment decision for an enterprise is not complicated, in most of the cases, it is dependent upon the cost and technology instrument.

For enterprise 1, obviously, the investment choices could be  $(0,0)$ ,  $(a_1, 0)$ ,  $(0, a_2)$ ,  $(a_1,a_2)$ . The decision can be made after the comparison of  $u_1(0, 0)$  and  $u_1(a_1, 0)$ , as well as  $u_1(0, a_2)$  and  $u_1(a_1, a_2)$ . From (6) we can obtain (Liang-qiao, 2007):

$$u_1(a_1,0) - u_1(0,0) = z_1 a_1 + \gamma_1 a_1 u^c + b_1 a_1 - \frac{1}{2} c_1 a_1^2 \quad (8)$$

$$u_1(a_1, a_2) - u_1(0, a_2) = w_1 a_1 a_2 - z_2 a_2 + \gamma_1 a_1 u^c + b_1 a_1 - \frac{1}{2} c_1 a_1^2 \quad (9)$$

In (8),  $z_1 a_1 < 0$ . From the presupposition of security investment cost  $v_i(a_i) > 0$ , we can know:

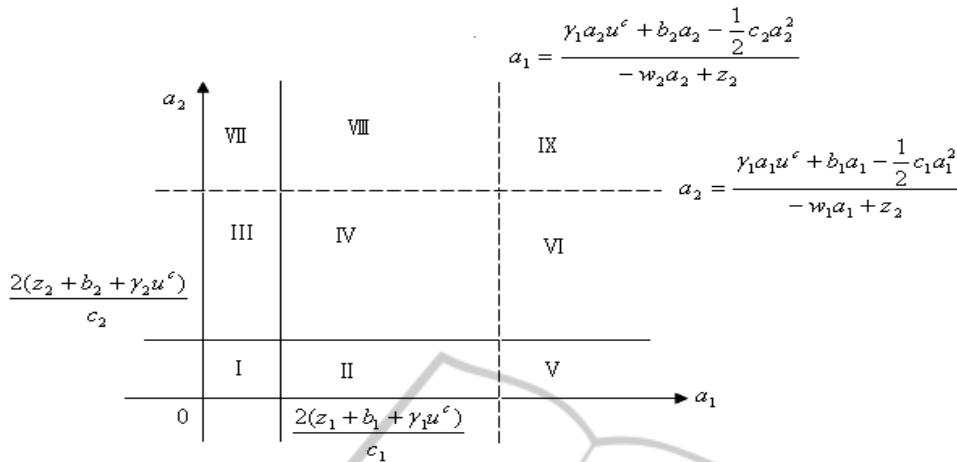


Figure 4: Integrated gaming zones.

$b_1 a_1 - \frac{1}{2} c_1 a_1^2 < 0$ . Under a certain level of investment  $a_1$ ,  $\gamma_1 a_1 u^c$  depends on the value of  $\gamma_1$  and  $u^c$ .  $\gamma_1$  is mainly affected by business environment and technology level, the higher the technical level requires, the greater value  $\gamma_1$  from the impact of security probability would be.  $u^c$  is related to the enterprise cost, a lower cost reflects a higher value of  $u^c$ . From the viewpoint of technology and cost components, a lower level of technology would cause a higher investment cost. Compared with  $z_a a_1$  and  $(b_1 a_1 - \frac{1}{2} c_1 a_1^2)$ , the value of  $\gamma_1 a_1 u^c$  is very small and can be ignored. Therefore,  $u_1(a_1, 0) - u_1(0, 0) < 0$ . In the situations of  $(0, 0)$  and  $(a_1, 0)$ , enterprise 1 would not make a security investment until an improvement of technology and profit occurs.

For expression (9),  $b_1 a_1 - \frac{1}{2} c_1 a_1^2 < 0$ ,  $w_1 a_1 a_2 - z_2 a_2 = (w_1 a_1 - z_2) a_2$ . As a matter of experience, under the unique condition of competition, the choice of  $(0, a_2)$  is better than  $(a_1, a_2)$  for enterprise 1 to improve the competition, this conclusion can also be derived from the analysis of utility functions. Compared with the case of  $(0, a_2)$ , the case  $(a_1, a_2)$  implies a smaller impact of the overall utility value for enterprise 1, and  $w_1 a_1 a_2 - z_2 a_2 < 0$ . The value of  $\gamma_1 a_1 u^c$  is very small compared with that of  $z_1 a_1$  and  $w_1 a_1 a_2 - z_2 a_2$ , so it can be ignored.  $u_1(a_1, a_2) - u_1(0, a_2) < 0$ , so we have the same conclusion for the situations of  $(0,0)$  and  $(a_1, 0)$  that enterprise 1 would not make a security investment until an improvement of safety instruments.

From the analysis of (8) and (9), under the condition of a low technology level and a high

security investment cost, the enterprise 1 thought it is not necessary to make a security investment because of the low utility value in the short run. Without a change of business environment, enterprise 1 would be gaming forever. Similarly, enterprise 2 also gets the same conclusion and does not want to invest neither. Consequently, the choices for enterprise 1 and enterprise 2 would be  $(0, 0)$  which located at zone IV in Figure 4, both enterprises are unwilling to make a security investment.

#### 4 CONCLUSIONS

The reason why enterprises are reluctant to make a security investment is partially due to a low level of technology, high investment cost as well as a result of ineffective supervision, thereof the problems of low input and technology instrument are not difficult to solve. However, the fact of inadequate government supervision and the pursuit of maximum profit leads to the enterprises' gaming behavior. To improve the security investment efficiency, some suggestions are summarized as follows:

- The managerial principle of profit maximization results in a shortage of security input, irrational gaming, inappropriate supervision, and even corruption. A task of top priority is to strengthen security supervision, add impetus to the marketization of public utilities, improve funds efficiency, and establish a rational institutional system. The enterprises may have to provide detailed statements timely to the administrative agencies.

- The government should encourage the enterprises to improve the technology level through making more R&D investment and accelerating the pace of imports and international cooperation in advanced technology.
- Facing with the problems of high security input cost and capital shortage in the context of the recent crisis in global financial markets and weakening of global economic activity, the government should make an effort to adjust the policy, including the government subvention, public subsidy, tax exemption and reduction.
- The enterprises must identify the objective and the use of security investment funds, including government subsidies, R&D, and the security fee deducted, setting up a proactive investment mechanism and risk preventive system.
- The enterprises should establish a budget and check framework to monitor progress and effectiveness of the security funds, as well as create an internal performance reporting structure to ensure the funds are in a virtuous cycle.

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