OPTIMAL ORDER LOT SIZING AND PRICING WITH CARBON TRADE

Guowei Hua
Department of Logistics Management, School of Economics and Management, Beijing Jiaotong University
100044, Beijing, P.R. China

Han Qiao
School of Economics, Qingdao University, 266071, Qingdao, P.R. China
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, 100190, Beijing, P.R. China

Jian Li
School of Economics and Management, Beijing University of Chemical Technology
100029, Beijing, P.R. China

Keywords: Carbon trade, EOQ model, Pricing, Carbon footprints.

Abstract: Carbon emission trading is one of the broadly adopted methods to curb the amount of carbon emission. This paper examines the optimal decisions of retailers under cap-and-trade. We derive the optimal order lot size and retail price under cap-and-trade when the demand is an additive function or multiplicative function of retail price, and analyze the impacts of carbon trade on the order decision, pricing decision, carbon emission and profit.

1 INTRODUCTION

In order to alleviate global warming, many measurements such as economics, legislation were taken to curb the total amount of carbon emissions. Carbon emission trading is generally accepted as one of the most effective market-based mechanisms, which has been broadly adopted by UN, EU, and many governments. For example, the Kyoto Protocol (UNFCCC, 1997) and the European Union Emission Trading System (EU-ETS) implement a mandatory "cap and trade" system in 183 countries and the 27 EU member countries (EU, 2009), respectively. More than 20 platforms for trading carbon are running in the world.

Facing the cap-and-trade, firms can optimize their strategic decisions such as supply chain design or operations decisions in production, transportation, and inventory to reduce carbon emissions. There are few studies on the operations decisions under carbon emission regulations. Cachon (2009) discusses how the new objective of reducing carbon footprints is likely to affect supply chain operations and structures. Hua et al. (2010) examined the optimal order quantity under carbon trade. Benjaafar et al. (2010) introduce a series of simply 3models to illustrate how carbon footprint considerations could be incorporated into operations decisions. Bonney and Jaber (2010) examined the importance of inventory planning to the environment and the possibility of using models to perform analyses. However, all the researches mentioned-above are not incorporated pricing into them.

Although there are plentiful studies of purchase decisions incorporating pricing (Chen and Simchi-Levi, 2010), they did not incorporate carbon footprints into them. To fill the gap, in this paper, we examine the optimal order lot sizing and pricing for retailers under carbon trade.

The rest of this paper is organized as follows: In Sections 2 we formulate EOQ model with pricing under carbon trade, derive the optimal order quantity and price. In Sections 3 and 4, we analytically and

Hua G., Qiao H. and Li J..
OPTIMAL ORDER LOT SIZING AND PRICING WITH CARBON TRADE.
DOI: 10.5220/0003597005330536
In *Proceedings of the 13th International Conference on Enterprise Information Systems* (DMLSC-2011), pages 533-536
ISBN: 978-989-8425-55-3

numerically the impacts of carbon trading on order decisions, pricing decision, carbon emissions, and total cost. Finally we conclude the paper and suggest topics for future research in Section 5.

2 THE MODEL

This section we will formulate the EOQ with pricing under carbon trade, and derive the optimal order lot sizing and pricing. Carbon trading is also known as cap and trade. A firm is allocated a limit or cap on carbon emissions. If its amount of carbon emissions exceeds the carbon cap, it can buy the right to emit extra carbon from the carbon trading market. Otherwise, it can sell its surplus carbon credit. We focus on the carbon emissions caused by logistics and warehousing activities in this paper.

The notation used in the paper is as follows:

· K= fixed ordering cost;
· T = the replenishment time interval;
· h= annual holding cost per unit, expressed as a percentage of the average inventory value;
· p= the retail price (a decision variable);
· Q = order lot size in units (a decision variable);
· $D(p)$ = annual demand or demand rate, which is a function of the retail price p;
· w = wholesale price per unit;
· α = carbon emission quotas per unit time;
· C = carbon price per unit (ton);
· CE=the amount of carbon emission;
·e= the amount of carbon emissions in executing an order;
·gQ = the amount of carbon emissions in holding Q units product, where g is the variable emission factor in warehouse;
· X = transfer quantity of carbon emissions (a decision variable);
· $\pi(Q,p)$ = total profit per unit time;

Following Abad and Aggarwal (2005), we suppose the demand function satisfies:

(i). $D(p)>0$ for $0<p\le p_{\max}$;

(ii). $D(p)$ decreases with increasing p, i.e., $D'(p)<0$;

(iii). the marginal revenue $\frac{d(pD(p))}{dD(p)}=p+\frac{D(p)}{D'(p)}$ is a strictly increasing function of p;

where $p_{\max}$ is a large number that the retail price does not exceed.

Notice that $CE=e\frac{D(p)}{Q}+g\frac{Q}{2}$, based on the classical EOQ model, we can formulate our problem as

$$\max \pi(Q,p)=(p-w)D(p)-K\frac{D(p)}{Q}-\frac{hQ}{2}+CX$$

$$s.t.\ e\frac{D(p)}{Q}+g\frac{Q}{2}+X=\alpha.$$

Substituting $X=\alpha-(e\frac{D(p)}{Q}+g\frac{Q}{2})$ into the objective function, we have

$$\max \pi(Q,p)=(p-w)D(p)$$
$$-\frac{(K+Ce)D(p)}{Q}-\frac{(h+Cg)Q}{2}+C\alpha. \quad (1)$$

The first-order condition for maximization yields the optimal retail price $p^*(Q)$ for a given Q. Let

$$\frac{\partial\pi(Q,p)}{\partial p}=D(p)+(p-w-\frac{K+Ce}{Q})D'(p)=0.$$

Namely,

$$p+\frac{D(p)}{D'(p)}=w+\frac{K+Ce}{Q} \quad (2)$$

Differentiating (2) with respect to Q, we have

$$\frac{dp^*(Q)}{dQ}\frac{(2D'^2-DD')}{D'^2}=-\frac{K+Ce}{Q^2}.$$

Based on the above analysis, we have the following theorems.

Theorem 1. For any given Q, the first-order condition (2) yields the unique maximum $p^*(Q)$.

Proof. If $p>p^*(Q)$, then

$$p+\frac{D(p)}{D'(p)}>w+\frac{K+Ce}{Q},$$

$$\frac{\partial\pi(Q,p)}{\partial p}=D(p)+(p-w-\frac{K+Ce}{Q})D'(p)>0.$$

If $p<p^*(Q)$, the $p+\frac{D(p)}{D'(p)}<w+\frac{K+Ce}{Q}$,

$$\frac{\partial\pi(Q,p)}{\partial p}=D(p)+(p-w-\frac{K+Ce}{Q})D'(p)<0,$$

which indicate that $p=p^*(Q)$ is the unique maximum of $\pi(Q,p)$ for a given Q. □

Next, we will derive the optimal order lot size and price.

Substituting $p = p^*(Q)$ into $\pi(Q,p)$, we have

$$\pi(Q,p^*(Q)) = -\frac{D^2(p^*(Q))}{D'(p^*(Q))} - \frac{(h+Cg)Q}{2} + C\alpha \tag{3}$$

Since $\frac{dp^*(Q)}{dQ}\frac{(2D'^2-DD')}{D'^2} = -\frac{K+Ce}{Q^2}$, we have

$$\begin{aligned}&\frac{d\pi(Q,p^*(Q))}{dQ}\\ &= -\frac{dp^*(Q)}{dQ}\frac{D(2D'^2-DD')}{D'^2} - \frac{h+Cg}{2}\\ &= D(p^*)\frac{K+Ce}{Q^2} - \frac{h+Cg}{2}\end{aligned} \tag{4}$$

Theorem 2.

(1) when $D(p) = a - bp, (a,b>0)$, then Q^* satisfies

$$(h+Cg)Q^3 - (a-bw)(K+Ce)Q + b(K+Ce)^2 = 0,$$

$$\pi(Q^*) = \max \pi(Q),$$

and $p^* = \frac{a}{2b} + \frac{w}{2} + \frac{K+Ce}{2Q^*}$.

(2) when $D(p) = ap^{-b}, (a>0, b>1)$, then Q^* satisfies

$$a\left(\frac{b-1}{b}\right)^b \frac{K+Ce}{Q^2}\left(w+\frac{K+Ce}{Q}\right)^{-b} - \frac{h+Cg}{2} = 0,$$

$$\pi(Q^*) = \max \pi(Q),$$

and $p^* = \frac{b}{b-1}\left(w + \frac{K+Ce}{Q^*}\right)$.

From (2) and (4), we can derive Theorem 2 easily, and we also can obtain

$$X^* = \alpha - \left(e\frac{D(p^*)}{Q^*} + g\frac{Q^*}{2}\right).$$

3 THE IMPACT OF CARBON TRADE ON DECISIONS

Due to the difficulty of the problem, in this section we will numerically examine the impact of carbon trade on the order quantity and price. First, we introduce the following theorem.

Notice that when C=0, our problem is the EOQ with pricing. In this case, the optimal order lot size and price can be found from the following formulas.

$$p + \frac{D(p)}{D'(p)} = w + \frac{K}{Q},$$

$$\pi(Q,p^*(Q)) = -\frac{D^2(p^*(Q))}{D'(p^*(Q))} - \frac{hQ}{2},$$

$$\frac{d\pi(Q,p^*(Q))}{dQ} = -\frac{dp^*(Q)}{dQ}\frac{D(2D'^2-DD')}{D'^2} - \frac{h}{2}$$

$$= D(p^*)\frac{K}{Q^2} - \frac{h}{2}$$

From the above formulas and (2), we can derive the following Theorem 3 easily.

Theorem 3. If the order quantity is the same as that without carbon trade, then the retail price should increase. And if the price is the same as that without carbon trade, then the order quantity should increase.

Theorem 3 shows that carbon trade increases the cost of retailer, if his order quantity keeps constant, he will increase his retail price in order to offset the increased carbon cost, in other words, the end-customers will partially pay the cost of low-carbon.

From Theorem 2, we have the following observations.

Theorem 4. The order quantity, retail price and the amount of carbon emission are decided by carbon price, and have nothing to do with carbon emission quotas.

From the following examples, we can obtain some new observations.

Example 1. Let $D(p)$=6000−30p, K=\$200/order, h=\$0.4/\$/year, w=50, e=500, g=2, $\alpha = 2000$, the results was summarized in Table 1 and Table 2.

Table 1: The results of Example 1 with increasing *C*.

C	(Q^*, p^*)	CE	*profit*
0	(1500,125)	2249.3	168450
0.2	(1299,125.1)	2163.7	168630
0.4	(1224.7,125.2)	2141.3	168820
0.6	(1186,125.2)	2132	169000
0.8	(1162,125.3)	2127	169190

Table 2: The results of Example 1 with increasing α.

α	(Q^*, p^*)	CE	*profit*
4000	(1299,125.1)	2143	169030
3000	(1299, 125.1)	2143	168830
2500	(1299, 125.1)	2143	168730
2000	(1299, 125.1)	2143	168630
1500	(1299, 125.1)	2143	168530

Example 2. Let $D(p) = 4000000p^{-2}$, K=\$200/order, and h=\$0.3/\$/year, w=50, e=500,

g=2, $\alpha = 3000$, the results was summarized in Table 3 and Table 4.

Table 3: The results of Example 2 with increasing *C*.

C	(Q^*, p^*)	CE	*profit*
0	(99.5,104)	3764.3	38454
0.2	(84.2,107.1)	4222.6	37940
0.4	(78.2,110.2)	4286.4	37489
0.6	(74.6,113.4)	4245.1	37069
0.8	(71.8,116.7)	4159.4	36675

Table 4: The results of Example 2 with increasing α .

α	(Q^*, p^*)	CE	*profit*
7000	(84.2,107.1)	4222.6	38740
6000	(84.2,107.1)	4222.6	38540
5000	(84.2,107.1)	4222.6	38340
4000	(84.2,107.1)	4222.6	38140
3000	(84.2,107.1)	4222.6	37940

Tables 1-4 show that the order quantity would decrease but retail price would increase with increasing the carbon price. The carbon emission would decrease in an additive demand function but increase in a multiplicative demand function with increasing the carbon price. The profit would decrease with increasing the carbon price, which is straightforward.

Tables 1-4 also show that the order quantity, retail price and the amount of carbon emission would keep constant with decreasing carbon emission quotas. However, the profit would decrease with decreasing carbon emission quotas since the carbon constraint is becoming strict.

4 CONCLUSIONS

To respond to the regulations on carbon emissions, a firm can optimize their operations decisions in production, transportation, and inventory to reduce carbon emissions. This paper examines the jointly inventory and price decisions with carbon trade, we derive the optimal order lot size and price based on the EOQ model. We theoretical analyze the impact of the carbon price and carbon emission quotas on the order and price decisions, the carbon emission and profit. We also present some interesting observations from numerical tests.

In this paper, we suppose that carbon price has nothing to do with carbon emission quotas, in fact, carbon price is effected by carbon emission quotas, if carbon emission quotas is small, which means the carbon policy is strict, generally speaking, the carbon price would increase. So, to examine the same question in this case is a good further research direction.

ACKNOWLEDGEMENTS

This research was supported by the NSFC under grant number 71071015, 71003057, and 70801003, the Ph.D. Programs Foundation of the Ministry of Education of China under grant number 20100009120009, and the Fundamental Research Funds for the Central Universities under grant number 2009JBZ010-3.

REFERENCES

Benjaafar S., Li Y., Daskin M., 2010. Carbon footprint and the management of supply chains: Insights from simple models. *http://www.ie.umn.edu/faculty/ faculty/ pdf/beyada-3-31-10.pdf.*

Bonney, M., Jaber, M.Y., Environmentally responsible inventory models: Nonclassical models for a non-classical era. *International Journal of Production Economics* (2010), doi:10.1016/j.ijpe.2009.10.033.

Cachon G. Carbon footprint and the management of supply chains. *The INFORMS Annual Meeting, San Diego, CA, 2009.*

Chen X.., Simchi-Levi D. 2011. Pricing and Inventory Management. To appear in *Handbook of Pricing*, eds. R. Philips and O. Ozalp.

EU, 2009. *http://ec.europa.eu/environment/climat/ emission/ets post2012 en.htm.*

Hua G.W., Cheng T.C.E., Wang S.Y., Managing carbon footprints in inventory management. *International Journal of Production Economics* (2011), doi:10.1016/j.ijpe.2011.03.024.

UNFCCC, 2009. Kyoto Protocol. *http://unfccc.int/kyoto protocol/items/2830.php.*