FUZZY MODEL BUILDING USING PROBABILISTIC RULES

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Abstract: Uncertainty in the attributes and uncertainty in frequency of their occurrences are inherent to the real world problems and an attempt is made here to tackle them together. The possible connections between the two facets of uncertainty are explored and discussed. This paper also looks at the role of possibility and probability in the context of decision making and in the process utilizes the existing fuzzy models by incorporating the multiple probabilistic outputs in the associated fuzzy rules. This is needed to obtain the net conditional possibility from the probabilistic fuzzy rules where the probabilistic information of the outputs is given. A novel approach is devised to compute net conditional possibility from the given probabilistic rules. The basis for extending the existing fuzzy models is also presented using the computed net conditional possibility. The enhanced fuzzy models accruing from the addition of the probabilistic information would usher in better decision making. The proposed approach is demonstrated through two case-studies.

1 INTRODUCTION

Zadeh (1978) first coined the term possibility to represent the imprecision in information. This imprecision is quite different from the frequentist uncertainty represented by well developed probabilistic approach. But if we could appreciate the real world around us, there is a constant interplay between probability and possibility–even though the two represent different aspects of uncertainty. Hence, if not all, in many a situation, the two are intricately interwoven in the linguistic representation of a situation or an event by a human brain. And often, it is possible to infer probabilistic information from possibilistic one and vice versa. Even though they are dissymmetrical and treated differently in literature, there is a need to make an effort towards exploring a unifying framework for their integration. We feel that these two different, yet complimentary formalisms can better represent practical situations, going hand in hand.

Besides the vast potential of this study in more closely representing the real world, we are also motivated by its roots in philosophy. Non-determinism is almost a constant feature in nature, and together probability and possibility can go farther in representing the real world situations. Even though, probability and possibility represent two different forms of uncertainty and are not symmetrical, but still both are closely related, and often needs to be transformed into one other, to achieve computational simplicity and efficiency. This transformation would pave the way for simpler methods for the computation of net possibility. The intelligent controllers utilizing these transformations would represent the requirements and situations of the real world more truly and accurately. They would also be more computationally efficient in terms of speed, storage and accuracy in processing of the uncertain information.

Such transformations bridge two different facets of uncertainty – statistical/probabilistic and imprecision (on account of vagueness or lack of knowledge). (Dubois et al., 1992; 1993) analyzed the transformations between the two and judged the consistency in the two representations.

This paper is concerned with devising a novel approach for application of some of the research results to the field of fuzzy theory under probabilistic setting, and using the same to enhance the existing fuzzy models to better infer the value of possibility in the light of probabilistic information available. It also relooks at the relevant results along with their interpretations in the context of

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probabilistic fuzzy theory. This paper basically addresses the following issues:

1. To amalgamate the field of fuzzy theory with the probability theory and to discover the possible linkages or connections between these two facets of uncertainty.
2. To apply the probabilistic framework on the existing fuzzy models for imparting the practical utility to them.
3. To devise an approach to calculate the output of the probabilistic fuzzy models.
4. To study the effect of probabilistic information on the defuzzified outputs of fuzzy rules.

The paper is organized as follows: In Section 2, relationship between probability and possibility is explored by identifying the body of work in this field and giving it a new look. This section also gives the preliminaries needed for the paper. In Section 3, a few mathematical relations are presented in order to calculate the output of probabilistic fuzzy rules (PFRs). The utility and advantages of (PFR) are also discussed. Section 4 discusses an algorithm to compute net conditional possibility from probabilistic fuzzy rules. In sections 5 and 6, two case studies are taken up to illustrate the algorithm. Finally, Section 7 gives the conclusions and the scope of further research in the area.

2 PROBABILITY AND POSSIBILITY: A RELOOK

The possible links between the two facets of uncertainty: probability and possibility are explored on the basis of the key contributions in the area.

The celebrated example of Zadeh (1978) “Hans ate X eggs for Breakfast” illustrates the differences and relationships between probability and possibility in one go. The possibility of Hans eating 3 eggs for breakfast is 1 whereas the probability that he may do so might be quite small, e.g. 0.1. Thus, a high degree of possibility does not imply a high degree of probability; though if an event is impossible it is bound to be improbable. This heuristic connection between possibility and probability may be called the *possibility/probability consistency principle,* stated as: If a variable x takes values $u_1, u_2, ..., u_n$ with respective possibilities $\prod = (\pi_1, \pi_2, ..., \pi_n)$ and probabilities $P= (p_1, p_2, .., p_n)$ then the degree of consistency of the probability distribution P with the possibility distribution II is expressed by the arithmetic sum as

$$\gamma = \pi_1 p_1 + \pi_2 p_2 + ... + \pi_n p_n$$

Note that the above principle is not a precise law or a relationship that is intrinsic to the concepts of possibility and probability; rather it is an approximate formalization of the heuristic observation that a lessening of the possibility of an event tends to lessen its probability, not vice-versa. In this sense, the principle is applicable to situations in which we know the possibility of a variable x rather than its probability distribution. This principle forms the most conceptual foundation of all the works in the direction of probability/possibility transformations having wide practical applications Roisenberg (2009).

Having deliberated on the consistency principle, we will look into: (i) Basic difference between possibility and probability, (ii) Inter-relation between possibility and probability and vice-versa, (iii) Infer probability from possibility and vice-versa, and (iv) Transformation of probability to possibility and vice-versa, with a view to tackle real life problems involving both probabilistic and possibilistic information.

2.1 Basic Difference between Possibility and Probability

In the perspective of example given by Zadeh, possibility is the degree of ease with which Hans may eat u eggs whereas probability is the chances of actual reality; there may be significant difference between the two. This difference is now elucidated by noting that the possibility represents ‘likelihood’ of a physical reality with respect to some reference whereas the probability represents the occurrences of the same. To put it mathematically,

$$\pi (A) \triangleq Sup_{u \in A} \pi_x(u) \quad (1)$$

where

A is a non fuzzy subset of U

II is possibility distribution of x

π (A) denotes the possibility measure of A in [0,1]

π_x (u) is the possibility distribution function of $\prod_x$.

Let A and B be arbitrary fuzzy subsets of U. In view of (1), we can write that

$$\pi(A \cup B) = \pi(A) \vee \pi(B) \quad (2)$$

The corresponding relation for probability is written as

$$P (A \cup B) \leq P (A) + P (B) \quad (3)$$

2.2 Inter-relation between Possibility and Probability

Any pair of dual necessity/possibility functions (N, ∏) can be interpreted as the upper and lower probabilities induced from specific convex sets of probability functions.

Let π be a possibility distribution inducing a pair of functions [N, ∏]. Then we define

$$\mathcal{P}(\pi) = \{P, \forall A\ measurable, N(A) \leq P(A)\} = \{P, \forall A\ measurable, P(A) \leq \Pi(A)\}$$

The family, $\mathcal{P}(\pi)$, is entirely determined by the probability intervals it generates. Any probability measure $P \in \mathcal{P}(\pi)$ is said to be consistent with the possibility distribution, π (Dubois, 1992); (De Cooman, 1999). That is

$$\sup_{P \in \mathcal{P}(\pi)} P(A) = \Pi(A) \tag{4}$$

A relevant work in this direction was carried out in Walley (1999). It is shown that the imprecise probability setting is capable of capturing fuzzy sets representing linguistic information.

2.3 Inference of Probability from Possibility and Vice-versa

In Zadeh (1978), Dubois (1982, 1992,1993), degrees of possibility can be interpreted as the numbers that generally stand for the upper probability bounds. The probabilistic view is to prepare interpretive settings for possibility measures. This enables us to deduce a strong interrelation between the two. This principle basically implies the following inferences:

High Probability → High Possibility
Low Probability ↛Low Possibility
Zero Possibility → Zero Probability
Zero Probability ↛ Zero Possibility
High Possibility → High Probability
Low Possibility ↛ Low Probability (5)

From Klir (2000) and from the above properties of possibility and necessity measures, we know that maximizing the degree of consistency brings about two strong restrictive conditions having a strong coherence: cloudiness is directly pointing at more probability of rain.

2.4 Transformation from Probability to Possibility

Any transformation from probability to possibility must comply with the following three basic principles as in (Dubois, 1993).

1.Possibility-probability consistency: $\gamma = \pi_1 p_1 + \pi_2 p_2 + ... + \pi_n p_n$
2.Ordinal faithfulness: π (u) > π (u′) iff p (u) > p (u′) (6)
3.Informativity: Maximization of information content of π

If P is a probability measure on a finite set U, statistical in nature then, for a subset, E of U, its possibility distribution on U, π_E (u) is given by (Dubois, 1982):

$$\pi_E(u) = \begin{cases} 1 & if\ u \in E, \\ 1 - P(E) & otherwise, \end{cases} \tag{7}$$

Also $\prod_E$ (A) ≥ P (A), ∀A ⊆ U

In other words, π_E = x ∈ E with the confidence at least P (E). In order to have a meaningful possibility distribution, π_E, care must be taken to balance the nature of complimentary ingredients in (7), i.e. E must be narrow and P (E) must be high.

There are quite a few ways, in which one can do it. The one used in Dubois (1982) chooses a confidence threshold α so as to minimize the cardinality of E such that P (E) ≥ α. Conversely, cardinality of E can be fixed and P (E) maximized. This way, a probability distribution P can be transformed into a possibility distribution π^P (Dubois, 1982). Take p_i as the probability distribution on U and X = {x_1, x_2,.., x_n} such that p_i = P ({x_i}). Similarly possibility distribution π_i = ∏ ({x_i}) and $p_1 \geq p_2 \geq ... \geq p_n$, then we have

$$\pi_i^P(u) = \sum_{j=i}^{n} p_j\ \forall i = 1, n \tag{8}$$

For a continuous case, if the probability density function so obtained is continuous unimodal having bounded support [a, b], say p, then p is increasing on [a, x_0] and decreasing on [x_0, b], where x_0 is the modal value of p. This set is denoted as D in Dubois (1982).

Let p be the probability density function (pdf) in D such that a function f: [a, x_0] → [x_0, b] is defined as f(x) = max {y| p(y) >= p(x)}. Then the most specific possibility distribution π (minimizing the integral of π on [a, b]) that dominates p is defined by

$$\pi(x) = \pi\left(f(x)\right) = \int_{-\infty}^{x} p(y)dy + \int_{f(x)}^{\infty} p(y)dy \tag{9}$$

3 PROBABILISTIC FUZZY MODELING

A probabilistic fuzzy rule (PFR), first devised by (Meghdadi, 2001), is an appropriate tool to represent a real world situation possessing both the features of uncertainty. In such cases, we often observe that for a set of inputs, there may be more than one possible output. The probability of occurrence of the outputs may be context dependent. In a fuzzy rule, there being only a single output, we are unable to accommodate this feature of the real world – multiple outputs with different probabilities. This ability is afforded with PFR. The PFR with multiple outputs and their probabilities is defined as:

Rule R_q:

If x is Aq then y is O_1 with probability P_1
& ...
& y is O_j with probability P_j
& ...
& y is O_q with probability P_n

$$\mathbf{P} = [P_1, P_2, P_1, P_3, P_4, .., P_n], \text{ with } P_1 + P_2 + P_1 + P_3 + P_4 + ... + P_n = 1 \quad (10)$$

Given the occurrence of the antecedent (an event) in (10), one of the consequents (output) would occur with the respective probability of occurrence, P. Therefore, y is associated with both qualitative (in terms of membership function, O) and quantitative (in terms of probability of occurrence, **P**) information. Therefore y is both a stochastic and fuzzy variable at the same time. The real outcome is a function of the probability, while the quality of an outcome is a function of the respective membership function. The probability of an event is having a larger role to play since it is the one that determines the occurrence of the very event. More the probability of an output event, more are the chances of its certainty which in turn gives rise to the respective possibility of the event (in terms of membership function) determining the quality of the outcome.

The above example illustrates the fact that both these measures of uncertainty (probability and possibility) are indispensable in fuzzy modelling of real world multi-criteria decision making, and may lead to incomplete and misleading result if one of them is ignored. So the original fuzzy set theory, if backed by probability theory could go miles in better representing the decision making problems and deriving realistic solutions.

Here, one question that naturally arises is: how about treating probabilities in the antecedents? This aspect is taken into account by having more than one fuzzy rule and probabilistic outcome in the consequent which is sufficient to handle the frequentist uncertainty in the probabilistic fuzzy event. For example in (10), the antecedent could be:

If x_1 is μ_1 and x_2 is μ_2.

Now, the range of probable values of occurrence of inputs is either Input_1 or Input_n etc. Thus for each occurrence of an antecedent condition, there is a corresponding probabilistic fuzzy consequent event in (9).

As per the scope of this paper, we would be considering similar PFRs with the same structure for a probabilistic fuzzy system under consideration. That is, any two PFRs would have the same order of probabilistic outputs.

$$\forall j, q, q': O_{q_j} = O_{q'_j} = O_j$$

where,

q and q′ represent two PFRs A_q and $A_{q'}$

O_{q_j} is j^{th} output in q^{th} rule;

$O_{q'_j}$ is j^{th} output in q'^{th} rule

O_j is j^{th} output that remains the same in any PFR

The mathematical framework follows from (Van den Berg, 2002).Assuming two sample spaces, say X and Y, in forming the fuzzy events A_i and O_j respectively, the following equations hold good,

$$\forall x: \sum_i \mu_i(x) = 1, \forall y: \sum_j \mu_j(y) = 1 \quad (11)$$

If the above conditions are satisfied then X and Y are said to be well defined.

3.1 Input Conditional Probabilities of Fuzzy Antecedents

Given a set of S samples (x_s, y_s), s = 1,.., S from two well-defined sample spaces X, Y, the probability of A_i can be calculated as

$$P(A_i) = \tilde{f}_{A_i} = \frac{f_{A_i}}{S} = \frac{1}{S}\sum_{x_s} \mu_i(x_s) = \hat{\mu}_i \quad (12)$$

where,

A_i: is the antecedent fuzzy event, which leads to one of the consequent events $O_{1, .., } O_n$ to occur.

$\tilde{f}_{Ai}$: Relative Frequency of fuzzy sample values $\mu_i(x_s)$ for the fuzzy event A_i

f_{Ai}: Absolute Frequency of fuzzy sample values $\mu_i(x_s)$ for the fuzzy event A_i

The fuzzy conditional probability is given by,

$$P(O_j \mid A_i) = \frac{P(O_j \cap A_i)}{P(A_i)} \approx \frac{\sum_s \mu_j(y_s)\mu_i(x_s)}{\sum_s \mu_i(x_s)} \quad (13)$$

The density function, $p_j(y)$ can be approximated using the fuzzy histogram [11] as follows:

$$p_j(y) = \frac{P(O_j)\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} \quad (14)$$

where denominator $\int_{-\infty}^{\infty} \mu_j(y)dy$ is a scaling factor.

3.2 Input Conditional Probabilities of Fuzzy Arbitrary Inputs

A input vector x, activates the firing of multiple fuzzy rules, q, with multiple firing rates $\mu_q(x)$, such that $\sum_q \mu_q(x) = 1$. In case this condition is true for a single rule, only one of the consequents O_q will occur with the conditional probability $P(O_j \mid x)$.

In the light of (13) and (14) we obtain,

$$P(O_j \mid x) = \sum_{q=1} \frac{\mu_q(x)\, P(O_j \mid A_q)}{\int_{-\infty}^{\infty} \mu_j(x)dx} \quad (15)$$

Extending the conditional probability $P(O_j|x)$ to estimate the overall conditional probability density function p (y | x), using (14), we get

$$p\,(y \mid x) = \frac{P(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} \quad (16)$$

where, probabilities $P(O_j|x)$ is calculated using (15).

In view of (4) and (8) we obtain,

$$\pi\,(y \mid x) = \sum_j \frac{\Pr(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} \quad (17)$$

This value for conditional possibility can be used in the expression for finding the defuzzified output of fuzzy models

3.3 Obtaining Defuzzified Output

The existing fuzzy models can be used to obtain the defuzzified output by replacing the conditional possibility obtained.

3.3.1 Mamdani-larsen Model

Consider a rule of this model as:

Rule q: If x is Aq then y is Bq.

Here, fuzzy implication operator maps fuzzy subsets from the input space A^q to the output space B^q (with membership function $\emptyset(y)$) and generates the fuzzy output B^q with the fuzzy membership

Rule q: $\phi(y) = \mu(x) \rightarrow \emptyset(y)$

The output fuzzy membership is:

$$\phi^o(y) = \phi^1(y) \vee \phi^2(y) \vee \phi^3(y) \vee \ldots\ldots \vee \phi^k(y) \quad (18)$$

In Mamdani-Larsen (ML) model, the output of rule q is represented by B^q (b_q, v_q), with centroid b_q and the index of fuzziness v_q given by

$$v_q = \int_y \phi(y)dy \quad (19)$$

$$b_q = \frac{\int_y y\phi(y)dy}{\int_y \phi(y)dy} \quad (20)$$

where

$\phi(y)$ is output membership function for rule q.

Now in the probabilistic fuzzy setting, the above expressions (19) and (20) need to be modified. Replacing the value of the output membership function from (8) into (19) and (20) we get

$$v_q = \int_y \sum_j \frac{P(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} dy \quad (21)$$

$$b_q = \frac{\int_y y \sum_j \frac{P(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} dy}{\int_y \sum_j \frac{P(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} dy} \quad (22)$$

where, v_q is index of fuzziness and b_q is Centroid.

The defuzzified output can be calculated in the ML model by applying the weighted average gravity method for the defuzzification. The defuzzified output value of y^o is given by

$$y^0 = \frac{\int_y y\phi(y)dy}{\int_y \phi(y)dy} \quad (23)$$

where $\phi(y)$ is the output membership function calculated using (18).

Also, the defuzzified output y^0 can be written as:

$$y^0 = \sum_{q=1}^{Q} \frac{\mu^q(x).\,v_q}{\sum_{q'=1}^{Q} \mu^{q'}(x).\,v_{q'}}.\,b_q \quad (24)$$

where,

$$b_q = \frac{\int_y y \sum_j \frac{P(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} dy}{\int_y \sum_j \frac{\Pr(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} dy}$$

$$v_q = \int_y \sum_j \frac{P(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty} \mu_j(y)dy} dy \quad (25)$$

3.3.2 Generalized Fuzzy Model

The Generalized fuzzy model (GFM) model Azeem (2000) generalizes both the ML model and the TS (Takagi- Sugeno) model. The output in GFM model has the properties of fuzziness (ML) around varying centroid (TS) of the consequent part of a rule. Let us consider a rule of the form

R^k: if x^k is A^k then y is B^k (f^k (x^k), v_k).

where B^k is the output fuzzy set,

v^k is the index of fuzziness \

f^k is the output function.

Using (23), we can obtain the defuzzified output y^0 as

$$y^0 = \sum_{q=1}^{Q} \frac{\mu^q(x).v_q}{\sum_{q'=1}^{Q}\mu^{q'}(x).v_{q'}}.f^q(x) \qquad (26)$$

where
$f^q(x)$ is a varying singleton. It may be linear or non-linear. The linear form is:

$$f^q(x) == b_{q_0} + b_{q_0}x_1 + \ldots + b_{q_D}x_D$$

Replacing the value of b_q from (22) into (26) we get

$$y^0 = \sum_{q=1}^{Q} \frac{\mu^q(x).\int_y \sum_j \frac{\Pr(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty}\mu_j(y)dy}dy}{\sum_{q'=1}^{Q}\mu^{q'}(x).\int_y \sum_j \frac{\Pr(O_j|x)\,\mu_j(y)}{\int_{-\infty}^{\infty}\mu_j(y)dy}dy}.f^q(x) \qquad (27)$$

4 COMPUTATION OF PROBABILISTIC POSSIBILITY FROM PROBABILITY FUZZY RULES

How to compute the probabilistic possibility from a probabilistic fuzzy rule is presented as an algorithm here.

Step 1: Determine the fuzzy rules that are applicable for the given test input, x.

Step 2: Evaluate the membership values of the input fuzzy sets.

Step 3: Determine the membership values of the output fuzzy sets.

Step 4: Calculate the conditional probability of each probabilistic output using (15).

Step 6: Find the net conditional possibility of the output using (17).

Step 7: This step is an optional step. The relations for finding the defuzzified output for the fuzzy models as in (25) and (27) may be used in case all the values of parameters are available besides the possibility term (as computed in Step 6).

5 CASE-STUDY 1

Let us contemplate the functioning of a fuzzy air conditioner example in Kosko (1993) described by five input linguistic terms/in the form of fuzzy sets on X, along with five output linguistic terms represented by fuzzy sets on Y:

- The input fuzzy sets on X are: **Cold**, **Cool**, **Just Right**, **Warm**, and **Hot**
- The output fuzzy sets on Y are: **Stop**, **Slow**, **Medium**, **Fast**, and **Blast**

The following fuzzy rules are framed from an expert's knowledge.

1. If temperature is **cold,** motor speed is **stop**

2. If temperature is **cool,** motor speed is **slow**

3. If temperature is **just** right, motor speed is **medium**

4. If temperature is **warm**, motor speed is **fast**

5. If temperature is **hot**, motor speed is **blast**

A realistic representation of the above in the garb of PFR when the probabilities are associated with the outputs is the main concern now. The corresponding PFR of Rule 1 is as follows:

If temperature is **cold** then
motor speed is **stop** with probability 70%
& motor speed is **slow** with probability 20%
& motor speed is **medium** with probability 8%
& motor speed is **fast** with probability 2%

Similarly, other PFRs can also be constructed. The first column in Table 1 gives the antecedent value for each rule. The remaining columns give the values of the possible outputs for each rule. The conditional possibility of the output, is calculated when the inputs are 63^oF and 68^oF.

Table 1: The Probabilistic Fuzzy Rule-set.

#	Temp(X)	P_{Stop}	P_{Slow}	P_{Medium}	P_{Fast}	P_{Blast}
1	Cold	0.7	0.2	0.08	0.02	0.0
2	Cool	0.1	0.7	0.1	0.08	0.02
3	Jt Right	0.05	0.1	0.7	0.1	0.05
4	Warm	0.02	0.08	0.1	0.7	0.1
5	Hot	0.0	0.02	0.08	0.2	0.7

5.1 Case: Input 63^oF

It may be noted that the output fuzzy set with the highest probability is only opted followed by the others in the line. The farther a fuzzy set is from this output, the lesser is its probability. We will elaborate on the steps using the above example. The input and output fuzzy sets for this example are shown in Fig. 1 and Fig. 2 respectively. The corresponding applicable PFRs are as follows:

If temperature is **just right**
then motor speed is **stop** with probability 5%
& motor Speed is **slow** with probability 10%
& motor speed is **medium** with probability 70%
& motor speed is **fast** with probability 10%
& motor speed is **blast** with probability 5%

If temperature is **cool**
then motor speed is **stop** with probability 10%
& motor speed is **slow** with probability 70%
& motor speed is **medium** with probability 10%

& motor speed is **fast** with probability 8%
& motor speed is **blast** with probability 2%

The fuzzy membership values for input fuzzy sets, $\mu_0(x)$ as noted from Fig. 1 and Fig. 2 are as follows:

μ_0(Just Right)**:** 80% (0.8) μ_0(Cool)**:** 15% (0.15)

The fuzzy membership values for output fuzzy sets, $\mu_1(x)$ as noted from Fig. 1 and Fig. 2 are as follows:

μ_1(Slow)**:** 15% (0.15) μ_1(Medium)**:** 80% (0.8)

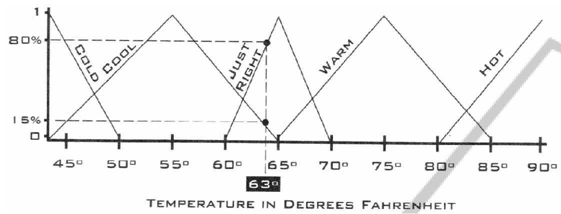

Figure 1: Input fuzzy sets and their membership.

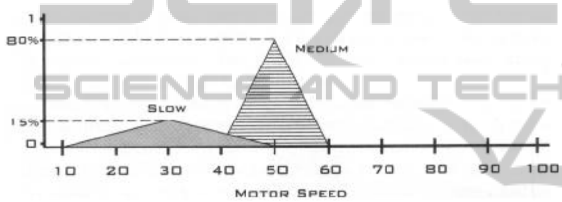

Figure 2: Output fuzzy sets and their membership values when input temperature is 63oF.

We apply (15) to calculate the conditional probability for each probabilistic output of a fuzzy rule that is applicable, given the input temperature is 63^oF.

In the light of (15), we have

$$P(O_j|\, x) = \sum_{q=1} \mu_q(x)\, P(O_j \mid A_q)$$

P(O_{Stop}|x) = (0.8 * 0.05) + (0.15 * 0.10) = 0.055
P (O_{slow} | x) = [(0.8 * 0.1) + (0.15 * 0.7) = 0.185
P (O_{Medium} | x) = [(0.8 * 0.7) + (0.15 * 0.1) = 0.575
P (O_{fast} | x) = [(0.8 * 0.1) + (0.15 * 0.08) = 0.092
P (O_{blast} | x) = [(0.8 * 0.05) + (0.15 * 0.02) = 0.043

The net conditional possibility for the output is calculated using (17) as

π (y | x) = (0 + (0.185 * 0.15) + (0.575 * 0.8) + 0 + 0) = 0.48775

Thus having got the value of the conditional probability, the same can be substituted along with other values in the relations for ML and GFM models as per (25) and (27) to obtain the defuzzified output.

Comparison of the Output with Basic Fuzzy Rules. We now use the above algorithm to estimate the effect of the probabilistic output on the net output conditional possibility. The fuzzy rules of interest are as follows:

1. If temperature is **cold** then motor speed is **stop**
2. If temperature is **cool** then motor speed is **slow**
3. If temperature is **just** right then motor speed is **medium**
4. If temperature is **warm** then motor speed is **fast**
5. If temperature is **hot** then motor speed is **blast**

The input and output fuzzy sets and their corresponding membership values are the same as above. The fuzzy sets for the given test input are shown in Fig.2 and the valid fuzzy rules are:

If temperature is **just right** then motor speed is **medium.**

If temperature is **cool** then motor speed is **slow.**

The conditional probability is computed using (15) as

$$P(O_j|\, x) = \sum_{q=1} \mu_q(x)\, P(O_j \mid A_q)$$

The conditional probabilities are evaluated as:

P (O_{Stop} | x) = 0
P (O_{slow} | x) = (0.15 * 1) = 0.15
P (O_{Medium} | x) = (0.8 * 1) = 0.8
P (O_{fast} | x) = 0 P (O_{blast} | x) = 0

The net conditional possibility is found using (17) as

π (y | x) = 0 + (1 * 0.15) + (1 * 0.8) + 0 + 0 = 0.95

5.2 Case: Input 68^oF

The fuzzy input and output membership values are:

μ_0(Warm)**:** 0.2 μ_0(Just Right)**:** 0.55

μ_1(Medium)**:**0.55 μ_1(Fast)**:** 0.2

Applying (15) and taking Table 1 into account, the conditional probability can be computed as in 5.1.

P(O_{Stop} | x) = 0.0315 P (O_{slow} | x) = 0.071
P (O_{Medium} | x) = 0.525 P (O_{fast} | x) = 0.075
P (O_{blast} | x) = 0.0475

The net conditional possibility is found using (17) as above in 5.1.

π (y | x) = (0.55 * 0.525) + (0.2 * 0.075) = 0.303

Comparison of the Output with Basic Fuzzy Rules when Input is 68^0F. The conditional probabilities in the case of basic fuzzy rules can be computed as

$P(O_{Stop} \mid x) = 0$ $\quad P(O_{slow} \mid x) = 0$
$P(O_{Medium} \mid x) = 0.55$ $\quad P(O_{fast} \mid x) = 0.2$
$P(O_{blast} \mid x) = 0$

The net conditional possibility is found using (17) as

$$\pi(y \mid x) = 0 + (1 * 0.55) + (1 * 0.2) + 0 + 0 = 0.75$$

It is pertinent to note that what we have here is the possibility in the probabilistic framework. So, in this example, the overall conditional possibility would converge to the sum of the individual possibilities, whereas in the case of probabilistic fuzzy rules, the conditional possibility is a factor of probabilities as well as possibilities.

6 CASE-STUDY 2

Consider designing a fuzzy controller for the control of liquid level in a tank by varying its valve position Meghdadi(2001). The simple fuzzy controller employs Δh and dh/dt as inputs and $d\alpha/dt$ (rate of change of valve position α , $\alpha \in [0,1]$) as the output, where h is the actual liquid level, h_d is desired value of the level, and $\Delta h = h_d - h$ is the error in the desired level.

Three Gaussian membership functions for three input fuzzy sets (**negative**, **zero**, **positive**) are applicable on the input variables Δh and dh/dt. The output fuzzy sets (**close-fast**, **close-slow**, **no-change**, **open-slow**, **open-fast**) have triangular membership functions. The following fuzzy rules are selected using a human expert's knowledge.

R1. If Δh is **zero** then dα/dt is **no-change**
R2. If Δh is **positive** then dα/dt is **open-fast**
R3. If Δh is **negative** then dα/dt is **close-fast**
R4. If Δh is **zero** and dh/dt is **positive** then dα/dt is **close-slow**
R5. If Δh is **zero** and dh/dt is **negative** then dα/dt is **open-slow**

In order to model the existing scepticism of humans' opinion in defining the optimal rule set, we may substitute each conventional rule with a probabilistic fuzzy rule with the output probability vector P defined such that the only output sets of the conventional fuzzy rules are the most probable from the probabilistic fuzzy rules. Also the neighbouring fuzzy sets in the PFR have smaller probabilities and the other fuzzy sets have zero probabilities. For example rule RI in the above rule set may be modified as follows:

RI. If Δh is **zero** then
dα/dt is **no-change** with probability 80%
& dα/dt is **close-slow** with probability 10%
& dα/dt is **open-slow** with probability 10%

The consequent part of the PFR can be thus expressed in a compact form using the output probabilities vector P. The sample probabilistic fuzzy rule set is given in Table 2.

Table 2: Probabilistic Fuzzy Rule-set for the Liquid Level Fuzzy Controller.

#	Q_1	V_1	Q_2	V_2	P_{c-f}	P_{c-s}	P_{n-c}	P_{o-s}	P_{o-f}
1	Δh	0			0	0.1	0.8	0.1	0
2	Δh	+			0	0	0	0.2	0.8
3	Δh	-			0.8	0.2	0	0	0
4	Δh	0	$\frac{dh}{dt}$	+	0.1	0.8	0.1	0	0
5	Δh	0	$\frac{dh}{dt}$	-	0	0	0.1	0.8	0.1

Let Input: Δh = 0.
The PFRs for the given input are as follows:

R1. If Δh is **zero** then dα/dt is **no-change** with probability 80%
& dα/dt is **close-slow** with probability 10%
& dα/dt is **open-slow** with probability 10%

R4. If Δh is **zero** and dh/dt is **positive** then dα/dt is **no-change** with probability 10%
& dα/dt is **close-slow** with probability 80%
& dα/dt is **close-fast** with probability 10%

R5. If Δh is **zero** and dh/dt is **negative** then dα/dt is **no-change** with probability 10%
& dα/dt is **open-slow** with probability 80%
& dα/dt is **open-fast** with probability 10%

The membership values, $\mu_{zero}(x)$, $\mu_{Positive}(x)$ and $\mu_{Negative}(x)$ for the given input are given as follows:

$\mu_{Zero}(\Delta h)$: 1 $\quad \mu_{positive}(\frac{dh}{dt})$: 1 $\quad \mu_{Negative}(\frac{dh}{dt})$: 0

The membership grades for the output fuzzy sets are given as follows:

$\mu_{NoChange}(\frac{d\alpha}{dt})$: 1 $\quad \mu_{Slow}(\frac{d\alpha}{dt})$: 0.15 $\quad \mu_{Fast}(\frac{d\alpha}{dt})$: 0.15

The conditional probability is calculated using (15) for each probabilistic output in each fuzzy rule that is applicable, given the input value.

$P(O_{no\text{-}change} \mid x) = [(1 * 0.8) + (1 * 0.1) + (0 * 0.1)]/2 = 0.45.$

Note:- The probability values are normalized by taking the number of the input fuzzy sets as denominator. Similarly,

$P(O_{close\text{-}slow} \mid x) = 0.45$ $\quad P(O_{close\text{-}fast} \mid x) = 0.1$
$P(O_{open\text{-}slow} \mid x) = 0.1$ $\quad P(O_{open\text{-}fast} \mid x) = 0$

We arrive at the net consolidated conditional possibility for the output using (17) as

π (y|x) = (0.45 * 1) + (0.45 * 0.15) + (0.1 * 0.15) + (0.1 * 0.15) + (0 * 0.15) = 0.5475

Thus having obtained the value of net membership, the same can be substituted in the ML and GFM models to obtain (v_q, b_q). It can also be noted that for the basic fuzzy rules the net conditional possibility for a given input is the sum of the memberships of the various output fuzzy sets that are applicable.

7 CONCLUSIONS

It is shown how a probabilistic fuzzy framework is more flexible and convenient than the conventional methodology. As a consequence of this the probabilistic possibility is derived from the applicable probabilistic fuzzy rules which constitute the probabilistic fuzzy system with the help of the fuzzy modelling. The utility of probabilistic fuzzy systems in representing real world situations is also highlighted. Its ability to represent fuzzy nature of situations along with corresponding probabilistic information brings it much closer to real-world.

Two examples dealing with the practical applications of an air-conditioner and a liquid level controller are taken up to demonstrate a probabilistic fuzzy system. It is noticed from this study how the probability of the output affects the net possibility for a particular test input.

It is observed that in the case of probabilistic fuzzy rules, the conditional output probabilistic possibility of an output fuzzy set for a given input spans over the applicable output fuzzy sets. A basic fuzzy rule is a special case of probabilistic fuzzy rule in which there is only one output for a fuzzy rule that translates into 100% probability for that particular output. The methodology proposed for calculating conditional probabilistic possibility for PFRs fits well with basic fuzzy rules and leads to the intuitively acceptable result. The proposed work provides functionality to process the probabilistic fuzzy rules that are better equipped to represent the real-world situations.

Another feature of probabilistic fuzzy rules is the enhanced adaptability in view of the outputs with varying probabilities. This is borne out of the fact that the outputs in the fuzzy rules are context-dependent hence vary accordingly.

The proposed approach to calculate the possibility from probability can be tailored to a specific application depending upon the output membership functions and their probabilities. This can also be extended to represent probabilistic rough fuzzy sets and other types of fuzzy sets so as to increase its utility in capturing the higher forms of uncertainty from probability since the probabilistic information along with possibility aids the decision making in the solution of the real-world problems. The proposed framework has the capability to address the uncertainty arising from fuzziness and vagueness in the wake of their random occurrences.

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