APPROXIMATE REASONING IN CANCER SURGERY

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Abstract: The compositional rule of inference, grounded on the *modus ponens* law, is one of the most effective fuzzy systems. We modify the classical version of the rule (Zadeh, 1973, 1979) to propose an original model, which concerns determining an operation chance for gastric cancer patients. The operation prognosis will be dependent on values of biological markers indicating the progress of the disease.

1 INTRODUCTION

One of the early systems, evolved by Zadeh as an approach to decision making in vague circumstances, was the technique of approximate reasoning (Zadeh, 1973, 1979). The compositional rule of inference found adherents who adapted the primary foundations of the theory to own models (Baldwin and Pilsworth, 1979; Mizumoto and Zimmermann, 1982; Zimmermann, 2002).

Some trials of technical use of approximate reasoning have been made, but it is still difficult to find a medical application based on the inference rule. The rule was once tested by the author in order to make decisions concerning operation chances for gastric cancer patients (Rakus-Andersson, 2009). The decisions were based on values of one biological marker C-reactive proteins *CRP*, regarded by physicians as the essential index of cancer progress.

In the current paper we wish to extend the number of clinical symptoms in the model. In practice we want to add a value of age to *CRP*-value (Do Kyong-Kim *et al*., 2009) to deduce a verbal evaluation of the operation chance for post-surgical survival in cancer diseases.

We discuss the approximate reasoning structures in Section 2. Fuzzy sets, taking place in the model, will be created in Section 3. Section 4 is added as a presentation of the algorithm prognosis made for an individual patient.

2 RULE OF INFERENCE

Surgical decisions are made with the highest thoughtfulness in the case of patients suffering from cancer. The physician wants to prognosticate the operation role positively; we therefore introduce the concept "operation chance" to determine the outcome of a surgery.

The most decisive clinical markers *CRP* and age found in an individual patient will constitute the input data in the approximate reasoning model to evaluate the operation chance for survival.

Let us state a logical compound tautology (Rakus-Andersson, 2009)

$$\begin{aligned}&(\mathrm{IF}\,(p_1\mathrm{AND}\cdots\mathrm{AND}p_p)\,\mathrm{AND}\,((\mathrm{IF}\,(p_1\mathrm{AND}\cdots\\&\mathrm{AND}p_p)\,\mathrm{THEN}\ q)\mathrm{ELSE}\,(\mathrm{IF}\,(\mathrm{NOT}\,(p_1\mathrm{AND}\cdots\\&\mathrm{AND}p_p\,))\mathrm{THEN}\,(\mathrm{NOT}\,q)))\,)\,\mathrm{THEN}\,q.\end{aligned} \tag{1}$$

In accordance with the generalized law *modus ponens* (Zadeh, 1973) we interpret (1) as a statement

$$\begin{aligned}&(\mathrm{IF}(\,p_1{}' \wedge\cdots\wedge p_p{}'\,)\mathrm{AND}((\mathrm{IF}(\,p_1\wedge\cdots\wedge p_p\,)\mathrm{THEN}\\&q)\mathrm{ELSE}(\mathrm{IF}(\mathrm{NOT}(\,p_1\wedge\cdots\wedge p_p\,))\mathrm{THEN}\ (\mathrm{NOT}q))))\\&\mathrm{THEN}\quad q`,\end{aligned} \tag{2}$$

provided that the semantic meaning of p_i and p_i\`, $i$ = 1,…, $p$, ($q$ and $q$\` respectively) is very close.

In (2) let p_i and p_i\` be mapped in fuzzy sets $P_i$ and $P_i$\` in the universes X_i and let q and q\` be expressed by fuzzy sets $Q$ and $Q$\` in the universe Y.

We now make a feedback to the medical task previously outlined to evaluate the operation chances as verbal expressions.

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Let $S_1,\dots,S_p$ denote clinical markers possessing the decisive power in the evaluation of the operation chance. We regard S_i, $i = 1,\dots, p$, like the symptoms whose growth levels are assimilated with codes. Values of the codes form the universes X_i = "S_i's *levels*" = $\{1,\dots,k_i,\dots,n_i\}$. Assume that level 1 is associated with the slightly heightened symptom values whereas level n_i indicates the dangerous symptom intensity (Rakus-Andersson, 2009).

Universe Y consists of words describing operation chance priorities. We set Y = "*operation chance priorities*" = {L_1 = "*none*", L_2 = "*very little*", L_3 = "*little*", L_4 = "*moderate*", L_5 = "*promising*", L_6 = "*very promising*", L_7 = "*totally promising*"} assuming that Y is experimentally restricted to seven chance priorities.

We assign p_i\`, $p_i$, $q$ and $q$\` to sentences (Rakus-Andersson, 2009)

p_i\` = "symptom S_i is found in patient on level k_i, $i = 1,\dots,p$",

p_i = "lower levels of S_i are essential for a positive operation outcome",

q = "operation chance can be estimated on the basis of S_1 and…and S_p"

and

q\` = "patient with the $k_1$-level of $S_1$ and…and the $k_p$-level of $S_p$ gets an estimated operation chance as this $L_l$, which has the highest degree in $Q$\`, $l = 1,\dots, 7$",

Rule (2) will thus become a scheme

(IF ("symptom S_1 is found in patient on level k_1" and…and "symptom S_p is found in patient on level k_p" = $P_1^{'} \cap \cdots \cap P_p^{'}$) AND ((IF "lower levels of S_1 are essential for a positive operation outcome" and…and "lower levels of S_p are essential for a positive operation outcome" = $P_1 \cap \cdots \cap P_p$ THEN "operation chance can be estimated on the basis of S_1 and…and S_p" = Q) ELSE (IF it is not true that "lower levels of S_1 are essential for a positive operation outcome" and…and "lower levels of S_p are essential for a positive operation outcome" = $C(P_1 \cap \cdots \cap P_p)$ THEN "operation chance cannot be estimated on the basis of S_1 and…and S_p" = CQ))) THEN "patient with the k_1-level of S_1 and…and the k_p-level of S_p gets an estimated operation chance as this L_l, which has the highest degree in Q\`, $l = 1,\dots, 7$" = $Q$\`.

$C(P_1 \cap \cdots \cap P_p)$ and CQ are complements of $P_1 \cap \cdots \cap P_p$ and Q.

3 DATA SETS IN *X* AND Y

The decision model, sketched in Section 2, includes operations on fuzzy sets P_i\`, $P_i$, $Q$ and $Q$\`. First we design fuzzy sets in X_i, $i = 1,\dots,p$, as structures

$$\begin{aligned}
&P_i^{'} = \\
&\{(1,\mu_{P_i^{'}}(1)),\dots,(k_i,\mu_{P_i^{'}}(k_i)),\dots,(n_i,\mu_{P_i^{'}}(n_i))\} \\
&= \{\dots,(k_i-2,\tfrac{n_i-2}{n_i}),(k_i-1,\tfrac{n_i-1}{n_i}),(k_i,1), \\
&(k_i+1,\tfrac{n_i-1}{n_i}),(k_i+2,\tfrac{n_i-2}{n_i}),\dots\}
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
&P_i = \\
&\{(1,\mu_{P_i}(1)),\dots,(k_i,\mu_{P_i}(k_i)),\dots,(n_i,\mu_{P_i}(n_i))\} = \\
&\{(1,\tfrac{n_i}{n_i}),\dots,(k_i,\tfrac{n_i-(k_i-1)}{n_i}),\dots,(n_i,\tfrac{1}{n_i})\}
\end{aligned} \tag{4}$$

referring to S_i due to the definitions of p_i and p_i\`.

The set Q is sophisticated to be stated as a fuzzy set since its support consists of other fuzzy sets L_l, $l = 1,\dots,7$, defined in a symbolic chance reference set $Z = [z_{\min}, z_{\max}] = [0,1]$. To find restrictions of L_l we study the technique of Rakus-Andersson (2010).

Suppose that $L_1,\dots,L_m$ are included in the linguistic list, where m is an odd positive integer greater or equal to 5. Supports of the restrictions $\mu_{L_l}(z)$, $l = 1,\dots,m$, will cover parts of the reference set $Z = [0,1]$. We introduce E to be the length of Z.

We divide all expressions L_l in three groups, namely, a family of "*leftmost*" sets $L_1,\dots,L_{\frac{m-1}{2}}$, the set $L_{\frac{m+1}{2}}$ "*in the middle*" and a collection of "*rightmost*" sets $X_{\frac{m+3}{2}},\dots,L_m$.

The "*leftmost*" family is given by

$$\mu_{L_t}(z) = \begin{cases}
1 \text{ for } z \le z_{\min} + (t-1)\frac{E}{m-1}, \\
1 - 2\left(\frac{z-\left(z_{\min}+(t-1)\frac{E}{m-1}\right)}{\frac{E}{m-1}}\right)^2 \text{ for} \\
z_{\min} + (t-1)\frac{E}{m-1} \le z \le \frac{E}{2(m-1)} + (t-1)\frac{E}{m-1}, \\
2\left(\frac{z-\left(\frac{E}{m-1}+(t-1)\frac{E}{m-1}\right)}{\frac{E}{m-1}}\right)^2 \text{ for} \\
\frac{E}{2(m-1)} + (t-1)\frac{E}{m-1} \le z \le \frac{E}{m-1} + (t-1)\frac{E}{m-1}, \\
0 \text{ for } z \ge \frac{E}{m-1} + (t-1)\frac{E}{m-1}
\end{cases} \tag{5}$$

for parameter t, $t = 1,\dots,\frac{m-1}{2}$.

To implement the "*rightmost*" functions we use

$$\mu_{L_{m-t+1}}(z)=\begin{cases} 0 \text{ for } z \le E-\left(\frac{E}{m-1}+(t-1)\frac{E}{m-1}\right), \\ 2\left(\frac{z-\left(E-\left(\frac{E}{m-1}+(t-1)\frac{E}{m-1}\right)\right)}{\frac{E}{m-1}}\right)^2 \text{ for} \\ E-\left(\frac{E}{m-1}+(t-1)\frac{E}{m-1}\right)\le z\le E-\left(\frac{E}{2(m-1)}+(t-1)\frac{E}{m-1}\right), \\ 1-2\left(\frac{z-\left(E-\left(z_{\min}+(t-1)\frac{E}{m-1}\right)\right)}{\frac{E}{m-1}}\right)^2 \text{ for} \\ E-\left(\frac{E}{2(m-1)}+(t-1)\frac{E}{m-1}\right)\le z\le E-\left(z_{\min}+(t-1)\frac{E}{m-1}\right), \\ 1 \text{ for } z\ge E-\left(z_{\min}+(t-1)\frac{E}{m-1}\right) \end{cases} \tag{6}$$

for $t = 1,\ldots,\frac{m-1}{2}$. The t-values are set in the formula (6) in the reverse order to generate the sequence $L_{\frac{m+3}{2}},\ldots,L_m$.

The function of $L_{\frac{m+1}{2}}$ is constructed as

$$\mu_{L_{\frac{m+1}{2}}}(z)=\begin{cases} 0 & \text{for } z\le\frac{E(m-3)}{2(m-1)}, \\ 2\left(\frac{z-\frac{E(m-3)}{2(m-1)}}{\frac{E}{m-1}}\right)^2 & \text{for } \frac{E(m-3)}{2(m-1)}\le z\le\frac{E(m-2)}{2(m-1)}, \\ 1-2\left(\frac{z-\frac{E}{2}}{\frac{E}{m-1}}\right)^2 & \text{for } \frac{E(m-2)}{2(m-1)}\le z\le\frac{E}{2}, \\ 1-2\left(\frac{z-\frac{E}{2}}{\frac{E}{m-1}}\right)^2 & \text{for } \frac{E}{2}\le z\le\frac{Em}{2(m-1)}, \\ 2\left(\frac{z-\frac{E(m+1)}{2(m-1)}}{\frac{E}{m-1}}\right)^2 & \text{for } \frac{Em}{2(m-1)}\le z\le\frac{E(m+1)}{2(m-1)}, \\ 0 & \text{for } z\ge\frac{E(m+1)}{2(m-1)}. \end{cases} \tag{7}$$

For the list "*operation chance priorities*" we state $z_{\min} = 0$, $m = 7$ and $E = 1$. Fuzzy sets L_l, $l = 1,\ldots,7$, are depicted in Fig. 1.

When defuzzifying the sets L_l we consider z-coordinates of the intersection points between $\mu_{L_l}(z)=1$ and $\mu_{L_l}(z)<1$. We denote the z-values by $z(L_1) = 0$, $z(L_2) = 0.166$, $z(L_3) = 0.338$, $z(L_4) = 0.5$, $z(L_5) = 0.668$, $z(L_6) = 0.834$, $z(L_7) = 1$ and we let them represent $L_1,\ldots,L_7$.

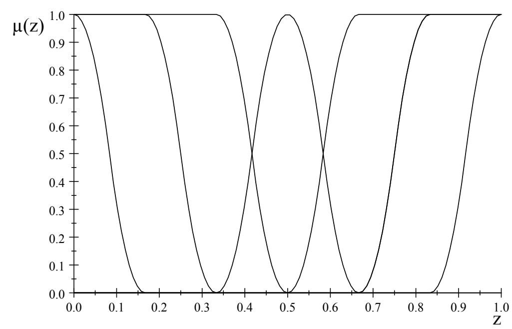

Figure 1: Fuzzy sets L_1–L_7.

Then we build the set "*numerical operation chance*", which gets the constraint $s(z, 0, 0.5, 1)$ over [0,1]. We compute the degrees of $z(L_l)$ via this constraint to obtain set Q in the form

$$Q=\{(L_1,0),(L_2,0.055),(L_3,0.22),(L_4,0.5),(L_5,0.78),(L_6,0.945),(L_7,1)\}. \tag{8}$$

Further, for all $(k_1,\ldots,k_p) \in X_1\times\ldots\times X_p$ we determine the intersections

$$\begin{aligned} &P_1' \cap\cdots\cap P_p' = \\ &\{((1,\ldots,1),\min(\mu_{P_1`}(1),\ldots,\mu_{P_p`}(1))),\ldots, \\ &((k_1,\ldots,k_p),\min(\mu_{P_1`}(k_1),\ldots,\mu_{P_p`}(k_p))),\ldots, \\ &((n_1,\ldots,n_p),\min(\mu_{P_1`}(n_1),\ldots,\mu_{P_p`}(n_p)))\} \end{aligned} \tag{9}$$

and

$$\begin{aligned} &P_1 \cap\cdots\cap P_p = \\ &\{((1,\ldots,1),\min(\mu_{P_1}(1),\ldots,\mu_{P_p}(1))),\ldots, \\ &((k_1,\ldots,k_p),\min(\mu_{P_1}(k_1),\ldots,\mu_{P_p}(k_p))),\ldots, \\ &((n_1,\ldots,n_p),\min(\mu_{P_1}(n_1),\ldots,\mu_{P_p}(n_p)))\}. \end{aligned} \tag{10}$$

In conformity with Zadeh (1973) and Zimmermann, (2002) we introduce the matrix R being a mathematical expression of the implication

(IF $P_1\cap\cdots\cap P_p$ THEN Q)

ELSE (IF $C(P_1\cap\cdots\cap P_p)$ THEN CQ).

The membership function of R is yielded by

$$\begin{aligned} &\mu_R((k_1,\ldots,k_p),L_l)= \\ &\min(1,(\mu_{C(P_1\cap\ldots\cap P_p)}(k_1,\ldots,k_p)+\mu_Q(L_l)), \\ &(\mu_{P_1\cap\ldots\cap P_p}(k_1,\ldots,k_p)+\mu_{CQ}(L_l))) \end{aligned} \tag{11}$$

for all $(k_1,\ldots,k_p) \in X_1\times\ldots\times X_p$ and all $L_l \in Y$.

Set $Q`$ will be formed as (Zadeh, 1973)

$$Q`=(P_1'\cap\ldots\cap P_p')\circ R. \tag{12}$$

$Q`$ is designated by the membership function

$$\begin{aligned} &\mu_{Q`}(L_l)=\max_{\substack{(k_1,\ldots,k_p)\in\\ X_1\times\cdots\times X_p}}(\min(\mu_{P_1'\cap\cdots\cap P_p'}(k_1,\ldots,k_p), \\ &\mu_R((k_1,\ldots,k_p),L_l))). \end{aligned} \tag{13}$$

By comparing magnitudes of membership degrees in set $Q`$ with respect to all $L_l$, $l = 1,\ldots,7$, we select this chance priority $L_l$, which assists the largest value of $\mu_{Q`}(L_l)$.

4 CHANCE DETERMINATION

The CRP-value and age are decisive markers of the prognosis in cancer surgery (Do-Kyong Kim *et al*., 2009).

The heightened values of *CRP* (measured in milligrams per liter) are discerned in levels

1 = "*almost normal*" for *CRP* < 10,
2 = "*heightened*" if 10 ≤ *CRP* ≤ 20,
3 = "*very heightened*" if 20 ≤ *CRP* ≤ 25,
4 = "*dangerously heightened*" for *CRP* > 25.

The age borders are decided as

1 = "*not advanced for surgery*" if "*age*" < 60,
2 = "*advanced for surgery*" if 60 ≤ "*age*" ≤ 80,
3 = "*dangerous for surgery*" if "*age*" > 80.

Suppose that in a seventy-year-old patient the *CRP*-value is measured to be 18.

Due to (4) and (10) sets P_1, P_2 and their intersection are expressed as

$$\begin{aligned} &P_1 = \{(1,1),(2,0.75),(3,0.5),(4,0.25)\}, \\ &P_2 = \{(1,1),(2,0.66),(3,0.34)\}, \\ &P_1 \cap P_2 = \{((1,1),1),...,((3,2),0.5),..., \\ &((4,3),0.25)\} \end{aligned} \tag{14}$$

while P_1\`, $P_2$\` and their cut are computed, with respect to (3) and (9), as

$$\begin{aligned} &P_1' = \{(1,0.75),(2,1),(3,0.75),(4,0.5)\}, \\ &P_2' = \{(1,0.66),(2,1),(3,0.66)\}, \\ &P_1' \cap P_2' = \{((1,1),0.66),...,((3,2),0.75),..., \\ &((4,3),0.5)\} \end{aligned} \tag{15}$$

provided that $X_1 = \{1,2,3,4\}$ and $X_2 = \{1,2,3\}$.

Matrix R, found in compliance with (11), is expanded as a two-dimensional table

$$R = \begin{array}{c} \\ (1,1) \\ \vdots \\ (3,2) \\ \vdots \\ (4,3) \end{array} \begin{array}{c} \begin{array}{ccc} L_1 & \cdots L_4 & \cdots L_7 \end{array} \\ \begin{bmatrix} 0 & \cdots 0.5 & \cdots 1 \\ \vdots & & \vdots \\ 0.5 & \cdots 1 & \cdots 0.5 \\ \vdots & & \vdots \\ 0.75 & \cdots 0.75 & \cdots 0.25 \end{bmatrix} \end{array}. \tag{16}$$

We in6sert R given by (16) and $P_1' \cap P_2'$ determined by (15) in (12) in order to estimate

$$\begin{aligned} &Q\text{`} = \{(L_1,0.66),(L_2,0.715),(L_3,0.72), \\ &(L_4,0.84),(L_5,0.88),(L_6,0.715),(L_7,0.66)\}. \end{aligned} \tag{17}$$

The largest membership degree in (17) points out chance L_5 = "*promising*" for a result of the operation on the elderly patient whose *CRP*-index is evaluated on the second growth level.

5 CONCLUSIONS

We have adapted approximate reasoning as a deductive algorithm to introduce the idea of evaluating the operation chance for patients with heightened values of biological indices in cancer diseases.

The formulas of membership functions in data sets have been expanded by applying a formal mathematical design invented by the author. The data sets involve parametric families of functions, which allow preparing a computer program. We have tested a large sample of patient data to get the results mostly converging to the physicians' prognoses. This confirms reliability of the system.

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REFERENCES

Baldwin, J. F., Pilsworth, B. W., 1979. A model of fuzzy reasoning through multi-valued logic and set theory. In *Int. J. of Man-Machine Studies 11*, 351-380.

Do-Kyong Kim *et al.*, 2009. Clinical significances of preoperative serum interleukin-6 and C-reactive protein level in operable gastric cancer. In *BMC Cancer 2009*, *9*, 155 -156.

Mizumoto, M., Zimmermann, H. J., 1982. Comparison of fuzzy reasoning methods. *FSS 8*, 253-283.

Rakus-Andersson, E. 2009. Approximate reasoning in surgical decisions. In *Proceedings of IFSA 2009*, Instituto Superior Technico, Lisbon, 225-230.

Rakus-Andersson, E., 2010. Adjusted *s*-parametric functions in the creation of symmetric constraints. In *Proceedings of ISDA 2010,* University of Cairo, 451-456.

Zadeh, L. A., 1973. Outline of a new approach to the analysis of complex systems and decision process. In *IEEE Transactions on Systems, Man, and Cybernetics*, *MSC-3(1)*, 28-44.

Zadeh, L. A., 1979. A theory of approximate reasoning. In *Machine Intelligence 9*, 149-174.

Zimmermann, H. J., 2002. *Fuzzy Set Theory and its Applications*, Kluwer, Boston Dortrecht London, 4th edition.