

INDIVIDUALLY AND COLLECTIVELY TREATED NEURONS AND ITS APPLICATION TO SOM

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Abstract: In this paper, we propose a new type of information-theoretic method to interact individually treated neurons with collectively treated neurons. The interaction is determined by the interaction parameter α . As the parameter α is increased, the effect of collectiveness is larger. On the other hand, when the parameter α is smaller, the effect of individuality becomes dominant. We applied this method to the self-organizing maps in which much attention has been paid to the collectiveness of neurons. This biased attention has, in our view, shown difficulty in interpreting final SOM knowledge. We conducted a preliminary experiment in which the Ionosphere data from the machine learning database was analyzed. Experimental results confirmed that improved performance could be obtained by controlling the interaction of individuality with collectiveness. In particular, the trustworthiness and continuity are gradually increased by making the parameter α larger. In addition, the class boundaries become sharper by using the interaction.

1 INTRODUCTION

Neurons in neural networks have been treated individually or collectively in different learning methods. No attempts have been made to examine the interaction of individuality with collectiveness. In this paper, we postulate that neurons should be treated individually and collectively and these two types of neurons should interact with each other to have special effect for neural learning. We, in particular, focus upon the self-organizing maps (SOM) (Kohonen, 1988), (Kohonen, 1995). Because only the collectiveness of neurons has been taken into account in the SOM, ignoring the properties of individual treated neurons. Thus, it is easy to demonstrate the effect of interaction using the SOM.

The SOM is a well-known technique for the vector quantization and the vector projection from high dimensional input spaces into low dimensional output spaces. However, it is hard to interpret final SOM knowledge from simple visual inspection. Thus, many different types of visualization techniques have been proposed, for example, the U-matrix and its variants (Ultsch, 2003b), (Ultsch, 2003a) visualization of component planes (Vesanto, 1999), linear and non-linear dimensionality reduction methods such as the principal component analysis (PCA) (Bishop, 1995), Sammon Map (Sammon, 1969) and many non-linear

methods (Joshua B. Tenenbaum and Langford, 2000), (Roweis and Saul, 2000), (Demartines and Herault, 1997), and the responses to data samples (Vesanto, 1999). Recently, more advanced visualization techniques were proposed such as gradient field and borderline visualization techniques (Georg Polzlbauer and Rauber, 2006), the connectivity matrix of prototype vectors (Tasdemir and Merenyi, 2009) and the gradient-based SOM matrix (Costa, 2010) and so on.

Even using these visualization techniques, it remains to be difficult to interpret final SOM knowledge. The detection of class or cluster boundaries is, in particular, a serious problem. If neurons on both sides of class boundaries should behave differently, it is easy to find the boundaries with some visualization techniques. However, cooperation processes in SOM diminishes the effect of the boundaries, because the cooperation processes aim to increase continuity over the output space. Intuitively, the continuity is contradictory to the boundaries. In the proposed method, the individuality as well as collectiveness of neurons is introduced. The introduction of individuality is related to the more explicit detection of class boundaries by reducing collectiveness.

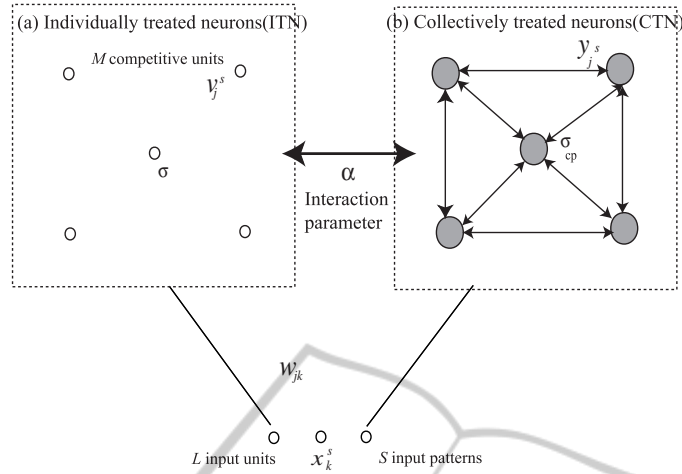


Figure 1: Concept of interaction of individuality with collectiveness.

2 THEORY AND COMPUTATIONAL METHODS

2.1 Interaction

The individuality and collectiveness are easily implemented in neural network architecture. Figure 1 shows a concept of neurons individually and collectively treated. Neurons are treated individually in Figure 1(a), and collectively in Figure 1(b). Two types of neurons are mediated by the interaction parameter α . In our view, in the conventional SOM, neurons have been treated collectively or cooperatively. Little attention has been paid to the individuality of neurons. For example, the good performance of the self-organizing maps has been evaluated by the trustworthiness and continuity on the output and input space (Kiviluoto, 1996), (Villmann et al., 1997), (Bauer and Pawelzik, 1992), (Kaski et al., 2003), (Venna and Kaski, 2001), (Polzlbauer, 2004), (Lee and Verleysen, 2008). No attempts have been made to evaluate the good performance of the self-organizing maps on the clarity of the obtained class structure.

The individuality and collectives can be considered in terms of neighborhood functions. As the range of the neighbors becomes smaller, the individuality can be considered. However, those neighborhood functions are only used for making cooperation processes smooth. Thus, we think that it is necessary to control or reduce the effect of cooperation among neurons, and much more attention should be paid to the extraction of explicit class boundaries.

2.2 ITN

When each neuron is individually treated, we can obtain individually treated neurons (ITN) as shown in Figure 1(a). In actual implementation, the method corresponds to our information-theoretic competitive learning (Kamimura et al., 2001a), (Kamimura, 2003). In this method, competition processes are supposed to be realized by maximizing mutual information between competitive units and input patterns.

Let us compute mutual information for a network shown in Figure 1(a). The j th competitive unit output can be computed by

$$v_j^s \propto \exp \left\{ -\frac{1}{2} (\mathbf{x}^s - \mathbf{w}_j)^T \Lambda (\mathbf{x}^s - \mathbf{w}_j) \right\}, \quad (1)$$

where \mathbf{x}^s and \mathbf{w}_j are supposed to represent L -dimensional input and weight column vectors, where L denotes the number of input units. The $L \times L$ matrix Λ is called a "scaling matrix," and the kl th element of the matrix denoted by $(\Lambda)_{kl}$ is defined by

$$(\Lambda)_{kl} = \delta_{kl} \frac{p(k)}{\sigma^2}, \quad k, l = 1, 2, \dots, L. \quad (2)$$

where σ is a spread parameter, and $p(k)$ shows a firing probability of the k th input unit and is initially set to $1/L$, because we have no preference in input units. The output is increased when connection weights become closer to input patterns. The conditional probability of the firing of the j th competitive unit, given the s th input pattern, can be obtained by

$$p(j | s) = \frac{v_j^s}{\sum_{m=1}^M v_m^s}. \quad (3)$$

The probability of the firing of the j th competitive unit is computed by

$$p(j) = \sum_{s=1}^S p(s)p(j|s). \quad (4)$$

With these probabilities, we can compute mutual information between competitive units and input patterns (Kamimura et al., 2001b). Mutual information is defined by

$$MI = \sum_{s=1}^S \sum_{j=1}^M p(s)p(j|s) \log \frac{p(j|s)}{p(j)}. \quad (5)$$

When this mutual information is maximized, just one competitive unit fires, while all the other competitive units cease to do so. Finally, we should note that one of the main properties of this mutual information is that it is dependent upon the scaling matrix, or more concretely, the spread parameter σ . As the spread parameter is decreased, the mutual information between competitive units and input patterns tends to be increased.

We can differentiate the mutual information and obtain update rules, but direct computation of mutual information is accompanied by computational complexity (Kamimura et al., 2001a), (Kamimura, 2003). To simplify the computation, we introduce free energy (Rose et al., 1990). The free energy F can be defined by

$$F = -2\sigma^2 \sum_{s=1}^S p(s) \log \sum_{j=1}^M p(j) \times \exp \left\{ -\frac{1}{2} (\mathbf{x}^s - \mathbf{w}_j)^T \Lambda (\mathbf{x}^s - \mathbf{w}_j) \right\}. \quad (6)$$

We suppose the following equation

$$p^*(j|s) = \frac{p(j)v_j^s}{\sum_{m=1}^M p(m)v_m^s}. \quad (7)$$

Then, the free energy can be expanded as

$$F = \sum_{s=1}^S p(s) \sum_{j=1}^M p^*(j|s) \|\mathbf{x}^s - \mathbf{w}_j\|^2 + 2\sigma^2 \sum_{s=1}^S p(s) \sum_{j=1}^M p^*(j|s) \log \frac{p^*(j|s)}{p^*(j)}. \quad (8)$$

This equation shows that, by minimizing the free energy, we can decrease mutual information as well as quantization errors. We usually set $p(j)$ into $1/M$ for simplification, and then

$$p^*(j|s) = \frac{v_j^s}{\sum_{m=1}^M v_m^s}. \quad (9)$$

By differentiating the free energy, we have

$$\mathbf{w}_j = \frac{\sum_{s=1}^S p^*(j|s) \mathbf{x}^s}{\sum_{s=1}^S p^*(j|s)}. \quad (10)$$

2.3 CTN

We can extend the information-theoretic competitive learning to a case where the collectiveness of neurons is taken into account. For the CTN, we try to borrow the computational methods developed for the conventional self-organizing maps, and then we use the ordinary neighborhood kernel used for SOM, namely,

$$h_{jc} \propto \exp(-\|\mathbf{r}_j - \mathbf{r}_c\|^2), \quad (11)$$

where \mathbf{r}_j and \mathbf{r}_c denote the position of the j th and the c th unit on the output space. Because the adjustment of individuality and collectiveness, namely, neighborhood relations, are realized by the interaction. The neighborhood function has no parameters to be adjusted.

The collective outputs can be defined by the summation of all neighboring competitive units

$$y_j^s \propto \sum_{c=1}^M h_{jc} \exp \left\{ -\frac{1}{2} (\mathbf{x}^s - \mathbf{w}_c)^T \Lambda_{ctn} (\mathbf{x}^s - \mathbf{w}_c) \right\}, \quad (12)$$

where the kl th element of the scaling matrix $(\Lambda_{ctn})_{kl}$ is given by

$$(\Lambda_{ctn})_{kl} = \delta_{kl} \frac{p(k)}{\sigma_{ctn}^2}, \quad (13)$$

where σ_{ctn} denotes the spread parameter for the collective neurons. The conditional probability of the firing of the j th competitive unit, given the s th input pattern, can be obtained by

$$q(j|s) = \frac{y_j^s}{\sum_{m=1}^M y_m^s}. \quad (14)$$

Thus, we must decrease the following KL divergence measure

$$I_{KL} = \sum_{s=1}^S \sum_{j=1}^M p(s)p(j|s) \log \frac{p(j|s)}{q(j|s)}. \quad (15)$$

As already mentioned in the above section, instead of the direct differentiation, we introduce the free energy. The free energy can be defined by

$$F = -2\sigma^2 \sum_{s=1}^S p(s) \log \sum_{j=1}^M q(j|s) \times \exp \left\{ -\frac{1}{2} (\mathbf{x}^s - \mathbf{w}_j)^T \Lambda (\mathbf{x}^s - \mathbf{w}_j) \right\}. \quad (16)$$

Then, the free energy can be expanded as

$$\begin{aligned}
F &= \sum_{s=1}^S p(s) \sum_{j=1}^M p^*(j|s) \|\mathbf{x}^s - \mathbf{w}_j\|^2 \\
&+ 2\sigma^2 \sum_{s=1}^S p(s) \sum_{j=1}^M p^*(j|s) \\
&\times \log \frac{p^*(j|s)}{q(j|s)}, \quad (17)
\end{aligned}$$

where

$$p^*(j|s) = \frac{q(j|s)v_j^s}{\sum_{m=1}^M q(m|s)v_m^s}. \quad (18)$$

By differentiating the free energy, we can have update rules

$$\mathbf{w}_j = \frac{\sum_{s=1}^S p^*(j|s)\mathbf{x}^s}{\sum_{s=1}^S p^*(j|s)}. \quad (19)$$

2.4 Interaction Procedures

In the interaction ITN with CTN, all neurons compete with other, because our method is based upon information-theoretic competitive learning. The degree of competition is determined by the spread parameter σ and σ_{ctn} for ITN and CTN. The spread parameter σ_{ctn} is computed by the competition parameter β

$$\sigma_{ctn} = \frac{1}{\beta}, \quad (20)$$

where β is larger than zero. As the competition parameter β is larger, competition among neurons becomes stronger.

The spread parameter σ is gradually decreased from β to a point by the interaction parameter α to control ITN and CTN. For simplicity's sake, we suppose that finally the spread parameter σ is proportional to the other parameter σ_{ctn} . Then, we have a relation

$$\sigma = \alpha\sigma_{ctn}, \quad (21)$$

where α is supposed to be greater than zero. As the interaction parameter α is larger, the spread parameter for ITN is larger. This means that the effect of ITN diminishes and that of CTN augments. Actually, the spread parameter σ is decreased from the value of β to $\alpha\sigma_{ctn}$.

3 RESULTS AND DISCUSSION

3.1 Experimental Setting

We present experimental results on the Ionosphere

Table 1: Quantization (QE), topographic (TE), training and generalization (gene) errors by the conventional SOM and the interaction method when the interaction parameter α is changed from one to fifty.

	QE	TE	Training	Gene
SOM	0.130	0.009	0.209	0.205
1	0.075	0.496	0.068	0.154
10	0.107	0.051	0.137	0.128
20	0.124	0.000	0.261	0.188
30	0.126	0.004	0.218	0.179
40	0.126	0.000	0.218	0.179
50	0.126	0.004	0.218	0.179

data from the machine learning database¹ to show how well our method performs. We use the SOM toolbox developed by Vesanto et al. (Vesanto et al., 2000), because it is easy to reproduce the final results presented in this paper by using this package. In the SOM, the Batch method is used, which has shown better performance than the popular real-time method in terms of visualization, quantization and topographic errors. To evaluate the validity of the final results, we tried to use the very conventional methods as well as modern methods for exact comparison. In the conventional methods, we used two types of errors, namely, quantization and topographic errors. The quantization error is simply the average distance from each data vector to its BMU (best-matching unit). The topographic error is the percentage of data vectors for which the BMU and the second-BMU are not neighboring units (Kiviluoto, 1996). For more modern techniques, we used trustworthiness and continuity (Venna and Kaski, 2001), (Venna, 2007) based upon the random method proposed by (Kiviluoto, 1996). In addition, we computed the error rate for training and testing data. The error rate was computed by using the k-nearest neighbor (k=1). For computing the generalization performance in the error rate, we divided the data into training (2/3) and testing (1/3) data.

3.2 Ionosphere Data

We applied the method to the ionosphere data from the machine learning database. This radar data was collected by a system in Goose Bay, Labrador. The data should be classified into "good" and "bad." The number of input units and patterns are 34 and 351 which is divided into the training (2/3) and testing data (1/3). Table 1 shows quantization, topographic, training and generalization errors by the conventional SOM and the interaction method. The quantization error by the conventional SOM is 0.130. On the other

¹<http://archive.ics.uci.edu/ml/>

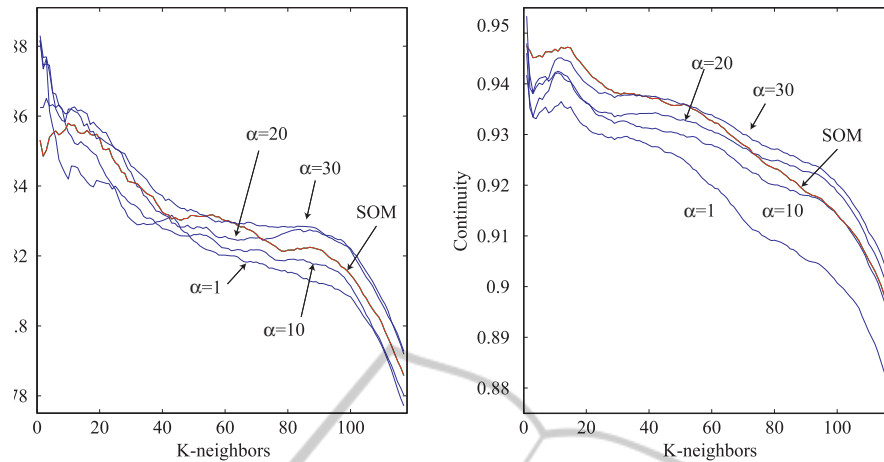


Figure 2: Trustworthiness (a) and continuity (b) as a function of k-neighbors.

hand, when the interaction parameter α is one, the error is 0.075. Then, the error is gradually increased to 0.126 when the parameter is 50. The topographic error is 0.496 when the parameter is one, which is much larger than 0.009 by the conventional SOM. However, when the parameter is 10, the error becomes 0.051. In addition, when the parameter is 20 and 40, the errors are completely zero. Training errors are the lowest (0.068) when the parameter is one. When the parameter is increased, the error becomes larger. The generalization error is the lowest when the parameter is ten and the largest when the parameter is 20. Compared with errors (0.205) by the conventional SOM, all errors by the interaction are much smaller.

Figure 2(a) shows trustworthiness as a function of k-neighbor. As can be seen in the figure, the trustworthiness is the lowest over almost all neighbors when the parameter is one. As the parameter is increased from 10 to 20, the trustworthiness is gradually increased. Then, when the k-neighbor is 30, the trustworthiness is higher than that by the conventional SOM in red. Figure 2(b) shows the continuity as a function of k-neighbor. When the parameter is one, the continuity is the lowest and far from the level by the conventional SOM. As the parameter is increased, the continuity is increased over almost all range of k-neighbor. When the k-neighbor is 30, the continuity is larger for the majority of k-neighbors.

Figure 3 shows U-matrices by the conventional SOM (a) and interaction (b)-(f). When the parameter is one in Figure 3(b), boundaries to be represented in warmer colors seem to be scattered over the matrix. When the parameter is increased to ten in Figure 3(c), the boundaries in warmer colors are located on the both sides. When the parameter is increased further to twenty in Figure 3(d), the two explicit boundaries in warmer colors can be seen on the lower side of the

map, which are very close to those obtained by the conventional SOM in Figure 3(a). When the parameter is further increased to thirty and forty in Figure 3(e) and (f), the two boundaries seem to be more explicit than those by the conventional SOM in Figure 3(a).

4 CONCLUSIONS

In this paper, we have proposed a new type of information-theoretical model in which the individuality and collectiveness of neurons are controlled by the interaction parameter α . As the interaction parameter α is increased, the effect of collectiveness becomes larger. We have applied the method to the production of the self-organizing maps with the ionosphere data from the machine learning database. Experimental results confirmed that improved performance could be observed in terms of all measures, namely, quantization, topographical, training and generalization errors by controlling the interaction parameter α . In addition, the trustworthiness and continuity over almost all ranges of k-neighbors become gradually larger as the interaction parameter α is larger. This means that the collectiveness can be used for neurons to cooperate with others like SOM. Finally, the feature maps obtained by our method showed sharper class boundaries compared with those by the conventional SOM.

The present experimental results are only preliminary ones with an initial condition. Thus, we need to compare the results more rigorously. However, we can at least show a possibility that the flexible interaction of ITN and CTN can be used to produce improved performance and explicit class structure.

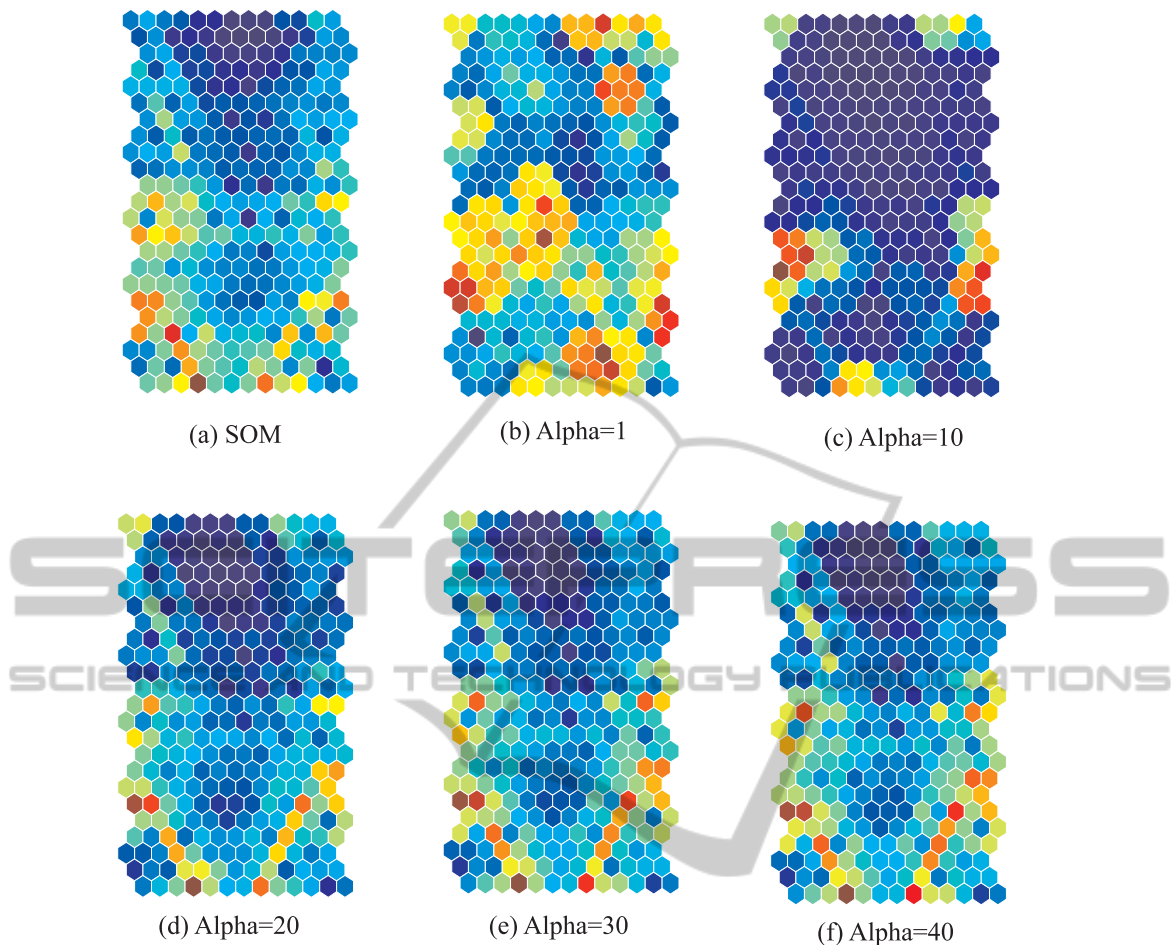


Figure 3: U-matrices by the conventional SOM (a), and interaction when the interaction parameter is changed from one (a) to 40 (f).

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