COMPLEX EXPONENT MOMENTS FFT ALGORITHM AND ITS APPLICATION

ZiLiang Ping[1] and YongJing Jiang[2]
[1]Centenary College, Beijing University of Post and Communication, Beijing, China
[2]Inner Mongolia Normal University, Huhhot, China

Keywords: Multi-distorted invariance, FFT algorithm, Compex Exponent Moments (CEMs), Human face recognition.

Abstract: A fast and accurate algorithm for computation of multi-distorted invariant Complex Exponent Moments (CEMs) is presented in the paper. An image function in polar coordinate system, $f_p(r,\theta)$, was divided into 2-D discrete image matrix in which the radial variables on lines and angle variables on columns. 2-D Fast Fourier Transform (FFT) was excuted for the matrix and the Complex Exponent Moments (CEMs) can be obtained. The multi-distorted invariance and the excellent performance of Complex Exponent Moments (CEMs) were demonstrated. The Complex Exponent Moments (CEMs) were applied in human face recognition.

1 PREFERENCE

Orthogonal multi-distorted invariant moments have been successfully used in the fields of image analysis, pattern recognition, object classification and image watermarking ete. (Papakostas and Boutalis, 2007; Kan and Srinath, 2002; Kim and Lee, 2003). Many research works devote to orthogonal invariant moments, such as Zernike Moments (Teague, 1980), orthogonal Fourier-Mellin Moments (Sheng and Shen, 1994), Chebyshev-Fourier Moments (Ping et al., 1748-1754), Radial-Harmonic-Fourier Moments (Ren and Ping, 2003) and Jacobi-Fourier Moments (Ping and Ren, 2007). All of those moments were calculated by the integral operation in former algorithm literature. Because of the transform between Cartesian coordinate system and polar coordinate system and calculating complex degree, the calculation of the moments is of time waste and those moments are lower precision. The fast and accurate algorithm for Complex Exponent Moments (CEMs), 2D FFT algorithm, was proposed in this paper.

2 THE DEFINITION OF COMPLEX EXPONENT MOMENTS (CEMS)

Radial-Harmonic-Fourier Moments (RHFMs) (Ren and Ping, 2003) is definited in polar coordinate system as:

$$\phi_{nm} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^1 f(r,\theta)T_n(r)\exp(-jm\theta)rdrd\theta \quad (1)$$

Here

$$T_n(r) = \begin{cases} \sqrt{\frac{1}{r}} & if \quad n=0 \\ \sqrt{\frac{2}{r}}\sin(n+1)\pi r & if \quad n \quad is\ odd \\ \sqrt{\frac{2}{r}}\cos n\pi r & if \quad n \quad is\ even \end{cases} \quad (2)$$

According to Euler’s fomular the radial harmonic function can be transform to complex exponential function:

$$A_k(r) = \sqrt{\frac{2}{r}}\exp(j2k\pi r) \quad (3)$$

And the relationship between $T_n(r)$ and $\mathrm{A}_k(r)$ is as following:

Ping Z. and Jiang Y..
COMPLEX EXPONENT MOMENTS FFT ALGORITHM AND ITS APPLICATION.
DOI: 10.5220/0003700904650468
In *Proceedings of the 4th International Conference on Agents and Artificial Intelligence* (ICAART-2012), pages 465-468
ISBN: 978-989-8425-95-9

$$\begin{cases} T_0(r) = \frac{1}{\sqrt{2}} A_0(r) & n = k = 0 \\ T_n(r) = \frac{1}{2j}(A_k(r) - A_{-k}(r)) & n = 2k-1 \quad k = 1,2,3\cdots \\ T_n(r) = \frac{1}{2}(A_k(r) + A_{-k}(r)) & n = 2k \quad k = 1,2,3\cdots \end{cases} \quad (4)$$

The Radial-Harmonic-Fourier Moments (RHFMs) can be rewritten as:

$$E_{k,m} = \frac{1}{4\pi}\iint_{x^2+y^2\le 1} f_c(x,y)\sqrt{\frac{2}{r_{x,y}}}\exp(-j2k\pi r_{x,y})\exp(-jm\theta_{x,y})dxdy \quad (5)$$

Here, $r_{x,y} = \sqrt{x^2+y^2}$, $\theta_{x,y} = \arctan\frac{y}{x}$, and the integral can be calculated in Cartesian coordinate system. The $E_{k,m}$ is definited as Complex Exponent Moments (CEMs) and can be calculated with FFT algorithm.

3 ALGORITHM OF THE COMPLEX EXPONENT MOMENTS(CEMS) AND IMAGE RECONSTRUCTION WITH CEMS

According to formular (5) the Complex Exponent Moments (CEMs) can be numerically calculated as following:

$$E_{km} = \frac{1}{M^2}\sum_{u=0}^{N-1}\sum_{v=0}^{M-1} F_p(r_u,\theta_v)\sqrt{\frac{r_u}{2\times N}}\exp(-j\frac{2\pi}{N}ku)\exp(-j\frac{2\pi}{M}mv) \quad (6)$$

The CEMs can also be calculated with 2D fast Fourier transform (FFT) algorithm. Uniformly sampling an image function in polar coordinate system to make image function being a discrete $M\times N$ numetrical image matrix first. Here N and M is the sample point number on the radial direction and angular direction respectively. Then taking 2D Fast Fourier Transform for this discrete $M\times N$ numetrical image matrix, the CEMs of the image can be obtained. FFT algorithm of the CEMs possesses the lower calculation complexity ($O(N^2\log_2 N)$) than integral algorithm ($O(N^4)$), so CEMs are more efficient algorithm and time-saving.

Because of the orthogonal performance of kernel function the CEMs is orthogonal image moments and an image can be reconstructed with its CEMs according to following equation:

$$f(r,\theta) = \sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} E_{km}A_k(r)\exp(jm\theta) \quad (7)$$

The similar degree between the reconstructed image and the original image will increase with the increasing of moment's numbers. Former research (Ren and Ping, 2003) works verified that RHFMs possess the best image reconstruction performance in all of orthogonal moments. Produced from RHFMs, CEMs possess the best reconstruction performance too. The experiments verified that CEMs calculated by FFT algorithm possess better reconstruction performance than calculated by integral algorithm. Fig.1 shows comparation of integral algorithm with FFT algorithm. From Fig.1 it can be seen that the quality of reconstructed images via FFT algorithm is much better than reconstructed images via integral algorithm.

4 THE ROTATION AND SCALING INVARIANT PERFORMANCE OF CEMS

The complex Fourier factor of the kernel function in angular direction will maintain the rotated-invariant performance of the modular value of the moments. Through unifying process the performance of CEMs will be scaled-distorted invariant and intensity-distorted invariant. So, Complex Exponent Moments (CEMs) are multi-distorded invariant

Figure 1: The image reconstruction with CEMs: (a)-(d) reconstructed images with integral algorithm, (e)-(h) reconstructed images with FFT algorithm.

(Ping et al., 1748-1754). Fig.2 shows the rotation-distorted invariant performance of Complex Exponent Moments (CEMs): The line-up is original image of Lena and its modular value distribution, the line-down is rotated image of Lena and its modular value distribution. From Fig.2 it can be seen that the modular values of CEMs for rotated image are invariant.

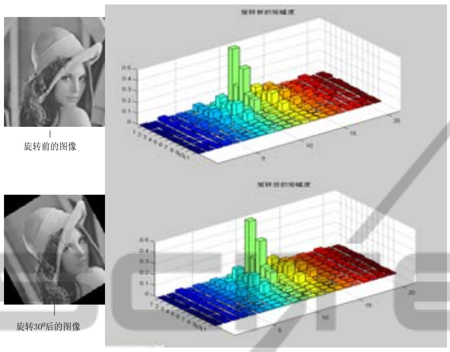

Figure 2: The rotated invariant performance of CEMs. The line up is standart image and its CEMs modular value distribution. The line down is rotated image and its CEMs value distribution.

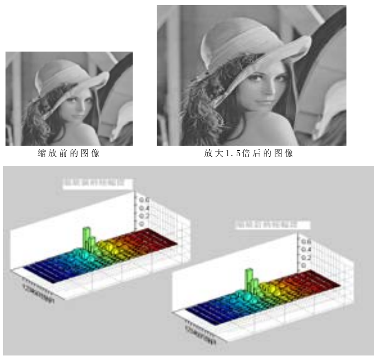

Figure 3: The scaling invariant performance of CEMs. The left column is normal image and its CEMs modular value distribution. The right column is scaled image and its CEMs modular value distribution.

Fig.3 shows the scaling invariant performance of modular value of CEMs

The left column is Lena normal image and modular value distribution of its CEMs, the right column is distorted image scaled 1.5 times and its modular value distribution of its CEMs. From Fig.3 it can be seen that the modular value of CEMs of scaled image possesses the same modular value distribution with the normal image.

5 APPLICATION OF CEMS FOR PATTERN RECOGNITION

Using the CEMs of image to be feature vectors for Suppot Vector Machine (SVM) algorithm pattern recognition was performed for human face of 20 persons, for whom each one has 10 various visions and different facial expression. Fig.4 shows the experiment images of two persons of the recognized human faces. Each face image was rotated for 10°、20°、30° and has 40 pieces of image for one person, and there are 800 pieces of variable image for 20 persons.

Take mass center of image to be original point of the image coodinate system and calculate the CEMs of the image via FFT algorithm. Using CEMs of the image to be feature vectors of Support Vector Machine (SVM), the human face recognition experiment was performed. The 24 pieces of face image for each person and 480 pieces of face image for 20 persons were taken as trining sample image set, and the other 320 pieces of face images were taken as testing sample image set, and the one to more SVM algorithm was applied in the experiment. Table-1 shows the experiment result. From the data of Table-1 it can be seen that taking 36 CEMs as image feature vectors the recognition rate is the highest, achieving 92.5%, as number of CEMs increasing the recognition rate of image is decreasing down, this is because of the over-learning problem for Suport Vector Machine(SVM)

Figure 4: The various face expression for two persons in experiment, upper: face of male person; lower: face of female person.

Table 1: The recognition rate human face via Complex Exponent Moments (CEMs).

Momen Number	Recognition rate （%）	Momen Number	Recognition rate （%）	Momen Number	Recognition rate （%）
9	85	30	90.75	56	87.97
16	88.75	**36**	**92.5**	64	81.25
20	88.75	42	90.63	72	78.59
25	90	49	90	81	76.25

6 CONCLUSIONS

In the paper a novel orthogonal multi-distorted invariant moments (CEMs) was presented and a fast and accurate algorithm, 2D Fast Fourier Transform (FFT) algorithm, was performed for this moments. The theoretical analysis and experiment results have verified that 2D FFT algorithm of CEMs possesses multi-distorted invariance, lower image reconstruction error, higher quality of reconstructed image and lower calculating complicity degree compared to the integral method. The CEMs were applied in human face recognition and experiment result has verified efficience of CEMs and the 2D-FFT algorithm for it.

REFERENCES

G. A. Papakostas, Y.S. Boutalis, A new class of Zernike moments for computer vision applications, *Inf.Sci.* 177(13) (2007) 2802-2819

Chao Kan, Mandyam D. Srinath, Invariant character recognition with Zernike and orthogonal Fourier-Mellin moments, *Pattern Recognition*, 35(2002) 143-154

Hyung Shin Kim, Heung-Kyu Lee, Invariant image watermark using Zernike moments, *IEEE Trans. Circuits Syst. Video Techn.* 13(8) (2003) 766-775.

M. R. Teague, Image analysis via the general theory of moments. *J. Opt. Soc. Am.* 70(1980)920-930.

Y. Sheng, L. Shen, Orthogonal Fourier-Mellin moments for invariant pattern recognition, *J. Opt. Soc. Am.* A 11 (6) (1994) 1748-1757

Z. L. Ping, R. Wu, Y. L. Sheng, Image description with Chebyshev-Fourier moments, *J. Opt. Soc. Am. A 19*(9) (2002) 1748-1754

H. P. Ren, Z. L. Ping, Multi-distorted invariant image recognition with Radial-Harmonic-Fourier Moments, *J. Opt. Soc. Am. A 20* (4) (2003) 631-637

Z. L. Ping, H. P. Ren, Generic orthogonal moments: Jacobi-Fourier moments for invariant image description, *Pattern Recognition*, 40 (2007) 1245-1254