INTERVAL AVAILABILITY ANALYSIS OF A TWO-ECHELON, MULTI-ITEM SYSTEM

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Abstract: In this paper we analyze the interval availability of a two-echelon, multi-item system. Modeling the system as a Markov chain we analyze the interval availability of the system. We compute in closed and exact form the expectation and, the variance of the availability during a finite time interval *[0,T]*. We use these characteristics together with the probability that interval availability is equal to one to approximate the survival function using a Beta distribution. Comparison of our approximation with simulation shows excellent accuracy, especially for points that are practically most relevant.

1 INTRODUCTION

Nowadays, the aftersales service business represents a considerable part of the economy and, moreover, is continuously growing (AberdeenGroup 2005; Deloitte 2006).

Advanced capital goods such as MRI scanners, lithography systems, baggage handling systems, and radar systems, are highly downtime critical. So the customers of these advanced goods are not just interested in acquiring these systems at an affordable price, but far more in a good balance between the resulting Total Cost of Ownership (TCO) and system productivity throughout the life cycle, including the limitation of downtime. For customers the system use rather than the system upkeep is their core business. Therefore, a major part of the system upkeep is preferably outsourced to the original manufacturer or to a service provider that can offer a good balance between the downtime and costs. For that reason, service contracts are made between the service provider and customers. These contracts specify the services provided by the supplier with their corresponding Service Level Agreements (SLAs), such as the time between system failure and time of fault resolution, and the system availability.

The SLAs are measured over a predetermined time window, e.g., a quarter or a year. For the service providers, it is essential that the service levels are attained, because in some cases penalties apply if an SLA target is violated. In case of a large scale service contract (the average performance over many systems is measured), the average performance should meet the target. If the number of systems covered by a contract is relatively small, we have inherent statistical variability and we need an additional buffer in performance to assure that the probability of not meeting the SLAs over the time window is still acceptable. We encountered such a situation at Thales Netherlands, a manufacturer of naval sensors and naval command and control systems. There, a service contract typically covers a few systems only. In the literature, this issue is usually neglected. In this paper, we are mainly interested in the logistical delay due to the unavailability of spare parts. Moreover, the focus will be on SLAs that are based on the system availability during a predetermined period of times.

In service parts logistics there is usually a tradeoff between the cost involved in keeping the stocks very close to the customers sites or at a central depot, which can supports multiple customers at the same time. Due to the risk pooling effect, it is more desirable from the point of a service provider to position the stocks of spare parts centrally. However, having a strict SLA, e.g., 99% availability, with a customer forces the service provider to move some spare parts closer to the customer sites. In addition, in order to reduce the system downtime and its critical consequences it is usually the case that the repair of failed system is done by replacing the failed part with a new part. The failed part is sent to the repair shop, i.e., the

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inventory is managed using the so-called base stock policy referred to as (s-1,s)-policy. (Sherbrooke 1968) was among the first to tackle the spare part optimization problem. He proposed the METRIC model that is based on the maximization of system availability subject to a constraint on the invested budget in spare parts. METRIC model is a good approximation in case of multi-echelon spare part network and especially in case of high availability. (Graves 1985) extended the METRIC model and proposed an improved approach called VARI-METRIC. We note that VARI-METRIC model is used in most commercial software tools.

It is worth to mention that both METRIC and VARI-METRIC and most spare parts management theory are based on the maximization of the steady state average system availability, i.e., the fraction of time the system is operational during a very long (infinite) period of time. However, in practice we often see that the agreed upon availability SLA is the average availability during a finite period, e.g., month, quarter, or year. Moreover, if the availability during a period of time is lower than a specific percentage the penalty rules then apply. This motivates us to analyze the availability during a finite period of time, the so-called interval availability in reliability theory defined as follows see, e.g., (Nakagawa and Goel 1973):

Definition: *The system interval availability is defined as the fraction of time a system is operational during a period of time [0,T].*

Note that as opposed to the steady state average availability the interval availability is a random variable (rv) that has a distribution.

2 RELATED LITERATURE

In this section we shall the review the existing literature on interval availability. (Takács 1957) was among the first to analyze the interval availability distribution function of an on-off stochastic process. Takács result is in the form of an infinite sum of terms, each consisting of multiple convolutions. This result is hard to compute numerically. Approximation by fitting the on and off periods by a phase type distribution with two phases was proven to give accurate result with small computation time, see e.g., (van der Heijden 1988). Another approximation based on fitting the approximated first two moments and the hundred percent and nil probability of the interval availability in a Beta distribution was proposed in (Smith 1997). For an on-off two states Markov chain the first two moments of the interval availability are derived exactly in (Kirmani and Hood 2008). We note that in all these previously mentioned studies the underlying assumption is that the on periods are independent and the off periods are independent, moreover, all the on and off period are independent of each other, i.e., the on-off process can be represented by a renewal process.

(De Souza e Silva and Gail 1986) derived in closed-form the cumulative sojourn time distribution in a subset of states of a Markov chain during a finite period of time. The subset of states may represent the operational states of a system. Therefore, the division of the cumulative sojourn time by the period length gives right away the system interval availability. We note that computing the full curve of the interval availability distribution using the result of (De Souza e Silva and Gail 1986) is time consuming. (Carrasco 2004) proposed an efficient algorithm to compute the interval availability distribution for the special case of the systems which can be modeled by an *absorbing* Markov chain. Note that in the latter two papers the renewal assumption of the on-off process is not anymore necessary.

In this paper, we shall propose a numerically efficient approach to compute the distribution function of the interval availability. Our approach builds on the result of (De Souza e Silva and Gail 1986) extensively in order to compute in closed-form the first two moments of the interval availability. Note that these two moments were not derived previously in the literature for a Markov chain with more than two states. Moreover, we will follow a similar approach to (Smith 1997) to approximate the interval availability by a Beta distribution using the first two moments in addition to the probability that interval availability is equal 1.

3 MODEL

We consider a two-echelon, multi-item supply network. There is a single depot that supports multiple identical systems which are located at different bases. A system consists of multiple items that are subject to breakdown. These items are repairable and belong to the class of expensive slow-movers, i.e., they have low failure rates. The depot and the bases hold a safety stock of spares for each item. Upon an item failure, the item is immediately sent to the depot for repair and at the same time a replenishment order is issued according to the (s-1,s)-policy, where s denotes the order-up-to level.

Note that it is possible to extend our model by allowing for repair of failed items at the bases. The unsatisfied demand of parts is backordered. When the replenishment order arrives at the base it is used to fill backorders, if any. Otherwise, it is added to the base stock. The time needed to transfer a spare from the depot to the base is assumed to be exponentially distributed. This assumption was validated in (Alfredsson and Verrijdt 1999). In Section 5, we shall numerically examine the impact of the assumption of exponential order-and-ship times on the interval availability distribution. We say that the system is operational if all the items are operational. Obviously, if an item fails and no spare is available at the base, the system will be malfunctioning and unavailable for use.

We consider a scenario inspired by a case study done at Thales Netherlands. There is one naval radar system at each of the N bases (frigate). A system consists of M items. We assume that the j-th item fails according to a Poisson process with rate λ_j, $j=1,\dots,M$. Moreover, the failure of item j is independent of the rest of items. We assume that the replenishment time of the i-th item at the depot is exponentially distributed with rate μ_j. The replenishment time includes the time to transport the failed item from the base to the depot and the time to repair the item at the depot. We model the depot repair shop as an ample server, i.e., it has an unrestricted repair capacity. We also assume that the transshipment time of a spare part from the depot to the system is exponentially distributed with rate μ_0. Let s_{ij}, $i=0,\dots,N$, $j=1,\dots,M$, denote the stock level of item j at location i, where i=0 represents the depot and i=1,..,M represents the i-th base. Under the above assumption it is easily seen that the behavior of the system over time can be modeled as a continuous-time Markov chain. More precisely, since there is a finite number of spare parts in the network the continuous-time Markov chain is of finite size. Comparing the assumptions of our model and (VARI-)METRIC the only difference is the exponentially distributed replenishment time and order-and-ship time, whereas order-and-ship times are deterministic and replenishment times have a general distribution in (VARI-)METRIC.

Let $A_i(T)$, $i=1,\dots,N$, denote the interval availability of system i during $[0,T]$. Our objective is to find the survival function of $A_i(T)$, i.e., the complementary cumulative distribution function of $A_i(T)$. For this reason, we first compute the mean and the second moment of the interval availability as well as the probability that the interval availability equals 1, i.e., $P(A_i(T)=1)$. Although we may also compute the probability mass in the point zero, $P(A_i(T)=0)$, this is not really useful: for practical relevant problem instances, it will be very close to zero. Next, using the three performance metrics as mentioned above we approximate the survival function of $A_i(T)$ by a mixture of a probability mass at one and a Beta distribution. Throughout this paper, we shall only focus on the interval availability of a tagged system. For this reason, we shall drop the index i in $A_i(T)$ and refer to it as $A(T)$: the interval availability of a tagged system at one of the bases. In addition, we shall refer to the stock level of item j in the tagged system as s_j.

Since the failure processes of the different items are independent of each other and the repair capacity is unrestricted, the different items on the tagged system behave mutually independent over time. Let $X_j(t)$ denote the state of item j in the tagged system at time t, i.e., $X_j(t)=1$ if the item is operational at time t and zero otherwise. Note that $X_j(t)=0$ if item j fails and there is no spare part available at the base to replace the malfunctioning item. Let $PL_{ij}(t)$ denote the item j pipeline of the tagged system i. That is, it is the total number of item j backorders of the tagged system at the depot or in transport from the depot to the tagged system. Note that the pipeline of item j depends on the stock on-hand at the depot. Furthermore, the depot stock depends on the failure processes of item j in *all* the systems in the installed base including the tagged system. Let us denote $N_j(t)$ the total number of failed items of type j in the depot repair shop. Note that backorders at the depot are served according to FIFO discipline. Therefore, if $N_j(t)\geq s_{0j}$, i.e., on-hand stock in the depot is equal to zero, it is also necessary to keep track of the position of the tagged system backorders in the depot backorders list. This is a complication that arises when computing the interval availability distribution which is not encountered in (VARI-)METRIC model. The previous complication makes a detailed Markov analysis difficult. For this reason, in the following section we shall propose an approximate two-dimensional finite-size Markov chain to represent the state evolution of item j.

The tagged system is operational at time t if $X_j(t)=1$, for all j=1,…,M. Let $O(T)$ denote the total sojourn time of the joint process $(X_1(t), X_2(t),\dots, X_M(t))$ in state $(1,..,1)$ during $[0,T]$. The interval availability of the tagged system can be written as $A(T)=O(T)/T$. Note that the processes $X_j(t)$, for j=1,…,M, are mutually independent and can be modeled as a Markov chain. Therefore, the joint process $(X_1(t),\dots, X_M(t))$ is also a Markov chain.

A word on notation: Given that A is a matrix, *A(i,j)* denotes the *(i,j)*-entry of *A*. We use $\otimes$ as the Kronecker product defined as follows. Let A and B be two matrices then A $\otimes$ B is a block matrix where the *(i,j)*-block is equal to *A(i,j)B*. We use *e* to denote a column vector with all entries equal to one.

4 APPROXIMATION

In this section, we first propose an approximate two-dimensional continuous-time finite-state Markov chain to model the evolution of $X_j(t)$ over time. Second, we represent the transition generator of the joint process $(X_1(t),\ldots, X_M(t))$ as function of the generators of $X_j(t)$, $j=1,\ldots,M$. The main approximations are as follow: the time to satisfy an item *j* backorder at the depot is equal to its time to repair. This means that it is exponential distributed with rate μ_j. If there is on-hand stock of item *j* at depot the time to satisfy a backorder at the base is equal to the minimum of the item repair time and the order-and-ship time. Moreover, we shall assume that all the systems in the installed base, excluding the tagged system, are always operational.

Let us consider the finite-state two-dimensional Markov chain $\left\{\left(PL_{ij}(t), N_j(t)\right): t \geq 0\right\}$, referred to as AMC_j. We note that the chain has a finite state space because of the finite number of stocks in the network. Recall that $PL_{ij}(t)$ is the item *j* pipeline of the tagged system *i* and $N_j(t)$ is the total number of *j*-th items in the depot repair shop. Note that $PL_{ij}(t) \in \{0,\cdots,s_{ij}+1\}$ and $N_j(t) \in \{0,\cdots,s_{0j}+s_{1j}+\cdots+s_{Mj}+M\}$. Figure 1 shows the transition diagram of AMC_j with $s_{0j}=2$ and $s_{ij}=1$. The process AMC_j has the following transitions:

- A failure of item *j* in the tagged system. In Figure 1, it represents the transition from $(PL_{ij}(t), N_j(t))$ to $(PL_{ij}(t)+1, N_j(t)+1)$ with rate λ_j.
- A failure of item *j* in one of the systems in the installed based excluding the tagged system. In Figure 1, it represents the transition from $(PL_{ij}(t), N_j(t))$ to $(PL_{ij}(t), N_j(t)+1)$, which occurs by assumption with rate $(N-1)\lambda_j$.
- A depot repair completion of an item *j* that is used to replenish a backorder for one of the systems in the installed based excluding the tagged system. In Figure 1, it represents the transition from $(PL_{ij}(t), N_j(t))$ to $(PL_{ij}(t), N_j(t)-1)$, which occurs by assumption with rate $(N_j(t)-PL_{ij}(t))\mu_j$.
- A depot repair completion of an item of type *j* that is used to replenish a backorder of the tagged system. In Figure 1, it is the transition from state $(PL_{ij}(t), N_j(t))$ to $(PL_{ij}(t)-1, N_j(t)-1)$, which occurs by assumption with rate $PL_{ij}(t)\mu_j$.
- A backorder replenishment from the stock on-hand at depot. In Figure 1, it is the transition from $(PL_{ij}(t), N_j(t))$ to $(PL_{ij}(t)-1, N_j(t))$ that is assumed to be equal to $PL_{ij}(t)\mu_0$. Note there is stock on-hand at the depot *if* $N_j(t)\leq s_{0j}$.

We emphasize that the previous four transitions rate are an approximation. The accuracy of these approximations shall be validated in Section 5.

Let G_j denote the transition generator of AMC_j. Since AMC_j is a finite state Markov that is aperiodic and irreducible we deduce that AMC_j has a steady state probability. Let $\pi_{m,n}(j)$ denote the steady state probability that AMC_j is in state *(m,n)*. We define the probability distribution row vector π as follows

$$\pi = \left(\pi_{0,\cdot}, \cdots, \pi_{s_{ij}+1,\cdot}\right),$$
$$\pi_{m,\cdot} = \left(\pi_{m,m}, \pi_{m,m+1}, \cdots, \pi_{m,s_{0j}+s_{1j}+\cdots+s_{Mj}+M}\right),$$
$$m = 0,\cdots, s_{ij}+1.$$

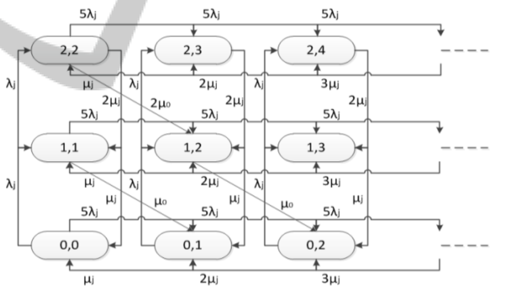

Figure 1: Transition diagram of AMC_j with $s_{0j}=2$, $s_{ij}=1$, and $N=6$.

In the case where AMC_j sojourns in states *(m,n)* with $m\leq s_{ij}$ item *j* in the tagged system is operational. This is because, for $m\leq s_{ij}$ there is no backorder of item *j* of the tagged system in the base. On the other hand, when $m=s_{ij}+1$ there is one item *j* backorder in the base. Therefore, item *j* in the tagged system is unavailable for $m> s_{ij}$. Let Ω_j denote the state space of AMC_j. We split Ω_j into to two disjoint subsets: Ω_j^o subset of operational states, i.e., states *(m,n)* with $m\leq s_{ij}$, and Ω_j^m subset malfunctioning states, i.e., states *(m,n)* with $m=s_{ij}+1$. The steady state probability that item *j* is operational in the tagged system gives

$$P(X_j=1) = \sum_{m=0}^{s_{ij}} \sum_{n=0}^{M+s_{0j}+\sum_{l=1}^{M} s_{lj}} \pi_{m,n}(j),$$

where X_j is the steady state of the process $X_j(t)$, i.e., $X_j = X_j(\infty)$. Throughout this paper, we shall assume that the AMC_j starts in steady state at time 0. Therefore, for all $t \in [0,T]$ the chain AMC_j, $j = 1, \dots, M$, will remain in steady state.

In the following, we shall use the uniformization method, which is extremely useful for computational purposes. The uniformization method transforms a continuous-time Markov chain with non-identical states leaving rate to an equivalent process in which the transition are generated by a Poisson process at a uniform rate (Tijms 2003). Let P_j denote the transition probability matrix of the uniformized process of AMC_j. The matrix P_j reads

$$P_j = I + \frac{1}{\nu} G_j,$$

where I is the identity matrix, and ν is given by: $\nu > \max(|G_j(l,l)|, l = 1, \dots, ||G_j||)$, where $||G_j||$ is the size of the matrix G_j. Let P_s denote the transition probability matrix of the joint uniformized process $(X_1(t), \dots, X_M(t))$. Then, P_s is equal $P_1 \otimes \dots \otimes P_M$, see, (Rausand and Høyland 2004).

4.1 Performance Metric

In this section, we derive in closed form the $E[A(T)]$, $Var[A(T)]$, and $P(A(T)=1)$. We refer the interested reader for results to (Al Hanbali and van der Heijden 2011).

Theorem 1: *The expected system interval availability during [0,T] is equal to the steady state system availability and is given by:*

$$E[A(T)] = \prod_{j=0}^{M} \sum_{m=0}^{s_{ij}} \sum_{n=0}^{M+s_{0j}+\sum_{l=1}^{M} s_{lj}} \pi_{m,n}(j).$$

Before reporting our result on the variance of $A(T)$, let us introduce some notation. Let γ_j denote a row vector that is defined as $\gamma_j = \left(\pi_{0,\cdot}, \cdots, \pi_{s_j,\cdot}, \mathbf{0}\right)$, where $\mathbf{0}$ denote a row vector with all entries equal to 0. Let f_j denote a column vector that is defined as $f_j = (e, \cdots, e, \mathbf{0})$.

Theorem 2: *The variance of the system interval availability during [0,T] is given by:*

$$Var[A(T)] = 2\sum_{n=1}^{\infty} e^{-\nu t} \frac{(\nu t)^n}{(n+2)!} \sum_{i=1}^{n} (n-i+1) \prod_{j=1}^{M} \gamma_j (P_j)^i f_j + 2E[A(T)] \frac{e^{-\nu t} + \nu T - 1}{(\nu T)^2} - E[A(T)]^2.$$

The process $X_j(t)$ is equal to one for all $t \in [0,T]$ if the time until absorption of ACM_j into the subset Ω_j^m is larger than T, given that ACM_j starts at time 0 in Ω_j^o. Let θ_j denote the row vector $\left(\pi_{0i}, \cdots, \pi_{s_{ij}}\right)$. Let O_j denote the transient generator of AMC_j under the assumption that the states of Ω_j^m are absorbing. That is, O_j is the matrix composed of the first $(s_{ij}+1) \times (s_{0j} + s_{1j} + \cdots + s_{Mj} + M)$ rows and columns of G_j. Let T_j^a denote the time until absorption into a state of Ω_j^m. It is then well known that, see, (Neuts 1981)

$$P(T_j^a \geq T) = \theta_j \exp(T\, O_j)\, e.$$

Theorem 3: *The Probability that A(T)=1is given by:*

$$P(A(T) = 1) = e^{-T\sum_{i=1}^{M} \nu_i} \prod_{j=1}^{M} \theta_j \sum_{n=0}^{\infty} \frac{(\nu_i T)^n}{n!} \left(P_j^a\right)^n e,$$

where $P_j^a = I + O_j/\nu_j$, *and* $\nu_j > \max(|O_j(l,l)|, l = 1, \dots, ||O_j||)$.

Note that the infinite sum in the previous Theorem 2 and 3 can be truncated with a predetermined truncation error bounds, see (De Souza e Silva and Gail 1986; Tijms 2003).

4.2 Approximation of $P(A(T) \geq y)$

Until now we have computed $E[A(T)]$, $Var[A(T)]$, and $P(A(T)=1)$. We shall report now how to fit these metrics in a probability distribution that is a mixture of probability mass at one and a Beta distribution. The choice for Beta distribution is made for the main reason that: the interval availability and a Beta rv both have finite support. The interval availability has probability mass at 0 and 1. However, in most practical cases with high expected interval availability $P(A(T)=0)$ is almost zero. For that reason, we shall neglect it in the following. We approximate the interval availability as follows: $A(T) = (1 - P(A(T) = 1) * B + P(A(T) = 1)$, where B is a Beta distributed rv bounded between 0 and 1. From the latter equation it readily seen that

$$E[\mathrm{B}] = \frac{E[A(T)] - P(A(T)=1)}{1 - P(A(T)=1)},$$

$$E[\mathrm{B}^2] = \frac{E[A(T)^2] - P(A(T)=1)}{1 - P(A(T)=1)}.$$

The probability density of a Beta rv reads

$$f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where $B(\alpha,\beta)$ is the Beta function. Given that expectation and the variance of B is known a simple algebra gives that

$$\alpha = \frac{(1 - E[\mathrm{B}]) * E[\mathrm{B}]^2}{Var[\mathrm{B}]} - E[\mathrm{B}], \text{and } \beta = \alpha\left(\frac{1}{E[\mathrm{B}]} - 1\right).$$

Finally, we conclude that

$$P(A(T) \geq y) = \ (1 - P(A(T) = 1) \int_x^1 f(x; \alpha, \beta) dx + P(A(T) = 1).$$

5 NUMERICAL VALIDATION

In this section, we compare the results of our model with the simulation as function of the average system availability. Moreover, we consider different cases with different number of items per system (*M*). The main scenario is as follows: One depot that serves six bases. We note this scenario and its input parameters value are inspired from a case study done at Thales Netherlands. A base is a system that is composed of *M*=10,30,50 items. All stocks are available at the depot and there is no possible repair at the bases. The repair time of item j is exponentially distributed with rate $\mu_j = 1/MTTR_j$, where $MTTR_j$ is the mean time to repair item j. The order of magnitude of the $MTTR_j$ is between few month to more than a year. The order-and-ship time is exponentially distributed with mean 120 hours. In the simulation, we shall assume that the order-and-system time is deterministic. Item j fails according to a Poisson process with mean time between failures ($MTBF_j$) equal to $1/\lambda_j, j = 1, \cdots, M$. The order of magnitude of λ_j is between few times per year to few times per hundred years. We are interested in the interval availability of a system during one year, i.e., *T*=8760 hours. The implementation of the simulation is done in Plant Simulation v8.2. We used Matlab v7.8 for the approximations. For details on the stock allocations see the Appendix.

In Figure 3, 4 and 5, we show the survival function of the interval availability using our model and the simulation with *M*=10, 30, 50, respectively. Note that the discontinuity points in the tail of *A(T)* using simulation are due to the deterministic assumption of the order-and-ship time. Observe that our model has the highest accuracy for the cases where *M*=10 and 30 and where *E[A(T)]* is larger than 80%. Our model predicts very well *E[A(T)]* for all the cases, see the second row in Table 1, and 2 for details. However, our model predicts *Var[A(T)]* with less accuracy. Moreover, it seems that the accuracy of *Var[A(T)]* has less impact on the survival function than the accuracy of *E[A(T)]*, see for example the results of cases 3, 6, and 9. Note that for all the different cases considered the difference of $P(A(T) \geq x)$, with $x \geq E[A(T)]$, obtained using the simulation and our model is larger than -0.04 and smaller than 0.07, as indicated in Table 1, and 2. Finally, note that the run time of our approximation is less 100ms for the considered cases.

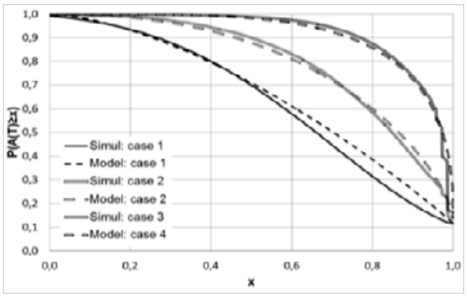

Figure 2: Interval availability survival function with *M*=10 in case: 1. *E[A(T)]* = 63%, 2. *E[A(T)]* = 79%, and 3. *E[A(T)]* = 92%.

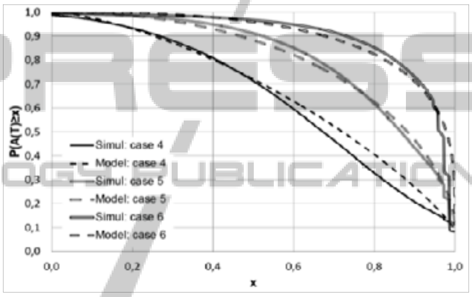

Figure 3: Interval availability survival function with *M*=30 in case: 4. *E[A(T)]* = 64%, 5. *E[A(T)]* = 81%, and 6. *E[A(T)]* = 90%.

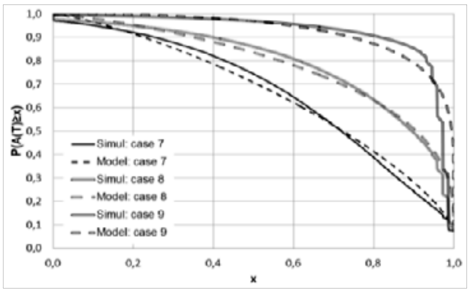

Figure 4: Interval availability survival function with *M*=50 in case: 7. *E[A(T)]* = 66%, 8. *E[A(T)]* = 79%, and 9. *E[A(T)]* = 92%.

Table 1: Relative absolute difference of *E[A(T)]* (resp., *Var[A(T)]*) obtained using our model and simulation for case 1,2, 3, and 4.

Case	1	2	3	4
Relative absolute difference *E[A]* (%)	3.68	0.17	0.35	3,67
Relative absolute difference *Var[A(T)]* (%)	8.08	12.42	20.45	7,39
Min difference of *P(A(T)>x), x<=E[A(T)]*	-0.04	-0.01	0	-0,04
Max difference *P(A(T)>x), x<=E[A(T)]*	0.01	0.02	0.02	0,01

Table 2: Relative absolute difference of *E[A(T)]* (resp., *Var[A(T)]*) obtained using our model and simulation for case 5, 6, 7,8 and 9.

Case	5	6	7	8	9
Relative absolute difference *E[A]* (%)	0,49	0,11	0,34	0,17	0,22
Relative absolute difference *Var[A(T)]* (%)	12,83	7,01	6,77	4,22	1,51
Min difference of *P(A(T)>x), x<=E[A(T)]*	-0,01	-0,01	-0,03	-0,02	-0.01
Max difference *P(A(T)>x), x<=E[A(T)]*	0,03	0,03	0,04	0,03	0.07

6 CONCLUSIONS

In this paper we analyzed the interval availability of a two-echelon network that supports multi-item systems. We proposed an analytical approximation that is based on a Markov chain analysis. We computed in closed and exact form the expected, the variance, and the probability of hundred percent interval availability of the system. Using the previous metrics we approximate the survival function of the interval availability. The simulation result shows that our model has accurate results especially for high expected interval availability.

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APPENDIX: SIMULATION DETAILS

Case 1: M=10, depot stock =(0 0 0 0 0 0 0 0 0 0). **Case 2:** M=10, depot stock =(1 0 0 0 1 0 0 0 0 0). **Case 3:** M=10, depot stock =(2 1 1 1 2 1 0 0 0 0). **Case 4:** M=30, depot stock =(0 1 2 2 0 1 1 1 1 0 1 1 1 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 0). **Case 5:** M=30, depot stock =(1 1 2 2 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 0). **Case 6:** M=30, depot stock =(7 1 2 2 2 1 1 1 1 0 1 1 1 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 1). **Case 7:** M=50, depot stock =(1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0). **Case 8:** M=50, depot stock =(7 1 2 2 2 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0). **Case 9:** M=50, depot stock =(7 1 2 2 4 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 1 1 1 0 0 0 0 0 4 4 4 1 1 1 1 1 1 2 1 1 0).