A LOCAL-GLOBAL MODEL FOR MULTIAGENT SYSTEMS
Sheaves on the Category MAS

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Keywords: Multiagent systems, Sheaves, Dynamic cooperation structures.

Abstract: In multiagent systems, each agent has its own local view of the environment. Nevertheless, agents try to cooperate to reach a common global goal. In this paper, we use a suitable Grothendieck topology and sheaves to model the agents' local data and their communication.

1 INTRODUCTION

Multiagent Systems (MASs) provide autonomous, distributed, and flexible problem solving capabilities for a wide field of problem areas. The present contribution elaborates how sheaf theory can provide the unification and abstraction to integrate cooperation structure, agents' local knowledge, and communication in a single model. The idea is to un-couple structural information and the agent's knowledge. Structural information comprises of all kinds of relations and cooperations between agents and is encoded in so called base diagrams. An agent's gathered knowledge is then encoded in a sheaf over base diagrams.

This paper is organized as follows. In Section 2, we describe the category $\mathbb{MAS}$, introduce our running example and some notions in sheafs. In Section 3, we apply the construction of sheaves to $\mathbb{MAS}$.

2 PRELIMINARIES

In this section, we introduce the category $\mathbb{MAS}$, our running example and some results for sheaves.

2.1 Base Diagrams

The category $\mathbb{MAS}$ (Pfalzgraf and Soboll, 2007) has as objects base diagrams representing the current cooperation structure of the underlying MAS, describing the agents, their properties and relationships. The morphisms in this category are maps respecting the structure of base diagrams. This is necessary to preserve the relational information encoded in there. For the remainder of this paper we use the following running example describing a set of agents that cooperate to weld two cubes. The cubes are delivered to the assembly agent by agents equipped with a gripper.
Agent Properties: $Ap = \{1,2,3\}$, agent is equipped with (1) a welding device, (2) a gripper, (3) agent can act as relay agent for communication. *Arrow types:* $At = \{c,d,r\}$ (c) communication channel, (d) delivery channel (dotted lines), (r) request channel.

In Fig. 1, the left hand side shows the actual robots, while the underlying base diagram is depicted on the right. Of the four robots, b, c, and d have a gripper (2), b is also a relay agent (3), and a is an assembly robot with a welding device (1). Agent b has a delivery channel to agent a and agent a has open communication channels to b and c. A $\mathbb{MAS}$ morphism F is depicted in Fig. 2. F is the obvious inclusion map, where the communication channels, the arrow types, as well as the object types are preserved.
Actions: The action types $Act = \{idle, wfr, weld, ed\}$ define of possible actions: Type 1 (welding agent) can execute $weld$ or $idle$. Type 2 (gripper agent) can execute wfr (*w*ait *f*or *r*esources, if no cubes are available) or $idle$, and it can set an outgoing delivery channel to ed (*e*xecute *d*elivery) or $idle$. Type 3 (relay agent) can only execute $idle$, but may act as a relay agent for communication and requests.

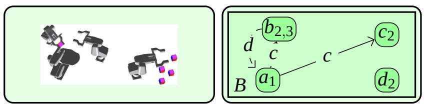

Figure 1: Example of a base diagram.

Soboll T. and Golas U..
A LOCAL-GLOBAL MODEL FOR MULTIAGENT SYSTEMS - Sheaves on the Category MAS.
DOI: 10.5220/0003742103310334
In *Proceedings of the 4th International Conference on Agents and Artificial Intelligence* (ICAART-2012), pages 331-334
ISBN: 978-989-8425-96-6

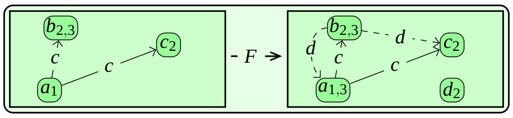

Figure 2: $\mathbb{MAS}$ morphism.

2.2 Applied Notions and Notations

Here we summarize notions, notations, and results from sheaf-theory (MacLane and Moerdijk, 1994; Kashiwara and Schapira, 2006). We will use these and want to introduce them here in an informal, (hopefully) intuitive and motivating manner.

Given some domain of (distributed) entities, like agents, a sheaf is a mathematical device providing the means to collate local information stored or gathered by each entity/agent in the system to a global view, if the junks of local information agree in overlapping areas. A presheaf is a very similar thing, but presheaves do not require the local observations to be collate-able to a unique global view, whereas sheaves do.

To be able to formalize the notion of overlapping areas we need some notions of intersection, union and covering, which are provided by a Grothendieck Topology (GT) of base diagrams. Given some base diagram B, we construct a subcategory $Sub(B)$ which is a collection of sub-diagrams of B together with associated inclusions. In this category we define what it means that a selection of sub-base diagrams covers B. Informally, this is the case if the union of a selection of sub-base diagrams results in B, using a GT.

Example 1. *In Fig. 3, a subcategory* $Sub(B)$ *is shown, where the base diagram* B *is depicted as the rightmost object. Bold arrows define the morphisms in* $Sub(B)$*. We can observe that the set of inclusion morphisms* $\{11,12\}$, $\{7,8\}$*, and* $\{9,10\}$ *cover* B, S_4*, and* S_5 *respectively. On the other hand, the inclusions* $\{4,5\}$ *do not cover* S_2 *because the arrow is missing.*

Given a sheaf F on $Sub(B)$, holding the observation gathered by the agents in B, for every subsystem S of $Sub(B)$, $F(S)$ holds all the information gathered or stored in S. We can perform a restriction of F to S denoted by $F|_S$, which is again a sheaf defined on $Sub(S)$. A sub-sheaf of F on $Sub(B)$ is simply a sheaf F' on $Sub(B)$ such that the information stored in F' is a subset of the information in F for every subsystem.

For a presheaf of observations, where for some or all observations there is no unique collation, we can perform sheafification. This operation provides for any presheaf P the "best" sheaf F you can get from P. F is obtained by identifying things that have the same restrictions and then adding in all the things that can be patched together (Mumford, 1999).

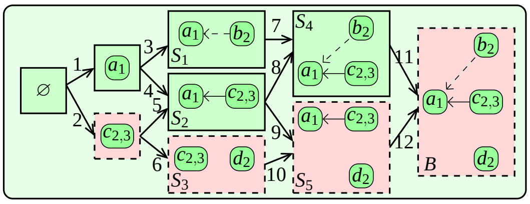

Figure 3: Example of a subcategory $Sub(B)$.

A very important notion is the gluing of sheaves. The main idea is that for sheaves, i.e.knowledge on different subsystems, where we explicitly allow intersections, we can collate the observations to a single sheaf if the corresponding "local" sheaves agree in the overlaps. This means that the restrictions of the "local" sheaves of the different subsystems to the intersection of the subsystems need to be equal.

Example 2. *Given the discrete topology on a set of agents* Ag*, for any subset* $U \subset Ag$ *the actual action assignments* $f : U \to Act$ *of the agents can be determined locally. For* $V \subset U$*, the restriction of* f *to* V*, denoted as* $f|_V : V \to Act$ *is the action assignment for the agents in* V*, this is a passage from global to local.*

3 SHEAVES ON $\mathbb{MAS}$

In this section, we apply the sheaf concepts to our base diagrams. Note that we allow in our running example that some arrows (here of type d) get actions assigned (ed and $idle$). Such arrows will be called action arrows (aA). For the other arrow types we do not introduce actions, because they do not influence the agent's knowledge in its local view.

We define the (pre)sheaves representing the agent's knowledge as a functor $P : Sub(B)^{op} \to \mathbf{SET}$. For all objects C of $Sub(B)$, $P(C)$ consists of a family of maps defined by $P(C) = \{f_i : Ag(C) \cup Aa(C) \to Act\}$, where $Ag(C)$ are the agents in C, $Aa(C)$ are the action arrows in C, Act is the set of actions and each map $f_i \in P(C)$ assigns to every agent and action arrow a single action of the set Act. Loosely speaking, each f_i represents a possible world compatible with the agents sensor and/or communication information.

3.1 Agent View

Each agent has sensors to allocate information in its environment, where the reading of each sensor results in a certain base diagram. We assume that an agent is capable of sensing the types of the agents and their identity and has knowledge about actions associated to these types. A suitable combination of all sensor information of an agent leads to its local view.

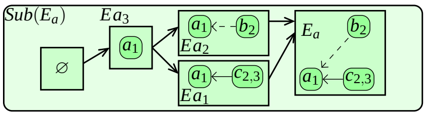

Figure 4: Env./Subcat. for agent a.

Definition 1 (Agent Environment)**.** *Given an agent a with n sensors, each sensor $i \in 1,2,\dots,n$ samples an environment Ea_i, where at least a is present. The* agent environment *E_a of a is the union of all environments $Ea_1,\dots,Ea_n$.*

Example 3. *Suppose agent a is equipped with two sensors which sample the environments Ea_1 and Ea_2. The agent environment E_a is obtained by the union of Ea_1 and Ea_2 (see $Sub(E_a)$ in Fig. 4).*

Table 1: Example: Type sensor readings.

t_1	a	c	t_2	a	b	$\dashrightarrow$
f	idle	wfr	j	idle	wfr	ed
g	idle	idle	k	idle	idle	ed
h	weld	wfr	l	weld	wfr	ed
i	weld	idle	m	weld	idle	ed
			o	idle	wfr	idle
			p	idle	idle	idle
			q	weld	wfr	idle
			r	weld	idle	idle

We distinguish type-sensors and action-sensors. This distinction is necessary to apply adequate "gluing" operations to collate the agent's knowledge. Type sensors collect type information, which allows an agent to deduce the possible actions the agents in its environment may execute. Action-sensors on the other hand capture observed actions. The combination of the sampled information provides the building block for local knowledge or the agent view.

Type sampling. For every agent i, we construct the presheaf $Pr_i : Sub(E_i)^{op} \to \mathbf{SET}$ representing the information gathered by sensors observing type information. Initially, Pr_i is the functor assigning to each object in $Sub(E_i)$ the empty set and to the empty base diagram the singleton containing the empty map only (which is a terminal object in **SET**). For every type-sensor, the sensor reading contains type information and results in a set of maps of the environment $E_s \in Sub(E_i)$. It is joined with the already available information in $Pr_i(E_s)$ and propagated into the sub-environments of E_s by restriction. By sheafification of Pr_i we construct the sheaf T_i.

Example 4. *Let $Pr_a : Sub(E_a)^{op} \to \mathbf{SET}$ be the initial presheaf. Agent a has two type sensors t_1, t_2 sampling the environments Ea_1, Ea_2 (see Fig. 4). The reading of t_1 in Ea_1 is "agent a has type 1 and agent c has type 2,3". From this, agent a deduces four possible action assignment maps f,g,h,i in Ea_1. The reading of t_2 in Ea_2 is "agent a has type 1 and agent b has type 2" and a delivery channel is recognized. This leads to eight possible action assignment maps j,k,l,m,o,p,q,r in Ea_2 (see Table 1).*

We update $Pr_a(Ea_1)$ to $Pr_a(Ea_1) \cup \{f,g,h,i\}$ and $Pr_a(Ea_2)$ to $Pr_a(Ea_2) \cup \{j,k,l,m,o,p,q,r\}$. Moreover, $Pr_a(Ea_3)$ with inclusions into Ea_1 and Ea_2 is updated. The sheafification of Pr gives the sheaf T_a with $T_a(Ea) = \{fj, fk, gj, gk, hl, hm, il, im, fo, fp, go, gp, ho, hp, io, ip\}$ (see Table 2).

Action Sampling. For a set of action-sensors $\{s_j | j = 1,\dots,k\}$, each sampling actions in a sub-environment E_j of E_i of agent i, we interpret the sensor reading as a sheaf Si_j on $Sub(E_j)$.

Example 5. *Assume agent a has two action sensors as_1 and as_3 sampling Ea_1 and Ea_3 such that the reading of as_1 results in the map $a_1 : \{a,c\} \to Act$ with $a_1(a) = idle$, $a_1(c) = wfr$ and the reading of as_3 is the map $a_3 : \{a\} \to Act$ with $a_3(a) = idle$. Note that the restrictions of these maps result in sheaves Sa_1 and Sa_2, respectively.*

Collating Type and Action Information. We collate the type information in T_i and the action information in $Si_1,\dots,Si_k$ leading to the sheaf $P_i : Sub(E_i)^{op} \to \mathbf{SET}$ by gluing sheaves.

Example 6. *Given the sheaves T_a, Sa_1, and Sa_2 from Ex. 4 and 5, we construct the sheaf P_a by gluing the maximal sub-sheaf T'_a. $P_a(E_a) = \{fj, fk, fo, fp\}$ (see Table 2). Including additional information that the agent has, e.g. that the delivery channel is in state ed, leads to only four possible action assignment maps in $P_a(E_a) = \{fj, fk\}$; for the sake of simplicity we will assume this for the remainder of the paper.*

Assume that agent c is waiting for resources, a local view P_c evaluated at E_c for c is then given by $P_c(E_c) = \{fh, fi, gh, gi\}$ as depicted in the right hand side of Table 2.

Definition 2 (Agent View)**.** *The* agent view *of agent i is the sheaf $P_i : Sub(E_i)^{op} \to \mathbf{SET}$. If the agent view P_i exists, i.e. the sensor information of i's different action sensors agree in the overlaps, we call the agent view* (locally) consistent.

3.2 Communication

Each communication arrow means that agents communicate selected information. Restrictions and sub-sheaves will be used for selecting specific information that shall be communicated. The collation of communication content is similar to the way sensor information was collated to form the agent view.

Definition 3 (Communication)**.** *Given the agent views P_i and P_k on $Sub(E_i)$ and $Sub(E_k)$, respectively,*

Table 2: Examples: Value of the sheaf T_a at E_a, P_a at E_a (bold) and P_c at E_c.

$T_a(E_a)$	a	b	c	$\dashrightarrow$		a	b	c	$\dashrightarrow$
fj	idle	wfr	wfr	ed	hl	weld	wfr	wfr	ed
fk	idle	idle	wfr	ed	hm	weld	idle	wfr	ed
gj	idle	wfr	idle	ed	il	weld	wfr	idle	ed
gk	idle	idle	idle	ed	im	weld	idle	idle	ed
fo	idle	wfr	wfr	idle	ho	weld	wfr	wfr	idle
fp	idle	idle	wfr	idle	hp	weld	idle	wfr	idle
go	idle	wfr	idle	idle	io	weld	wfr	idle	idle
gp	idle	idle	idle	idle	ip	weld	idle	idle	idle

$P_c(E_c)$	a	c	d
fh	idle	wfr	idle
fi	weld	wfr	idle
gh	idle	wfr	wfr
gi	weld	wfr	wfr

and a communication arrow from agent k to agent i in the environment of k. k selects a sub-sheaf K of a restriction E'_k of its environment and sends it to agent i, i computes the union of the environments E_i and E'_k denoted as $C_{i,k}$ and called communication-environment. *Agent i computes the maximal sub-sheaves K' and P'_i of the communicated sheaf K and its agent view P_i, respectively, such that K' and P'_i can be glued to the sheaf $Comm_{i,k} : Sub(C_{i,k})^{op} \to$ **SET**. $Comm_{i,k}$ holds the information of agent i including the communicated knowledge of agent k.*

Definition 4 (Communication Consistency). *Given communication arrows from agents $a_1,...,a_n$ to an agent i. We call the communication consistent if for all sheaves $Comm_{i,a_1}$, ...,$Comm_{i,a_n}$ there exist sub-sheaves $Comm'_{i,a_1},...,Comm'_{i,a_n}$ that can be glued to form the sheaf $Comm_i$ on the union C_i of all $C_{i,a1}$ to $C_{i,an}$ holding all information available to agent i.*

Table 3: Example: Value of sheaf $Comm_a$ at C_a.

$Comm_a(C_a)$	a	b	c	d	$\dashrightarrow$
fjfh	idle	wfr	wfr	idle	ed
fjgh	idle	wfr	wfr	wfr	ed
fkfh	idle	idle	wfr	idle	ed
fkgh	idle	idle	wfr	wfr	ed

Example 7. *Assume agent c communicates its entire agent view P_c to agent a. The resulting sheaf representing agent a's knowledge including communication is $Comm_a = Comm_{a,c}$ on C_a depicted by the dashed elements in Fig. 3, since c is the only agent communicating to a. This is a sheaf in case the local observations of a and c are not contradictory. In Ex. 6, 6 we have defined the sheaves P_a, P_c with $P_a(E_a) = \{fj, fk, fo, fp\}$ and $P_c(E_c) = \{fh, fi, gh, gi\}$. P'_a with $P'_a(E_a) = \{fj, fk, fo, fp\}$ and P'_c with $P'_c(E_c) = \{fh, gh\}$ are compatible leading to the gluing sheaf $Comm_a : Sub(C_a)^{op} \to$ **SET** . It represents the observation of a including the communicated data, where $Comm_a(C_a) := \{fjfh, fjgh, fkfh, fkgh, fofh, fogh, fpfh, fpgh\}$ (see Table 3).*

P_i and $Comm_i$ on $Sub(E_i)$ and $Sub(C_i)$, respectively, describe the knowledge available to agent *i* locally without and with communication. During the construction of $Comm_i$ specific maps can be eliminated (by building a subsheaf of P_i) using the additional information available to the agent. The elimination of maps by building sub-sheaves reduces the set of possible worlds, and hence this is in fact a method to gain knowledge rather than to loose it.

4 CONCLUSIONS AND OUTLOOK

We have demonstrated how sheaves can be applied to model local-global dependencies within a Multiagent System based on structural information of its base diagram and using a suitable Grothendieck topology for $\mathbb{MAS}$. Sheaves allow us to collate the local observations through communication for a "wider" view of a single agent and to construct group knowledge. The sheaf model implicitly checks for inconsistency in overlapping observations.

Future work includes the integration of cooperation rules describing rule-based changes of the base diagrams. The possible worlds that are generated by the sheaves represent an agent's knowledge, where based on this information it can decide whether and how to execute specific cooperation rules. As a next step, one could include additional information like more specific resource data in the model by including resources as agent properties or by defining additional sheaves representing the distribution of resources.

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