SEMIDEFINITE RELAXATIONS FOR THE SCHEDULING NUCLEAR OUTAGES PROBLEM

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Abstract: We investigate semidefinite relaxations for solving a MIQP (Mixed-Integer Quadratic Program) formulation of the scheduling of nuclear power plants outages, which is extremely hard to solve with CPLEX. Based on our numerical experiments, results obtained with semidefinite relaxations improve those obtained with continuous relaxation: the gap between the optimal solution and the continuous relaxation is on average equal to 1.80% whereas the semidefinite relaxation yields an average gap of 1.56%. These bounds are then used to obtain a feasible solution with a randomized rounding procedure.

1 INTRODUCTION

The French electrical production facilities is characterized by a high number of nuclear power plants, which have to be shut down regurlarly in order to proceed to refueling and maintenance operations. Optimizing the scheduling of these outages is therefore a key factor for an efficient economic performance.

This mid-term management problem consists in determining, on the five years ahead i) the dates for outages to refuel nuclear power plants, ii) the amount of supplied fuel and iii) the nuclear power plants production planning which satisfy the demand at minimal cost, while respecting numerous technical constraints.

This real-life problem is far too difficult to be tackled exactly, due to its huge size, its non-linear constraints, and because uncertainties affecting both production and demand. Finally, modelling the online/offline state of the plants requires the introduction of binary variables, which make the problem combinatorial.

Many approaches for this problem have been investigated (Khemmoudj et al., 2006), (Porcheron et al., 2009). In this paper, we deal with a deterministic version of the problem and emphasize its combinatorial nature in order to investigate efficiency of semidefinite relaxations.

It is organized as follows. In Section 2, we derive our model for the problem. In section 3, we outline the semidefinite relaxations we use. We report some numerical results in section 4 before concluding and giving prospects for future work.

2 MODELING THE NUCLEAR OUTAGES SCHEDULING PROBLEM

We will consider in this paper a deterministic version of the problem where only the most significant technical constraints are taken into account.

2.1 Key Operating Features of Nuclear Power Plants and Modeling

The nuclear park is composed of several sites, where each site is a set of 2, 4 or 6 nuclear power plants. The operation life of a nuclear power plant is decomposed into cycles, each cycle being made up of a phase of production, called *production campaign*, followed by an outage.

Let's introduce some convenient notations: in what follows, x and y will denote respectively the binary and continuous variables. (i,j) refers to the j-th cycle of the plant i. The index of the first and last cycle of each plant are respectively 1 and J_i. $\{1,\cdots,N_s\}$ and $\{1,\cdots,N_\nu\}$ are the set of sites and plants of the nuclear park. If P is a set of nuclear plants, $(i,j) \in P$ denotes the whole cycles of the plants of this set. We denote $i \in k$ the fact that a plant belongs to the k-th site and $(i,j) \in k$ the cycles of the plants of this site. Finally, a time step t corresponds to a week and the horizon time is composed of N_t weeks.

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2.1.1 Phase I - Campaign

During this phase, the nuclear power plant produces either in *standard* mode (at full power) or in *modulation* mode (at lower level). The standard mode, that is producing at maximal power W_i (expressed in MW), is the best operating level for the plants. On the contrary, when a nuclear plant doesn't produce at full power, it is said to "modulate". This mode of production may alter the state of the plant, which requires more maintenance afterwards. That is why the quantity of modulation, which is measured as the amount of "non-produced" energy, is limited. Let $y^{\mu}_{i,j}$ be the continuous variable that represents the modulation of the cycle (i,j):

$$\forall (i,j) \in N_\nu,\ y^{\mu}_{i,j} \in [0, M_{i,j}] \tag{1}$$

2.1.2 Phase II - Outage

A nuclear power plant shall be stopped regularly for refueling and maintenance operations. The duration of the outage of the cycle (i,j) is denoted by the number of weeks $\delta_{i,j}$. The scheduling of the outages requires to define a binary variable for each possible beginning date of each outage: $\forall t \in E_{i,j},\ x^{\nu}_{i,j,t} \in \{0,1\}$ where $E_{i,j}$ is the set of possible beginning dates for the outage of the cycle (i,j). Among the variables $x^{\nu}_{i,j,t}$ of the cycle (i,j), only the one for which t is the actual beginning date of the outage shall be equal to 1. Consequently, we impose the so-called *uniqueness constraint*:

$$\forall (i, j < J_i) \in N_\nu,\ \sum_{t \in E_{i,j}} x^{\nu}_{i,j,t} = 1 \tag{2}$$

With this modeling, the beginning date of the outage (i,j) can be easily computed using the formula $\sum_{t \in E_{i,j}} t x^{\nu}_{i,j,t}$ and the state of the plant i at week t, i.e. 1 if the plant is online, 0 otherwise, can be expressed as follows: $1 - \sum_{j=1}^{J_i} \sum_{t'=t-\delta_{i,j}+1}^{t} x^{\nu}_{i,j,t'}$. Note that for the sake of simplicity, we sometimes drop the notation $t \in E_{i,j}$ for $x^{\nu}_{i,j,t}$ and consider it implicitly.

It comes that the maximal capacity of production of the nuclear park y^{κ}_t at week t, a state variable, can be computed as:

$$\begin{aligned} &\forall t = 1, \cdots, N_t, \\ &\quad y^{\kappa}_t = \sum_{i \in N_\nu} W_i \big(1 - \sum_{j=1}^{J_i} \sum_{t'=t-\delta_{i,j}+1}^{t} x^{\nu}_{i,j,t'}\big) \end{aligned} \tag{3}$$

Refueling and Final Stock of Energy. For safety reasons, the stock of energy that remains in the reactor of a nuclear plant at the beginning of an outage (i,j), denoted $y^{\sigma}_{i,j}$, must lie within the interval $[\underline{F}_{i,j}, \overline{F}_{i,j}]$, except for the last cycle, for which only the lower bound is required since the outage is not attained:

$$\begin{aligned} \forall (i, j < J_i) \in N^\nu, &\quad \underline{F}_{i,j} \le y^{\sigma}_{i,j} \le \overline{F}_{i,j} \\ \forall i \in N^\nu, &\quad \underline{F}_{i,J_i} \le y^{\sigma}_{i,J_i} \end{aligned} \tag{4}$$

For the sake of concision, let's just say that $y^{\sigma}_{i,j}$ can be computed as a affine combination of the previous final stock $y^{\sigma}_{i,j-1}$, of the outages binary variables $x^{\nu}_{i,j,t}$ and of the variable $y^{\mu}_{i,j}$ and $y^{\rho}_{i,j}$ denoting the modulation and the amount of the reload carried out during outages respectively. Without detailing the particular case of the first and last cycles, we have the following formula for the final stock:

$$\begin{aligned} &\forall (i,\ 1 < j < J_i) \in N_\nu, \\ &\quad y^{\sigma}_{i,j} = W_i \delta_{i,j-1} + \beta_i y^{\sigma}_{i,j-1} + y^{\rho}_{i,j-1} + y^{\mu}_{i,j} \\ &\qquad - W_i \big(\sum_{t \in E_{i,j}} t x^{\nu}_{i,j,t} - \sum_{t \in E_{i,j-1}} t x^{\nu}_{i,j-1,t} \big) \end{aligned} \tag{5}$$

Besides, for each cycle $(i, j < J_i)$, the variable $y^{\rho}_{i,j}$ shall respect a maximal value $\bar{R}_{i,j}$, corresponding to the maximal capacity of the reactors:

$$\forall (i, j < J_i) \in N_\nu,\ y^{\rho}_{i,j} \in [0, \bar{R}_{i,j}] \tag{6}$$

Managing Resources for Outages. On a nuclear site, in order to manage the limited resources required for the refueling and maintenance operations, we impose a maximal number of parallel outages at each time step and a maximal lapping between outages.

Let N^{par}_k be the maximum authorized number of outages in parallel on site k. Then, the related constraint can be written:

$$\begin{aligned} &\forall k = 1, \cdots, N_s,\ \forall t = 1, \cdots, N_t, \\ &\quad \sum_{(i,j) \in k} \sum_{t'=t-\delta_{i,j}+1}^{t} x_{i,j,t'} \le N^{par}_k \end{aligned} \tag{7}$$

Let N^{lap}_k be the maximum authorized lapping between the outages of site k. A negative value represents a minimum space. Then, for each concerned pair $(i,j),(i',j')$, there are two possibilities: either (i,j) starts before (i',j'), or it doesn't. The computation of the lapping depends of the effective configuration: let $\Delta_{i,j,i',j'}$ denotes the space between beginning of outages, the lapping might be:

$$\delta_{i,j} + \Delta_{i,j,i',j'} \text{ or } \delta_{i',j'} - \Delta_{i,j,i',j'} \tag{8}$$

This disjonction requires the introduction of new binary variable: $x^{\lambda}_{i,j,i',j'}$ that codes 0 in the first case and 1 otherwise. Let $\tilde{M}$ be a sufficiently large number. Then both following constraints must be respected:

$$\begin{array}{llll} \delta_{i,j} & +\Delta_{i,j,i',j'} & -\tilde{M}x^{\lambda}_{i,j,i',j'} & \leq N_k^{lap} \\ \delta_{i',j'} & -\Delta_{i,j,i',j'} & -\tilde{M}(1-x^{\lambda}_{i,j,i',j'}) & \leq N_k^{lap} \end{array} \quad (9)$$

2.2 Constraints Related to Demand Satisfaction

In our problem, the production portfolio made up of N_ν nuclear power plants and N_θ fossil-fuel power plants has to satisfy the electrical demand on the two following periods of each time step (e.g. a week):

- A *peak* period when the demand is high and can not be satisfied by nuclear production (fossil-fuel production is needed) ;
- An *off-peak* period when the demand is low (for example, during the night) and can be satisfied by nuclear production only.

2.2.1 Peak Demand

At peak time, the whole capacity of production of the park $y_t^\kappa + \sum_{i\in N_\theta} U_{i,t}$, where $U_{i,t}$ is the capacity of production of the fossil-fuel power plant i at time step t, should satisfy the peak demand D_t^+:

$$\forall t = 1, \cdots, N_t, \quad \sum_{(i,j)\in N_\nu} y_t^\kappa \geq D_t^+ - \sum_{i\in N_\theta} U_{i,t} \quad (10)$$

2.2.2 Off-Peak Demand

At off-peak time, the demand constraint comes to limiting the whole modulation of the park at each time step. Here, we make a reasonable simplification consisting in respecting the sum of these constraints on the time horizon. This allows us to gather the modulation throughout the cycles, so the constraint can be written:

$$\sum_{(i,j)\in N_\nu} y^\mu_{i,j} \leq D^- \quad (11)$$

2.3 The Objective Function

Our aim is to minimize the global cost of production which is the sum of the nuclear production cost and the fossil-fuel production cost. The first one is proportional to the amount of reloads and the fossil-fuel production, which is computed as the difference between the peak demand and the nuclear production, has a quadratic cost, so the global cost function is:

$$\sum_{(i,j)\in N_\nu} \gamma_{i,j} y^\rho_{i,j} - \sum_{i\in N_\nu} \gamma_{i,J_i-1} y^\sigma_{i,J_i} + \sum_{t=1}^{N_t} \gamma_t^\theta [D_t^+ - y_t^\kappa]^2 \quad (12)$$

3 RESOLUTION

Finally, gathering equations (1), (2), (3), (4), (5), (6), (7), (9),(10), (11), (12) and introducing matrix formulation leads to the compact form:

$$(P) \left\{ \begin{array}{ll} \min_{x,y} & x^t Q x + p^t x + q^t y \\ \text{subject to} & Ax + By \leq c \\ & y \leq \bar{y} \\ & x \in \{0,1\}^{N_x},\ y \in \mathbb{R}_+^{N_y} \end{array} \right. \quad (13)$$

Our problem is therefore a mixed quadratic optimization problem with linear constraints, where the quadratic terms of the objective function involve only binary variables. This kind of problem is difficult to solve, even with a powerful commercial solver like CPLEX. For this reason, we investigate semidefinite relaxations in the view of obtaining better bounds of the solution than we can obtain when using continuous relaxations.

3.1 Semidefinite Relaxations

Semidefinite programming (SDP) is a subfield of convex optimization which deals with the optimization of a linear function over an affine subspace of the cone of the semidefinite matrices. With $A \bullet B$ denoting the Frobenius inner product, it has the following form:

$$(SDP) \left\{ \begin{array}{ll} \min_{X\in\mathbb{S}^n} & A_0 \bullet X \\ \text{subject to} & A_i \bullet X = b_i,\ i = 1, \cdots, m \\ & X \succcurlyeq 0 \end{array} \right. \quad (14)$$

This area of mathematical programming has undergone a rapid development in the last decades, spurred by the development of efficient resolution algorithms (see (Helmberg et al., 1996), (Helmberg and Rendl, 2000)) and by the discovery of widespread applications, in particular to relaxation of combinatorial problems. For further reading on the subject, see for example the surveys of Boyd and Vandenberghe (Vandenberghe and Boyd, 1994) and Todd (Todd, 2001) or the corresponding handbook (Wolkowicz et al.,). See also the survey of Laurent ((Laurent et al., 2005)) on the related relaxation of combinatorial problems.

Here we apply SDP to the relaxation of the previously described MIQP (13). For this, we introduce the following symmetric matrix:

$$X = \left(\begin{array}{cc|c} xx^t & x & * \\ x & 1 & \\ \hline \multicolumn{2}{c|}{*} & \mathrm{Diag}(y) \end{array} \right) \quad (15)$$

where $\mathrm{Diag}(y)$ stands for the diagonal matrix made up with vector y and $*$ means that any value can be taken. Let's note that the first submatrix include the vector x and the associated quadratic matrix xx^T. Then, by defining the appropriate matrices C_i, we can express the objective quadratic function (because the quadratic terms involve only x) and the linear constraints as $C_i \bullet X$.

About the binary constraints, we define the matrices D_i such that $x_i^2 - x_i = 0 \Leftrightarrow D_i \bullet X = 0$. Furthermore, a matrix E is used to impose that the last component of the first submatrix be equal to 1.
So, we have the following equivalent formulation for our problem:

$$(Q)\begin{cases} \min & C_0 \bullet X \\ \text{s.t.} & C_i \bullet X \leq 0,\ i = 1,\cdots,M \\ & D_i \bullet X = 0,\ i = 1,\cdots,N_x \\ & E \bullet X = 1 \\ & X_{i,j} = X_{i,N_x+1}X_{j,N_x+1},\ i,j = 1,\cdots,N_x \end{cases}$$

The last constraint comes to impose to X the specific form described at (15). This constraint, which is neither linear or convex, is what makes the problem NP-hard. Such as matrix is necessarily semidefinite positive, so the semidefinite relaxation is obtained by replacing this constraint by a constraint on its semidefiniteness, which is convex. Consequently, the associated SDP is:

$$(SDP)\begin{cases} \min & C_0 \bullet X \\ \text{s.t.} & C_i \bullet X \leq 0,\ i = 1,\cdots,M \\ & D_i \bullet X = 0,\ i = 1,\cdots,N_x \\ & E \bullet X = 1 \\ & X \succcurlyeq 0 \end{cases}$$

This relaxation gives rise to a lower bound of the optimal solution p^* of the initial problem, which can be used either within an exact search, typically a Branch & Bound procedure, or to compute an approximate solution of the problem, for example via a randomized rounding scheme. It is the latter alternative we are using here. We will compare the approximate solution obtained by applying this procedure (described in the paragraph below) to the solution of the Quadratic Program obtained by relaxing the integrality constraint (denoted here continuous relaxation). This program can be solved with CPLEX since the objective function is convex. In a second step, we will try to improve the SDP bound by adding some cuts based on the Sherali-Adams approach (Sherali and Adams, 1990).

3.2 Randomized Rounding Procedure

Randomization has proved to be a powerful resource to yield a feasible binary solution from a fractional one. The basic idea is to interpret the fractional value as the probability of the variable to take the value 1. Then the values of the binary variables are drawn according to this law and this process is iterated until the solution satisfies the constraints.

Here, we slightly change this principle, in order to find more easily a feasible solution: instead of deciding successively if a binary variable is 0 or 1, for each cycle, we choose one date among the possible beginning date for the associate outage, by using the fractional value as probabilily, since their sum is equal to one from the uniqueness constraint. Then, the values of the lapping variables x^λ follow. About the continuous variables, for the modulation x^μ, we keep the value of the relaxation and for the reload x^ρ, we take the minimal values that respects the final stock constraint.

3.3 Tightening Semidefinite Relaxation with Quadratic Cuts

Adding some valid appropriate equalities or inequalities may improve the bound of the semidefinite relaxation. Here, we apply the Sherali-Adams (Sherali and Adams, 1990) principle: let $Ax = b$ be a set of linear constraints and x_i a binary variable, the constraints $Axx_i = bx_i$ is valid. We apply this idea to the uniqueness constraint (2), with all the variables x_i that appear in the constraint. By using $x_i^2 = x_i$ it comes:

$$\forall (i, j < J_i) \in N_\nu,\ \forall t \in E_{i,j}, \quad \sum_{t' \in E_{i,j},\ t' \neq t} x^\nu_{i,j,t} x^\nu_{i,j,t'} = 0 \qquad (16)$$

4 NUMERICAL EXPERIMENTS

Numerical experiments have been performed on a three years time horizon (156 weeks), with one outage per year for each plant and two nuclear parks (respectively 10 and 20 nuclear power plants for the data set 1 to 12, and 13 to 24). Each park is declined into two versions which differ from the maximum amount of reload ($\overline{R}_{i,j}$) and modulation ($M_{i,j}$).

Finally, six instances have been tested for each data set, varying by the size of the search spaces associated to the outages dates variables (7 to 17 possibles dates).
All the computations have been made on an Intel(R) Core(TM) i7 processor with a clock speed of 2.13 GHz. In order to compare the solutions in the same conditions, the CPLEX results are obtained without activating the preprocessing. For each data set we computed:

Table 1: Results of exact search, relaxations and randomized rounding.

Data set	Nb of bin. var.	Opt		RelaxQP			RelaxSDP			RelaxSDP-Q		
		Obj	Time	Gap	Time	RR	Gap	Time	RR	Gap	Time	RR
D-1	215	3 343	1	0.73	0.02	2.35	0.54	12	2.35	0.26	12	0.70
D-2	278	3 254	21	0.80	0.00	3.88	0.64	19	1.49	0.46	21	1.70
D-3	341	3 174	183	0.94	0.02	4.86	0.82	31	2.43	0.65	36	3.25
D-4	406	3 110	1 286	1.10	0.02	4.23	0.97	44	5.04	0.83	54	5.14
D-5	469	3 051	7 200	1.18	0.02	11.70	1.08	63	3.72	0.96	79	4.04
D-6	530	2 994	5 780	1.17	0.03	14.56	1.09	81	3.35	1.00	108	4.73
D-7	215	3 297	2	1.24	0.02	3.31	1.03	5	2.82	0.68	6	0.82
D-8	278	3 223	8	1.89	0.03	10.28	1.72	8	7.15	1.38	11	3.35
D-9	341	3 176	39	2.94	0.08	11.31	2.81	15	9.95	2.49	64	2.11
D-10	406	3 133	169	3.91	0.13	14.69	3.80	26	11.94	3.52	98	8.98
D-11	469	3 070	76	3.87	0.18	13.56	3.78	38	13.81	3.53	147	11.79
D-12	530	3 024	232	4.25	0.20	14.47	4.17	53	17.98	3.95	236	16.20
D-13	539	12 580	7 200	0.85	0.05	3.16	0.77	154	3.28	0.61	171	2.08
D-14	698	12 431	7 200	0.95	0.10	3.47	0.89	252	3.76	0.76	286	4.06
D-15	852	12 290	7 200	1.13	0.14	5.78	1.08	373	4.58	0.99	436	4.83
D-16	1 011	12 156	7 200	1.14	0.14	6.16	1.09	578	5.19	1.02	750	5.29
D-17	1 170	12 034	7 200	1.15	0.22	5.72	1.12	791	5.77	1.08	1008	6.36
D-18	1 322	11 939	7 200	1.35	0.27	6.47	1.32	1030	5.67	1.30	1308	7.00
D-19	537	12 679	7 200	1.21	0.16	2.80	1.16	68	2.95	1.07	310	4.48
D-20	695	12 464	7 200	1.57	0.54	5.96	1.52	137	6.56	1.44	447	6.31
D-21	853	12 289	7 200	1.98	0.94	9.28	1.94	242	8.91	1.85	805	6.74
D-22	1 008	12 159	7 200	2.37	1.90	9.15	2.33	382	7.47	2.27	1113	8.80
D-23	1 165	12 034	7 200	2.65	2.95	7.87	2.62	628	7.70	2.58	2106	6.86
D-24	1 316	11 915	7 200	2.87	3.65	10.93	2.84	823	9.89	2.80	2231	8.52
Av.	651.83	7700.85	4224.91	1.80	0.49	7.75	1.71	243.88	6.41	1.56	493.46	5.59

- *Opt*: the best solution found within the time limit (2 hours) by using CPLEX-Quadratic 12.1. The time value 7200 means that the time limit has been reached, so the obtained integer solution is not optimal ;
- *RelaxQP*: the continuous relaxation computed with CPLEX-Quadratic 12.1;
- *RelaxSDP*: the SDP relaxation computed with the SDP solver CSDP 6.1.1 (cf (Borchers, 1999));
- *RelaxSDP-Q*: the SDP relaxation computed with CSDP 6.1.1 with quadratic cuts (cf 3.3) ;

For each data set, the table 1 reports the number of binary variables, the value of the objective function (in currency unit), the computational time in second and, for each kind of relaxation, the associated gap (Gap) and the relative gap of the randomized rouding (RR), whose formula are given below. The last line (Av.) gives the average of the previous lines.

$$Gap = \frac{p_{opt} - p_{relax}}{p_{relax}} \qquad RR = \frac{p_{RR} - p_{opt}}{p_{opt}} \tag{17}$$

Analysis of the Results

First we observe that CPLEX reaches the limited time for relatively small instances (e.g. 469 binary variables). This is in line with our expectations that this kind of problem is very hard for CPLEX, despite a quite small gap attained with continuous relaxation.

This may be related to the fact that, due to the demand constraint, the variable part of the objective function is very small w.r.t the absolute value of the cost. In other words, the optimal value is high, even with a "perfect" outages scheduling. Let's denote P the best possible objective value for a given data set, computed by considering the largest possible search space, and let's consider the variable part of the objective function, that is $p - P$, if p is the objective value. Then, the gap would increase, as shown in the following formula:

$$\frac{p_{opt} - p_{relax}}{p_{relax} - P} > \frac{p_{opt} - p_{relax}}{p_{relax}} \tag{18}$$

This illustrates the importance of considering the relative improvement of the gap achieved by semidefinite relaxation, rather that its absolute value.

For example, on the data set D-1, the gap is almost divided by three. Unfortunately, this ratio decreases as the number of binary variables raises, whereas the gap increases. This can be explained by the fact that the "exact solution" provided here is not optimal, considered that the computational time of CPLEX is limited. Let's denote $p'_{opt} > p_{opt}$ this value: then the ratio computed with this value is greater than the ratio computed with p:

$$\frac{p_{opt} - p_{relaxCPLEX}}{p_{opt} - p_{relaxSDP}} > \frac{p'_{opt} - p_{relaxCPLEX}}{p'_{opt} - p_{relaxSDP}} \qquad (19)$$

On average, the gap improves from 1.80% to 1.71% with original SDP relaxation and 1.56% with addition of valid equalitites. This latter improvement is promising, even though it comes at high additional computational cost, particularly on the larger instances. This can be ascribed to the fact that SDP solvers are only in their infancy, especially compared to a commercial solver like CPLEX.

Finally, the randomized rounding yields satisfying results: due to the random aspect of the procedure, there are still some data set where the continuous relaxation gives better results than the semidefinite relaxation, but on average the loss of optimality reduces from 7.75% to 6.41% and 5.59%, which is significant when considering the huge amount at stake.

5 CONCLUSIONS AND PROSPECTS

We investigated in this paper, semidefinite relaxations for a MIQP (Mixed-Integer Quadratic Program) version of the scheduling of nuclear power plants outages. Comparison of the results obtained on significant data sets shows the following main results. First, our MIQP is extremely hard to solve with CPLEX. Second, semidefinite relaxations provide a tighter convex relaxation than the continuous relaxation. In our experiments the gap between the optimal solution and the continuous relaxation is on average equal to 1.80% whereas the semidefinite relaxation yields an average gap of 1.56%. Third, the computational time for computing these semidefinite relaxations is reasonable. Exploiting those results in a randomized rounding procedure instead of the result of the continuous relaxation leads to a significant improvement of the feasible solution.

In the view of these preliminary results, additional investigations will concern i) introduction of more valid inequalities, ii) evaluation of others SDP resolution techniques, for instance *Conic Bundle* for facing problems of huge size.

REFERENCES

Borchers, B. (1999). Csdp, a C library for semidefinite programming.

Helmberg, C. and Rendl, F. (2000). A spectral bundle method for semidefinite programming. *SIAM Journal on Optimization*, 10(3):673–696.

Helmberg, C., Rendl, F., Vanderbei, R. J., and Wolkowicz, H. (1996). An interior-point method for semidefinite programming. *SIAM Journal on Optimization*, 6:342–361.

Khemmoudj, M. O. I., Bennaceur, H., and Porcheron, M. (2006). When constraint programming and local search solve the scheduling problem of electricité de france nuclear power plant outages. In *12th International Conference on Principles and Practice of Constraint Programming (CP'06)*, pages 271–283. LNCS.

Laurent, M., , , and Rendl, F. (2005). Semidefinite programming and integer programming. In K. Aardal, G. N. and Weismantel, R., editors, *Handbook on Discrete Optimization*, pages 393–514.

Porcheron, M., Gorge, A., Juan, O., Simovic, T., and Dereu, G. (2009). Challenge roadef/euro 2010: A large-scale energy management problem with varied constraints. Technical report, EDF R&D. http://challenge.roadef.org.

Sherali, H. D. and Adams, W. P. (1990). A hierarchy of relaxations between the continuous and convex hull representations for zero-one programming problems. *SIAM Journal on Discrete Mathematics*, 3(3):411–430.

Todd, M. J. (2001). Semidefinite optimization. *Acta Numerica*, 10:515–560.

Vandenberghe, L. and Boyd, S. (1994). Semidefinite programming. *SIAM Review*, 38:49–95.

Wolkowicz, H., Saigal, R., and Vandenberghe, L., editors. *Handbook of Semidefinite Programming: Theory, Algorithms, and Applications*, volume 27.