DETECTION AND RECOGNITION OF SUBPIXEL TARGETS WITH HYPOTHESES DEPENDENT BACKGROUND POWER

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Abstract: We consider the problem of detecting and recognizing the subpixel targets in sea background when the background power may be different under the null hypothesis – where it is assumed to be known – and the alternative multiple hypotheses. This situation occurs when the presence of the target triggers a decrease in the background power (subpixel targets). We extend the formulation of the Matched Subspace Detector (MSD) to the case where the background power is only known under the null hypothesis using the generalized likelihood ratio test (GLRT) for the multiple hypotheses case. The obtained multiple hypotheses test is based on the Modified MSD test (MMSD). We discuss the difference between the two detection and recognition systems: based on the MSD and MMSD tests. Numerical simulations attest to the validity of the performance analysis.

1 INTRODUCTION

Among the various frameworks in which pattern recognition has been traditionally formulated, the statistical approach has been most intensively studied and used in practice (Webb, 2002). Target detection and recognition in the remotely sensed image sequences can be conducted spatially, temporally or spectrally. The need for subpixel temporally (or spectrally) detection-recognition in remotely sensed image sequences arises from the fact that the targets sampling distances are generally larger than the sizes of targets of interest. In this case, the target is embedded in a single pixel sequence and cannot be detected or recognized spatially. As a result, traditional spatial-temporal analysis-based image sequence processing techniques are not applicable. Matched subspace detection-recognition is used to recognize the mulptiple hypotheses of different targets presence or absence of targets that are expected to lie in particular subspaces of the measurements. Standard approach in this case bases on calculating for each possible target of the GLR and determination of the target with maximum value of the GLR (Izenman, 2008). The common drawback of this approach is the assumption that the background power under hypothesis H_0 remains the same one as under hypotheses H_k. In digital optical systems, it is typically that the background has the same covariance structure under hypotheses H_0 and H_k, but different variances (Manolakis and Shaw, 2002), which is directly related to the fill factor, that is, the percentage of the pixel area occupied by the background. Because the background power is changed if any of the targets is present, the detection-recognition system is not optimum and, therefore, it is necessary to modify the MSD (Golikov, Lebedeva 2011). As a result, we assume that the proposed detection-recognition system can achieve a significant performance advantage against conventional one.

In this paper, we focus on the detection-recognition of small targets in the case of unknown power of Gaussian background under hypothesis H_i. We assume that different targets have the different subspace dimensions. In section 2, we formulate the subpixel detection-recognition problem using the linear mixing model and the concepts of targets and background subspaces. We derive the GLRT for the problem at hand and the distributions under the hypotheses. In Section 3, we investigated the detection-recognition performance losses in the case of background power variations between multiple hypotheses in a Gaussian environment for proposed and canonical detection-recognition systems in the presence of a mismatch between the designed and actual background power. Here, the numerical simulations are included to verify the validity of the

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theoretical analysis. Brief conclusions end the paper.

2 GENERALIZED LIKELIHOOD RATIO TEST

The problem addressed here is the detection-recognition of a K possible targets response $\boldsymbol{s}_k$ for a measurement $\boldsymbol{x}\sim N[\mu \boldsymbol{H}_k \boldsymbol{\theta}_k, \sigma_k^2 \boldsymbol{R}]$ in Gaussian background with covariance structure $\sigma_k^2 \boldsymbol{R}$, k=1,2,…,K. The problem is to decide between the null hypothesis (H_0) and the alternative hypotheses

$$(H_k)\text{: } H_0\text{: } \boldsymbol{x}=\boldsymbol{c}_0, \qquad H_k\text{: } \boldsymbol{x}=\mu \boldsymbol{s}_k+\boldsymbol{c}_k\,. \tag{1}$$

When the background covariance matrix $\boldsymbol{R}$, scaling σ_0^2, target subspace matrix $\boldsymbol{H}_k$, and the location parameter $\boldsymbol{\theta}_k$ are known, the appropriate detection-recognition statistics is presented in the MSD form (Scharf, 1991):

$$T_{kn}(\boldsymbol{x}) = (1/N\sigma_0^2)\max_k \boldsymbol{x}^H \boldsymbol{R}^{-1}\boldsymbol{H}_k\boldsymbol{\theta}_k[(\boldsymbol{H}_k\boldsymbol{\theta}_k)^H\boldsymbol{R}^{-1}\boldsymbol{H}_k\boldsymbol{\theta}_k]^{-1}(\boldsymbol{H}_k\boldsymbol{\theta}_k)^H\boldsymbol{R}^{-1}\boldsymbol{x} \tag{2}$$

We accept the hypothesis H_k when the statistics (2) achieves the maximum. The parameter $\boldsymbol{\theta}_k$ locates the target response $\mu \boldsymbol{s}_k=\mu \boldsymbol{H}_k \boldsymbol{\theta}_k$ in the target subspace spanned by the $p_k<N$ columns of a known matrix $\boldsymbol{H}_k$, $\boldsymbol{H}=C^{N\times p_k}$, which is the linear space of ($N\times p_k$) complex matrices. Let define the whitened targets mode matrix $\boldsymbol{\Phi}_k=\boldsymbol{R}^{-1/2}\boldsymbol{H}_k$ and the whitened measurements $\boldsymbol{y}=\boldsymbol{R}^{-1/2}\boldsymbol{x}$. We want to derive the detection-recognition test in the case of unknown parameters σ_k^2 using the generalized likelihood ratio of the conditional probability density functions (PDF). The maximized ratio of PDFs is obtained by replacing the unknown parameters by their estimators according to maximum likelihood (ML) criterion in such form:

$$L=\frac{\max_{k,\sigma_k^2} p(\boldsymbol{y};\boldsymbol{\theta}_k,\sigma_1^2/H_k)}{p(\boldsymbol{y};\sigma_0^2/H_0)} = \frac{\max\limits_{k,\sigma_k^2}[\pi^{-N}\sigma_1^{-2N}\exp(-(1/\sigma_1^2)(\boldsymbol{y}-\boldsymbol{\Phi}_k\boldsymbol{\theta}_k)^H(\boldsymbol{y}-\boldsymbol{\Phi}_k\boldsymbol{\theta}_k))]}{\pi^{-N}\sigma_0^{-2N}\exp[-\left(\frac{1}{\sigma_0^2}\right)\boldsymbol{y}^H\boldsymbol{y}]}, \tag{3}$$

where the numerator are maximized by parameter σ_k^2. The ML estimates (Jolliffe, 2002) of the σ_k^2 is obtained by solving such equations:

$$\frac{\partial L}{\partial \sigma_k^2}=0. \tag{4}$$

We designate the target subspaces matrix with a maximum number p_{max} of columns as $\boldsymbol{H}_{max}$ and $\boldsymbol{\Phi}_{max}$. It is well known (Scharf, 1991) that the estimate of the background variance is obtained as:

$$\hat{\sigma}_k^2 = \frac{\boldsymbol{y}^H\boldsymbol{P}_{\Phi_{max}}^{\perp}\boldsymbol{y}}{N-p_{max}}, \tag{5}$$

where $\boldsymbol{P}_{\Phi_{max}}^{\perp}=\boldsymbol{I}-\boldsymbol{P}_{\Phi_{max}}$ and $\boldsymbol{P}_{\Phi}=\boldsymbol{\Phi}_k(\boldsymbol{\Phi}_k^H\boldsymbol{\Phi}_k)^{-1}\boldsymbol{\Phi}_k^H$. Next, the maximum of (3) with respect to σ_k^2 is found for $\hat{\sigma}_k^2$, resulting in

$$L=\max_k\left\{\frac{e\boldsymbol{y}^H\boldsymbol{P}_{\Phi_{max}}^{\perp}\boldsymbol{y}\, exp[-(N)^{-1}\sigma_0^{-2}\boldsymbol{y}^H\boldsymbol{y}]}{\sigma_0^2(N-p_{max})}\right\}^{-N}. \tag{6}$$

Computing the logarithm of the N-th root of (6), we obtain the decision statistics:

$$T_{un}(\boldsymbol{y})=\max_k\left[\frac{\boldsymbol{y}^H\boldsymbol{y}}{N\sigma_0^2}-ln\frac{\boldsymbol{y}^H\boldsymbol{P}_{\Phi_{max}}^{\perp}\boldsymbol{y}}{(N-p_{max})\sigma_0^2}-1\right] =\max_k\left[\frac{A\boldsymbol{y}^H\boldsymbol{P}_{\Phi_k}\boldsymbol{y}}{N\sigma_0^2}+\frac{\boldsymbol{y}^H\boldsymbol{P}_{\Phi_k}^{\perp}\boldsymbol{y}}{N\sigma_0^2}-ln\frac{\boldsymbol{y}^H\boldsymbol{P}_{\Phi_{max}}^{\perp}\boldsymbol{y}}{(N-p_{max})\sigma_0^2}-1\right], \tag{7}$$

where A is the factor of the recognition sensitivity.

3 PERFORMANCE ANALYSIS

In this section, we derive the asymptotic distribution of the test statistic T_{un} with a view to evaluate it performance in terms of probability of detection, the probability of the recognition error and probability of false alarm. Moreover, we analyze numerically the difference of the performance between conventional statistics $T_{kn}(\boldsymbol{x})$ and the proposed statistics $T_{un}(\boldsymbol{x})$. It is well known that the distribution of the statistics $T_{kn}(\boldsymbol{x})$ is following:

$$T_{kn}(\boldsymbol{x})=\begin{cases}\dfrac{1}{2N}\chi_{2p_k}^2 & \text{under } H_0\\ \dfrac{\sigma_k^2}{2N\sigma_0^2}\chi_{2p_k}(2\lambda_k) & \text{under } H_k\end{cases}, \tag{8}$$

$$\lambda_k=\frac{\mu^2}{\sigma_k^2}\boldsymbol{\theta}_k^H\boldsymbol{H}_k^H\boldsymbol{R}^{-1}\boldsymbol{H}_k\boldsymbol{\theta}_k. \tag{9}$$

By analogy, we observe that the first term in (7) is equal to $T_{kn}(\boldsymbol{x})$, the second term has $\frac{1}{2N}\chi_{2(N-p_k)}^2$ central distribution with 2(N-p_k) real degrees of freedom under H_0 and $\frac{\sigma_k^2}{2\sigma_0^2 N}\chi_{2(N-p_k)}^2$ central distribution under H_k. In order to come up with manageable expressions, we investigate an asymptotic approach, assuming that the parameter N is large. In this case, it is well known that the chi-square distribution $\chi_n^2(0)$ converges to a Gaussian distribution with mean n and variance 2n (Scharf, 1991). Then, using the fact that the third term Q($\boldsymbol{y}$)= $ln\frac{\boldsymbol{y}^H\boldsymbol{P}_{\Phi_{max}}^{\perp}\boldsymbol{y}}{(N-p_{max})\sigma_0^2}$ has the chi-square distribution, one can write the following asymptotical expression:

$$Q(\boldsymbol{y})\sim\begin{cases} N\left(1,\frac{1}{N-p_{max}}\right) & \text{under } H_0 \\ N\left(b_k,\frac{b_k^2}{N-p_{max}}\right) & \text{under } H_k \end{cases}, \quad (10)$$

where $b_k=\frac{\sigma_k^2}{\sigma_0^2}$. Using a Taylor series expansion of $lnQ(\boldsymbol{y})$ around 1, it is easy to obtain that

$$Q(\boldsymbol{y})-1-ln[Q(\boldsymbol{y})] \approx (1/2)[Q(\boldsymbol{y}) -1]^2 . \quad (11)$$

We used the latter approximation and found that

$$Q(\boldsymbol{y})-1-ln[Q(\boldsymbol{y})]$$
$$\sim\begin{cases} \frac{1}{2(N-p_{max})}\chi_1^2(0) & \text{under } H_0 \\ \frac{b_k^2}{2(N-p_{max})}\chi_1^2\left(\frac{(N-p_{max})[b_k-1)]^2}{b_k^2}\right) & \text{under } H_k \end{cases}. \quad (12)$$

Since the first term and $Q(\boldsymbol{y})$ are independent, the asymptotic distribution of $T_{\text{un}}(\boldsymbol{x})$ is given by as follows:

$$T_{\text{un}}(\boldsymbol{x})$$
$$\sim\begin{cases} \frac{1}{2(N-p_{max})}\chi_1^2(0)+\left(\frac{1}{2N}\right)\chi_{2p}^2(0) & H_0 \\ \frac{b_k^2}{2(N-p_{max})}\chi_1^2\left(\frac{(N-p_{max})(b_k-1)^2}{b_k^2}\right)+\frac{b_k}{2N}\chi_{2p_k}^2(2\lambda_k). & H_k \end{cases}$$

The distributions derived above enable one to obtain the receivers operating characteristics (ROC). In order to come up with exploitable expressions, we examine a further approximation:

$$T_{\text{un}}(\boldsymbol{x})=\begin{cases} \frac{1}{2N}\chi_{2p_k+1}^2(0) & \text{under } H_0 \\ \frac{b_k}{2N}\chi_{2p_k+1}^2(\lambda_0+\lambda_k) & \text{under } H_k, \end{cases} \quad (13)$$

where $\lambda_0=\frac{(N-p_{max})(b_k-1)^2}{b_k^2}$. This expression holds for large N, p_k<<N and b_k=1-p_k/N. One can calculate the ROC using the following expression:

$$B(\eta, n, \lambda)=\int_\eta^\infty T_{\chi_n^2(n,\lambda)}(x)dx . \quad (14)$$

Also, one can obtain the threshold $\eta=B^{-1}(F,n,\lambda)$ using its inverse function. Then, the probability of false alarm F and probability of detection D can be written in such form:

$$F(T_{un})=B(2N\eta, 2p_k+1{,}0), \quad (15)$$

$$D(T_{un})=B(2\eta b_k^{-1}, 2p_k+1, \lambda_0+\lambda_k). \quad (16)$$

If we fix the false alarm rate F it is obvious that the increase of the factor of the detector sensitivity A augments the threshold η. In order to comprehend how and why the proposed statistics T_{un} may outperform the T_{kn}, let provide a qualitative analysis of the differences between these systems. The T_{kn} depends only on the data projection on the targets subspaces; the proposed statistics T_{un} depends on the projection on the background subspace. Notice that the only information used by T_{un} to modify T_{kn} is the power in the background subspace. One can then expect the different behavior of the T_{un}, each time the estimated background power is different from the expected one. The projection onto the targets subspaces will decrease and the projection onto the background subspace will increase. The developed system could recover a part of the energy having moved from one subspace to the other and try to maintain the test performance. Note that the $Q(\boldsymbol{y})=\frac{\hat{\sigma}_k^2}{\sigma_0^2}$ and then $\Delta(\boldsymbol{y})=\frac{1}{2}\left(\frac{\hat{\sigma}_k^2-\sigma_0^2}{\sigma_0^2}\right)^2$ is approximately zero for b=1, and is a monotonically increasing function when the parameter b decreases. This corrective term $\Delta(\boldsymbol{y})$ estimates the background variance for H_k and calculates the difference with the presumed one (σ_0^2). The mean of the statistics T_{kn} diminishes under the assumption that the parameter b decreases and then, when it is close to zero target amplitude μ, the detection probability can be much less than the presumed value of the false alarm probability. Therefore, performance of the T_{kn} suffers a remarkable degradation. The additional corrective term $\Delta(\boldsymbol{y})$ increases the value of statistics and, therefore, increases the probability of detection. When the targets have the same size and hence the same pixel fill factor b, the recognition performance of the T_{kn} and T_{un} is approximately equal, but in the case of the targets of the different size and therefore with the different pixel fill factor the corrective term $\Delta(\boldsymbol{y})$ is different and this term causes the decreases the recognition errors. At the numerical analysis stage, one should specify the background and target models properties. Let model the target mode matrix $\boldsymbol{H}$ is a Vandermonde matrix (Scharf, 1991). In the literature, it is often assumed (Scharf, 1991) that background has an exponential covariance matrix structure with one-lag correlation coefficient ρ. The parameter $\boldsymbol{\theta}$ is unknown in practice but for our scenario it is possible to use the appropriate deterministic approximation $\boldsymbol{\theta}=[1,1,\cdots,1]^T$. In order to limit the computational burden, the false alarm probability is chosen as F=10^{-3}. Figs.1, 2 illustrate the relation between the detection probability D and signal-to-background ratio ($\mu^2/\sigma_0^2\, dB$) under the chosen system constraint resulting from 10^6 Monte Carlo trials. Comparing figures 1 and 2 we notice, that the system in the case of the correlated background with a known covariance matrix in comparison with the uncorrelated one requires the smaller SBR for achieving of good detection. Recognition errors for different targets depend on difference between their subspace dimensions. In this example the recognition error between the first target with p=5 and the second with p=20 is equal to 6% and between the first and third targets

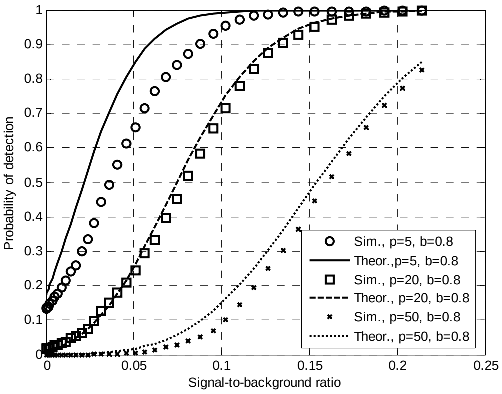

Figure 1: Probability of detection versus SBR_{in} for proposed system for target fill factor b =0.8. The lines depict the analytical results, whereas the markers show Monte Carlo simulation trial results. The false alarm rate F=10^{-3}, number of measurements N=200, 3 targets with subspace dimensions: p=5, 20, 50, uncorrelated background ρ= 0; A=1.

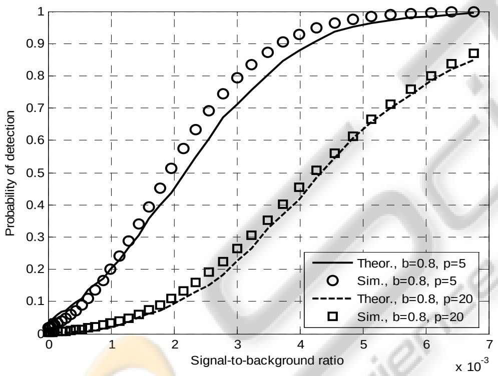

Figure 2: Probability of detection versus SBR_{in} for proposed system for target fill factor b =0.8. The lines depict the analytical results, whereas the markers show Monte Carlo simulation trial results. The false alarm rate F=10^{-3}, number of measurements N=200, 2 targets with subspace dimensions: p=5 and 20, correlated background ρ= 0.9; A=1.

about 2% in presence of uncorrelated background; these errors is equal to 8% and 3% in presence of correlated background with ρ= 0.9. The figure 3 shows the comparison in the detectability by the two systems. One can see that at the pixel fill factor b<1 the known system has losses in SBR with respect to the proposed system. Quality of recognition by the proposed system slightly is better, than by the known system.

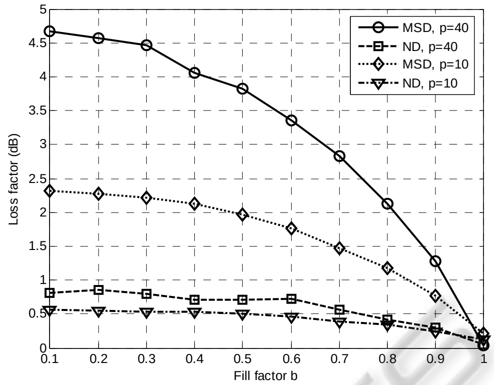

Figure 3: Loss factor of detection versus fill factor of target b for proposed system (ND) and well known (MSD). 2 targets with dimensions: p =10 and 40. Simulation results for F=10^{-3}, number of measurements N=200, uncorrelated background; A=1.

4 CONCLUSIONS

In this work, we intend to extend the detection-recognition problem in the case of the subpixel targets and Gaussian environment. We derived the GLRT for the problem at hand and carried out a performance analysis of the proposed system. The synthesized system modifies the well known by adding the corrective term proportional to the square of the background power variation. This term compensates a priori background power uncertainty in the case of the target's presence. It has been shown analytically and via statistical simulation that the performance of the proposed system considerably outperforms the known system performance.

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