

# SHAPE RECOGNITION USING THE LEAST SQUARES APPROXIMATION

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**Keywords:** Shape, Curvature Points, Approximation, Least-Squares method, Matching.

**Abstract:** This paper represents a novel algorithm to represent and recognize two dimensional curve based on its convex hull and the Least-Squared modeling. It combines the advantages of the property of the convex hulls that are particularly suitable for affine matching as they are affine invariant and the geometric properties of a contour that make it more or less identifiable. The description scheme and the similarity measure developed take into consideration technique for shape similarity. According to this method, the contours are extracted and decomposed into portions of curves. Each portion curve is approximated by some explicit curve using the Least Squares approximation. The obtained cubic curves are normalized in order to make the method invariant to scale change. Finally the resulting curves are used to compare and to compute similarity between shapes in images database using the Hausdorff distance. The proposed algorithm has been tested and its performance is found favourable as compared to other matching techniques.

## 1 INTRODUCTION

Object representation and recognition is a very difficult problem with many applications including computer vision. Computer vision researchers aim to capture image information in feature vectors which describe shape, texture and color properties databases of the image. These vectors are indexed or compared to one another during query processing to find similar images from the database. Considerable amount of information exists in two dimensional boundaries of objects since humans can readily recognize an object using the shape of its boundary. As a result, shape similarity retrieval plays an important role in content based image database systems.

Many techniques have been developed in the literature to represent the shape of a free form 3D object based on 2D silhouettes and most of them can be classified into two categories: surface-based methods and contour-based methods. Surface-based methods extract features from the whole shape region and are usually easy to compute and resistant to noise and shape distortions. Different moments, such as Zernike moments (Hwang et al., 2006) (Chong et al., 2003) and Legendre moments (Yang et al., 2006) have been demonstrated to achieve excellent performance. These methods are not

suitable for object recognition in the presence of occlusion. Unlike the contour-based methods explore boundary shape information and are more complicate requiring sophisticated implementations. They are low, but more suitable than surface-based methods for recognizing partially visible objects. In this category we find: invariant features extracted from boundaries of the object silhouette (Matusiak et al., 1998), 2D boundary curves of silhouette using Curvature Scale Space (CSS) (Mokhtarian et al., 1992) and (Dudek et al., 1997). The polygonal approximation has been used as a representation for recognizing objects (Carmona-poyato et al., 2010). Shape context (Belongie et al., 2002) is a method for describing shapes and finding the correspondence between point sets. Another shape descriptor is the Medial Axis Transform, which was presented by Blum (Blum, 1967) and later Sebastian and al (Sebastian et al., 2004) used this descriptor for shape recognition. Other techniques consist of approximate the shape contour by Fourier descriptors (Zahn et al., 1972) and B-spline (Paglieroni, 1985).

The notion of a part-based representation has played an important role in object recognition. In (Argawal et al, 2004), informative patches in the images are derived from the training examples and are used as fragments. Daliri and Torre (Daliri et al., 2010) proposed a representation for shape-based

recognition based on the extraction of the perceptually relevant fragments. Therefore, our aim is to develop a recognition system which requires two components: part-based representation and matching method. The part-based silhouette representation we use is built only on curves. Our shape matching algorithm is done by introducing the convex hull of the shape.

## 2 GEOMETRIC DESCRIPTION

Shape representation is one of the most challenging aspects of computer vision because shapes are often more complex than color and texture. The problem remains difficult in similarity retrieval in image databases. In This section, we present our approach for representing shape by using Least Squares approximation. The shape contour is first segmented into several curve segments. Each curve segment is then approximated by a cubic explicit curve using the Least- Squares method.

### 2.1 Extracting the Local Boundary Features (Parts)

In this section we describe how to extract the different curve segments. The decomposition process can be started by taking into account some features of boundaries which exert a crucial role in attracting the attention of an observer. Examples of such features, closely related to those considered in an early version of this paper are the high curvature points which give important clues for shape representation and analysis.

The basic step in our proposed algorithm is to extract the curvature points using the Chetverikov algorithm (Chetverikov, 2003) and locate the main points that may preserve the object shape as concave and convex points. Using the selected concave points, the shape boundary is then decomposed into a set of portions of curves as illustrated in Fig. 1.



Figure 1: Shape with different parts.

### 2.2 Curves Modelling

Our approach consists first in the use of the minimum rectangle MR that encloses the outline

shape (Graham, 1972). OXY is the referential attached to MR chosen such as the origin O is the left top edge of MR and the OX (resp. OY) axis corresponds to the width (resp. length) of the outline shape (see Fig. 2).



Figure 2: The minimum rectangle MR including the shape.

Given a shape contour  $\Omega$ . We assume that the original contour  $\Omega$  is close,  $\Omega$  is traversed in a counter-clockwise sense (the object is to the left).

$\Omega$  Consists of a finite number of an ordered list of parts that define the shape of the object silhouette (see Fig. 1). A curve modelling should be applied in order to facilitate matching and recognizing object shapes. In this paper the Least Squares model is employed to approximate each cut (part)  $C_i$  by an explicit cubic curve. The least squares curve of order 3 is defined by

$$C_i(x) = \sum_{j=0}^3 a_j x^j = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad (1)$$

Where the polynomial factors  $a_i)_{i=0,1,2,3}$  are computed such that the sum of the quadratic errors between the discrete data and their corresponding least squares curve is minimized.

#### 2.2.1 Normalization

In this subsection, we introduce the convex hull of the shape to generate an invariant representation. Convex hulls have some properties that make them suitable for recognition and representation tasks (Preparata et al., 1985).

The boundary  $\Omega$  of any object consists of a finite number of an ordered sequence of points that define the shape of an object:  $\Omega = \{p_1, p_2, \dots, p_n\}$  consisting of n two-dimensional points. Let  $C_H$  denotes the convex hull for the set  $\Omega$ . Let  $(x_i, y_i), i = 1, 2, \dots, m$  be the ordered vertices forming the convex hull. Using the Green's theorem (Gope et al., 2007) the centroid of  $C_H$  denoted by  $C_g = (c_x, c_y)$  can be expressed as:

$$\begin{cases} c_x = \frac{1}{6A} \sum_{i=1}^{m-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \\ c_y = \frac{1}{6A} \sum_{i=1}^{m-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \end{cases} \quad (2)$$

Where  $A$  is the area of  $C_H$  given by

$$A = \frac{1}{2} \sum_{i=1}^{m-1} (x_i y_{i+1} - x_{i+1} y_i) \quad (3)$$

In order to make the representation invariant to scale change we carry out a transformation on the approximation points. This can be accomplished by converting each approximation point  $(x'_i, y'_i)$  of each curve to another point by the transformation:

$$\begin{cases} x'_i \rightarrow \frac{x'_i}{d_{Max}} \\ y'_i \rightarrow \frac{y'_i}{d_{Max}} \end{cases} \quad (4)$$

Where  $d_{Max} = \max\{d(C_g, p_i)\}$  represents the maximal distance from the centroid of the convex hull of the shape to the boundary curve. This transformation allows us to bring back the different cubic curves approximating the original boundary curve at the different sizes on the same neighborhood

### 3 SHAPE MATCHING

In this section, we describe the basic concepts of our matching algorithm which compares images of the database with a query image. Consider that the features here are related to the convex hull and the normalized curves.

#### 3.1 Boundary Signature Matching

The boundary signature  $\gamma$  extracted from an object's boundary that characterizes the shape of an object is defined as the ratio of the minimal Euclidean distance between the centroid of the convex hull and the boundary shape to the maximal Euclidean distance  $d_{Max}$ . Matching between query shape to models is accomplished by comparing their boundary signatures.

### 3.2 Computing Shape Similarity

A necessary condition to match two shapes (query and model shapes) is the similarity between of their all normalized curves.

#### 3.2.1 Matching using the Normalized Curves

In this section, we explain how to match two curves. Hausdorff distance is used for matching two different curves. Given two normalized curves  $C$  and  $C'$  of a query shape  $Q$  and a reference shape  $M$  respectively, the Hausdorff distance is defined as:

$$H(C, C') = \max(h(C, C'), h(C', C)) \quad (5)$$

Where

$$h(C, C') = \max_{c \in C} \min_{c' \in C'} \|c - c'\| \quad (6)$$

and  $\|\cdot\|$  is a norm defined on the curve, such as the  $L_2$  norm.

A valid match between two normalized curves is found if the maximal difference between them (the similarity measure defined above) is under a threshold defined experimentally  $E_C$ ; otherwise they are different.

## 4 EXPERIMENTAL RESULTS

Our method is tested on ETH-80 database of 80 objects built by Leibe and B.Schiele (Leibe et al., 2003). Each object is represented by some views spaced evenly over the upper viewing hemisphere. The method possesses the important property of rotation and scale change. Invariance to rotation is achieved by computing using the referential defined by the minimum rectangle that encloses the shape. Using the maximal distance from the centroid of the convex hull, the representation is invariant to scale change. Some of the matching results are shown in Figure 3. The queries shapes are in the first row (at the left of each row). The similar shapes that have been matched by the proposed algorithm are shown in the rest rows. In the examples shown in Figure 3 there is a difference in the view angle between the query shape and the similar shapes. This examples show the robustness of our approach to orientation changes of the shapes.

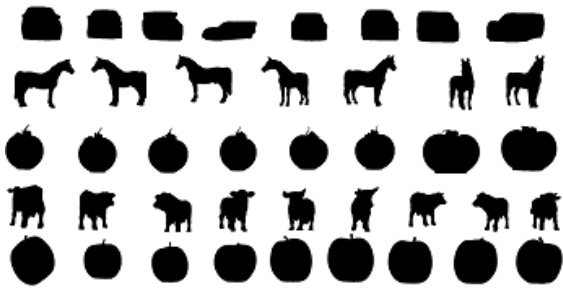


Figure 3: More matching of results.

We have summarized recognition rates for some different approaches which are tested on the ETH-80 database and cited in (Daliri et al., 2009).

Table 1: Some recognition rates for different algorithms tested on ETH-80 database.

| Algorithm                      | Recognition rate (%) |
|--------------------------------|----------------------|
| SC greedy                      | 86.40                |
| Decision tree                  | 93.02                |
| Fragment-based approach        | 86.40                |
| Kernel-edit-distance           | 91.33                |
| Robust symbolic representation | 89.03                |
| Proposed algorithm             | 92.50                |

## 5 CONCLUSIONS

In this paper, we have presented a new approach to represent the shape of the projection of a 3D object which enables similarity search. A key characteristic of our approach is the use of the geometric description of different parts constituting the outer closed boundary of the shape using a set of cubic curves. These curves enable us comparisons between different shapes. A shape matching technique, using the Hausdorff distance between two curves has been proposed. In our experiments, we have demonstrated invariance to similarity transformations: rotation and scaling. The results are encouraging. The proposed approach achieves a recognition rate equal to 92.5%.

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