AUTOMATIC ESTIMATION OF MULTIPLE MOTION FIELDS USING OBJECT TRAJECTORIES AND OPTICAL FLOW

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Abstract: Multiple motion fields are an efficient way of summarising the movement of objects in a scene and allow an automatic classification of objects activities in the scene. However, their estimation relies on some kind of supervised learning e.g., using manually edited trajectories. This paper proposes an automatic method for the estimation of multiple motion fields. The proposed algorithm detects multiple moving objects and their velocities in a video sequence using optical flow. This leads to a sequence of centroids and corresponding velocity vectors. A matching algorithm is then applied to group the centroids into trajectories, each of them describing the movement of an object in the scene. The paper shows that motion fields can be reliably estimated from the detected trajectories leading to a fully automatic procedure for the estimation of multiple motion fields.

1 INTRODUCTION

Video surveillance systems aim to detect and track moving objects (e.g., pedestrians), and to characterize their behaviors in the scene. Surveillance systems should be able to learn typical behaviors from video data in an unsupervised fashion, without using specific knowledge about the actions performed by humans in the monitored environment. Such systems typically involve the following steps: (a) detection of moving objects; (b) extraction of informative features (e.g. position, motion, shape); (c) tracking of the extracted features; and (d) classification of the observed behavior based on the extracted features (Turaga et al., 2008).

In outdoor applications, the object trajectories play an important role since they allow the system to characterize typical behaviors and discriminate abnormal ones. One way of modeling trajectories is the recently proposed approach using multiple motion fields, each representing a specific type of motion(Nascimento et al., 2009). The estimation of the motion fields is done automatically once we have a set of trajectories containing all typical activities in the scene. This model was applied with success to several problems. However, the training trajectories were hand edited to compensate for object detection and tracking errors.

This paper aims to overcome this difficulty. It proposes an automatic object tracking algorithm and its application to the estimation of multiple motion fields. The goal is to automate the estimation of multiple motion fields from the video stream.

The paper is organized as follows. Section 2, describes related work in this area. Section 3, presents the proposed automatic trajectory extraction algorithm. Section 4, describes the comparison between the detected trajectories and manually extracted ones, used as ground truth. Experimental results with real data are presented in Section 5.

2 RELATED WORK

2.1 Segmentation

Segmentation is a key step in most video surveillance systems. It aims to detect objects of interest in the video stream, using their visual and motion properties. It plays a key role since it reduces the amount of information to be processed by higher processing levels and locates the position of the targets. There is a large spectrum of segmentation methods that can be broadly classified into the following classes: (i) *statistical approaches*, (ii) *non-statistical approaches*, and (iii) *spatio-temporal approaches*.

The methods of the first class adopt statistical models for the background image, *e.g.*, each pixel of the background is modeled as a Gaussian distribution

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(Wren et al., 1997) or a mixture of Gaussians (Stauffer et al., 2000; McKenna and Gong, 1999). Also, minimization of Gaussian differences has been used (Ohta, 2001). Dynamic belief network (Koller et al., 1994) is another type of statistical method. This class of works also comprises a combination of frame differences and statistical background models (Collins et al., 1999).

Non-statistical based approaches adopt deterministic descriptions of the background image. The simplest approach is called background subtraction method, in which, a pixel-wise difference between the current frame and the background model is performed. Foreground/backgorund pixels are determined if the resulting difference is above/under a pre-determined threshold, respectively (Gonzalez and Woods, 2002). Other methods consider an admissible range for each pixel intensity, maximum rate of change in consecutive images or the median of largest inter-frames absolute difference (Haritaoglu et al., 2000).

The third class is based on spatio-temporal segmentation of the video signal. These methods try to detect moving regions taking into account not only the temporal evolution of the pixel intensities and color but also their spatial properties. Segmentation is performed in a 2D+T region of image-time space, considering the temporal evolution of neighboring pixels. This can be done in several ways, e.g. by using spatio-temporal entropy (Ma and Zhang, 2001), or the 3D structure tensor defined from the pixels spatial and temporal derivatives (Souvenir et al., 2005).

2.2 Optical Flow

Optical flow is a measure of the apparent motion of all image pixels from one frame to the next. With some exceptions, it is an estimate of the motion field (Barron et al., 1994), (Simoncelli, 1993). Denoting the image intensity at position (x,y) and time t by $\mathbf{I}(x,y,t)$, the optical flow equation is

$$(\nabla_x \mathbf{I})(x,y,t)u + (\nabla_y \mathbf{I})(x,y,t)v + (\nabla_t \mathbf{I})(x,y,t) = 0, \tag{1}$$

where x,y indicates a pixel location, and $\nabla_x \mathbf{I}$, $\nabla_y \mathbf{I}$, and $\nabla_t \mathbf{I}$ respectively denote the image gradient along the x,y and t directions. This equation is not enough to obtain the velocity vector (u,v) associated to each image point since it has an infinite number of solutions. This is known as the *aperture problem*. Additional constraints like smoothness have to be used.

There are several methods for computing the optical flow. Gradient based methods that impose smoothness constraints on the field of velocity vectors, the Horn and Schunck method (Horn and Schunck, 1981) that uses a global smoothness constraint or the Lucas-Kanade method (Lucas and Kanade, 1981) that assumes the velocity is locally constant combining local constraints over local regions. These methods are considered more accurate than the ones based on template matching, but they can only be used if the displacements is small (Barron et al., 1994).

There are several ways to deal with large displacements. One approach is region-matching based methods which do not actually solve (1), but try to find the most likely position for an image region in the next frame. Yet, another method is the Bayesian multiscale coarse to fine algorithm (Simoncelli, 1993). A coarse to fine warping scheme involving two nested fixed point iterations for energy minimization functional was proposed in (Brox et al., 2004) and a variant of this method (Sand and Teller, 2008), wherein video motion is represented by a set of particles and particle trajectories yielding the displacements as well as trajectories representing their motion.

2.3 Multiple Motion Field Model

Multiple motion fields were recently proposed as a tool to summarize objects motion in a scene and to statistically characterize their trajectories (Nascimento et al., 2009). Each trajectory is assumed to be a sequence of segments, each of them being generated by one of the motion fields, the so-called active field. Model switching is performed based on a probabilistic mechanism whose parameters depend on the position of the object in the image. This model is flexible enough to represent a wide variety of trajectories and allows the representation of space-varying behaviors as required.

Each trajectory is a length-n sequence of positions $\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_n)$ with $\mathbf{x}_t \in \mathbb{R}^2$. Each position represents the centroid of the object in the image. We assume each trajectory is the output of a switched dynamical system

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{T}_{k_t}(\mathbf{x}_{t-1}) + \mathbf{w}_t, \tag{2}$$

where $k_t \in \{1,\dots,M\}$ is the label of the active field at time t; $\{\mathbf{T}_1,\dots,\mathbf{T}_M\}$ are M vector fields which describe typical motion patterns and $(\mathbf{w}_1,\dots,\mathbf{w}_n)$ are independent samples of a zero-mean Gaussian random vector with identity covariance.

The label sequence $\mathbf{k} = (k_1, ..., k_n)$ is assumed to be a Markov chain of order one with space-dependent transition probabilities

$$Pr\{k_t = j | k_{t-1} = i, \mathbf{x}_{t-1}\} = b_{ij}(\mathbf{x}_{t-1}) \tag{3}$$

Matrix $\mathbf{B}(\mathbf{x}_{t-1}) = (b_{ij}(\mathbf{x}_{t-1}))$ is a space-varying stochastic matrix. The model parameters (motion fields, noise variances, switching matrix field) can be retrieved from observed object trajectories using the Expectation-Maximization (EM) algorithm.

3 PROPOSED APPROACH

In this section, we describe the segmentation and feature extraction processes. We first perform background subtraction to detect the active regions in a frame, and then estimate the velocity fields at the centroid of each active region by computing the optical flow using the template matching algorithm. The result of these steps is a sequence of vectors containing the spatial coordinates of the centroids of the active regions, over the entire set of frames, and also the corresponding velocity fields.

3.1 Active Region Detection

We represent a frame at time t with M rows and N columns by a matrix $\mathbf{I}_t \in \mathbb{R}^{M \times N}$. Given a sequence of K frames $\{\mathbf{I}_t, t = 1, ..., K\}$, we estimate the background image $\mathbf{B} \in \mathbb{R}^{M \times N}$ by taking a subset of frames and applying a median filter to the evolution of each pixel in time. We segment each frame $\mathbf{I}_t$, by subtracting the background and thresholding the difference image with a predefined positive value λ, to produce a binary image $\mathbf{J}_t \in \{0,1\}^{M \times N}$ with the value at pixel m,n,

$$\mathbf{J}_t(m,n) = \begin{cases} 1, & \text{if } |\mathbf{I}_t(m,n) - \mathbf{B}(m,n)| > \lambda, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

For multiple moving objects, this requires finding the connected pixels above this threshold. We do not assume beforehand, the number of moving objects. We therefore apply a clustering algorithm on this binary image $\mathbf{J}_t$ to find connected regions (assuming 8 neighbors), each cluster corresponding to a moving object.

In practice, because of the empirical nature of the threshold value λ, there may be false active regions (*i.e.* the threshold being exceeded where there is no motion) or the active region corresponding to a single moving object may get split into two or more clusters. We solve the second problem by dilating the binary image $\mathbf{J}_i$ before clustering, and solve the problem of false active regions by discarding the clusters with small area. The number of active regions detected at the frame t,will be denoted by n_t.

3.1.1 Optical Flow using Template matching

Region based matching approaches for computing the optical flow define the velocity vector of an object moving across successive frames as the vector of displacements (i, j) that produces the best fit between image regions at different times. The best fit means that a distance measure between a region in a frame, $\mathbf{I}_t$, and its possible location in the next frame, $\mathbf{I}_{t+1}$, is minimum for the vector (i, j), or a similarity measure such as cross-correlation is maximum. These methods are intuitively simple and relatively easy to implement, and the computational load is low since it has to be computed only for the active regions.

Let the bounding box of the $k-th$ active region be $I_k \in \mathbb{Z}^2$. Then assuming suitable boundary conditions, the velocity vector is computed by solving the optimization problem,

$$(u_k, v_k) = \arg \min_{(i,j)\in(-d_m,d_m)} E(i,j), \tag{5}$$

$$E(i,j) = \sum_{(m,n)\in I_k} (I_t(m,n) - I_{t+1}(m+i,n+j))^2 + \alpha(|i|+|j|). \tag{6}$$

where d_m is the maximum displacement, and $\alpha > 0$ is the regularization parameter that controls the relative weight of the penalty term. Since the entire block is assumed to be displaced by the vector (u_k, v_k), we consider this the optical flow at the centroid of the k^{th} region.

Figure 1 illustrates the optical flow between two frames at times t and $t+1$, for a man walking. The velocity vector is shown superimposed over the frame at time t and its binary image, at the centroid of the active region corresponding to the man.

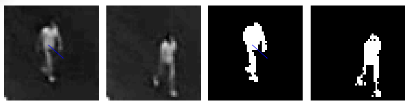


Figure 1: Optical flow between 2 frames. (a),(b) grayscale frames at times t and $t+1$, (c), (d) active regions corresponding to the moving person in the two frames.

3.2 Motion Correspondence

At the end of the optical flow step, we have a set of n_t centroid locations $\{(x_i, y_i), i = 1, ..., n_t\}$ and their respective motion vectors $\{(u_i, v_i), i = 1, ..., n_k\}$. We therefore need to apply a region association algorithm to integrate the centroids in successive frames into trajectories. This differs from the approach presented in (Nascimento et al., 2010), where the pre-processing consists of two steps: active region detection using the Lehigh Omnidirectional Tracking System (LOTS) algorithm followed by region association. The approach herein proposed is threefold regarding the preprocessing in [Nascimento et al., 2010]: (1) it computes directly the vector fields, (2) it reduces drastically the errors that may arise in the region association mechanism, and (3) consequently avoids manual corrections to obtain the trajectories. In this paper, we directly apply a motion correspondence algorithm, namely, the GOA tracker (Veenman et al., 2001) on the sequence of centroid positions.

Let the vector $\mathbf{x}_k^t = [x_k, y_k]^T$ denote the position vector of the centroid of active region k in frame t. For the centroids of two regions i and j in two successive frames t and $t+1$, the association cost is

$$C_t(i,j) = \|\mathbf{x}_j^{t+1} - (\mathbf{x}_i^t + \mathbf{v}_i^t)\|_2, \tag{7}$$

where $\mathbf{v}_k^t = [u_k, v_k]^T$ is the velocity vector at the centroid of active region k in frame t.

Let $\tilde{\mathbf{C}}_t$ be the size $n_{t+1} \times n_t$ matrix whose entries are computed using the above cost function. In general, the number of centroids in successive frames is not the same, that is, $n_{t+1} \neq n_t$. A centroid in frame t may have a corresponding point in frame $t+1$ belonging to its trajectory or it may be the last point belonging to that particular trajectory. Likewise, a centroid in frame $t+1$ may belong to a trajectory having an associated point in frame t, or it could be the starting point of a new trajectory. To be able to account for the "birth" or "death" of trajectories, we pad the matrix $\tilde{\mathbf{C}}_t$ with entries equal to γ to form a square cost matrix $\mathbf{C}_t$ of size $(n_t + n_{t+1}) \times (n_t + n_{t+1})$,

$$\mathbf{C}_t = \begin{bmatrix} \tilde{\mathbf{C}}_t, \gamma\mathbf{1}_{(n_{t+1} \times n_{t+1})} \\ \gamma\mathbf{1}_{(n_t \times n_t)}, \gamma\mathbf{1}_{(n_t \times n_{t+1})} \end{bmatrix}, \tag{8}$$

where $\mathbf{1}$ stands for a matrix whose entries are all 1, $\gamma > 0$ is the cost of starting a new trajectory or ending an existing one. A column of this matrix corresponds to a centroid in frame t and a row corresponds to a centroid in frame $t+1$. The Hungarian matching algorithm (Kuhn, 1955) is then used to solve the matching problem by minimizing the assignment cost

$$\mathcal{C} = \sum_{i,j=1}^{n_t+n_{t+1}} b_{ij} C_{ij}, \tag{9}$$

where $b_{ij} \in \{0,1\}$ and have the constraints

$$\sum_{i=1}^{n_t+n_{t+1}} b_{ij} = 1, \ \forall\, j \ \ ; \sum_{j=1}^{n_t+n_{t+1}} b_{ij} = 1, \ \forall\, i\,. \tag{10}$$

4 COMPARISON WITH GROUND TRUTH

Once we have computed a trajectory of an object in a given video sequence as described in the previous section, we would like to compare it with the trajectory for the same object from the set of previously computed trajectories which we consider the ground truth. We would like to do this automatically, without the user having to manually locate the sequence of frames corresponding to the trajectory. This has the possible problems that the trajectory and its counterpart from the ground truth may not exactly coincide in terms of starting and ending frames, may have some missed detections in some frames, or a single trajectory may have multiple trajectories in the ground truth corresponding to different segments (e.g. an object moving in loops).

We denote an automatically detected trajectory by $\{\mathbf{x}\}_{t\in T}$, defined over a subset of frames $T = \{t_0, \ldots, t_M\}$. Similarly, a trajectory i from the ground truth set is denoted as $\{\mathbf{x}^{gt,i}\}_{t\in T_i}$, with T_i being the set of relevant frames. If there are some frames in common between T and T_i, we define the set of overlapping frames, $T_{OL,i} = T \cap T_i$, and compute the mean square error between the two trajectories at the times corresponding to the overlapping frames

$$E(i) = \frac{1}{\#T_{OL,i}} \sum_{t\in T_{OL,i}} \|\mathbf{x}_t - \mathbf{x}_t^{gt,i}\|_2. \tag{11}$$

We then find all trajectories in the ground truth for which the MSE is within a limit of κ times the minimum value, $E_{\min}$, $\{i : E(i) \leq \kappa E_{\min}\}$. From this set, the trajectories which have no overlapping frames are the ones corresponding to different continuous segments of the trajectory $\{\mathbf{x}\}_t$.

5 EXPERIMENTAL RESULTS

We illustrate our proposed approach on a set of video sequences of a university campus (Instituto Superior Técnico, Lisbon) recorded with a single static camera. The total number of frames was 69596, acquired at a rate of 25 frames per second. For our computation of optical flow, the sequence was subsampled at a rate of 5 frames. The frame size is 432×540. There were 7 pedestrian activities of interest in this sequence, which are listed in Table 1.

Figure 2 shows two trajectories corresponding to people walking, detected over a segment of 2000 frames, and their respective velocity vectors superimposed. The velocities were scaled by a factor of 10 and subsampled by a factor of 8 for display.

Figure 3 shows the two trajectories from Figure 2 and the corresponding ground truth trajectories, retrieved by the library search. The trajectory in Figure 3(a) corresponds to a single trajectory from the ground truth library. The other trajectory of a person leaving the building and entering it again, which appears as a loop in Figure 3(b) has two non-overlapping matches in the ground truth (corresponding to activities "entering" and "leaving"), which are 70 frames apart. Therefore, for the estimation of the motion fields using the EM algorithm, we split this trajectory into two segments corresponding to the two ground truth trajectories for these activities.

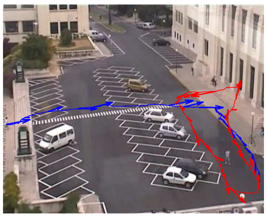
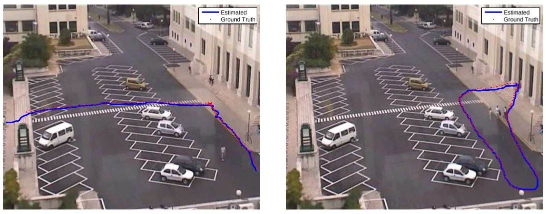


Figure 2: Motion fields computed using optical flow superimposed over the corresponding trajectories.



(a) (b)

Figure 3: Extracted trajectories using the proposed approach and their corresponding ground truth trajectories.

We validate our method by computing the statistics of the root mean square error (RMSE) between the trajectories obtained using our method and the ground truth trajectories from (Nascimento et al., 2010). We present the mean and variance of the error for each class of activities and overall values, in Table 1.

Table 1: Statistics of the Error in the Trajectories.

Trajectory class	Error Mean (pixels)	Error Standard Deviation (pixels)
entering	4.03	0.99
leaving	3.80	1.07
walking along	2.46	0.35
crossing park up	4.12	1.29
crossing park down	5.33	1.80
passing through cars	4.49	0.87
browsing	3.23	0.82
Overall	4.31	1.59

We also validate the trajectories by estimating the motion fields from the model (2) with parameters estimated by the EM method (Nascimento et al., 2009). It is not possible to directly compare the vector fields obtained from both sets of trajectories since the field estimates depend on the initialization of the EM method. We therefore, evaluated each set of vector fields by measuring its ability to predict the target position at the next time instant. For this dynamic model, the predictor and prediction error are given by

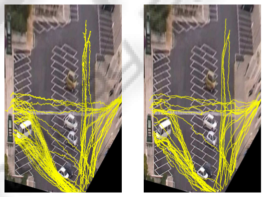
$$\hat{\mathbf{x}}_t = \mathbf{x}_{t-1} + \mathbf{T}_{k_t}(\mathbf{x}_{t-1}), \quad (12)$$

$$\hat{\mathbf{e}}_t = \mathbf{x}_t - \mathbf{x}_{t-1} - \mathbf{T}_{k_t}(\mathbf{x}_{t-1}). \quad (13)$$

In practice, we do not know the active field k_t, and therefore select the error with the smallest norm (ideal switching). We can now define an SNR measure

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sum_{t=2}^{L}\|\mathbf{x}_t - \mathbf{x}_{t-1}\|^2}{\sum_{t=2}^{L}\min_k\|\mathbf{x}_t - \mathbf{x}_{t-1} - \mathbf{T}_k(\mathbf{x}_t)\|^2}\right). \quad (14)$$

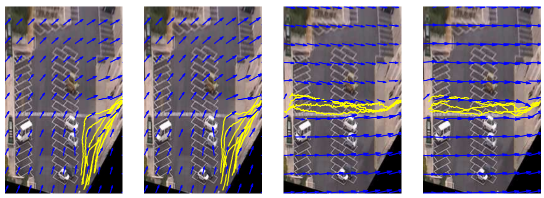
The same number of trajectories per activity was used to train the model for both the ground truth and our extracted trajectories, with a total of 69 trajectories. Figure 4 shows both sets of trajectories after applying a projective transformation (homography) between the image and a plane parallel to the ground. This is done to achieve viewpoint invariance, by projecting all image measurements onto a view orthogonal to the ground plane (the so-called birds eye view).



(a) (b)

Figure 4: Trajectories from (a) the ground truth and (b) obtained using the proposed approach, superimposed on the starting frame, after applying a homography.

Figure 5 shows the motion fields estimated using the EM algorithm, trained with trajectories corresponding to a single activity, with one model. Figures 5(a) and 5(b) show the motion fields for the activity "entering" using the trajectories from the ground truth set, and those obtained using the proposed approach, respectively. Similarly, Figures 5(a) and 5(b) present the corresponding motion fields for the activity "passing through cars".



(a) (b) (c) (d)

Figure 5: Motion fields for the activities "Entering" and "Passing through cars", estimated from (a),(c) the GT trajectories, and (b),(d) extracted trajectories.

Table 2 shows the best SNRs obtained for the

Table 2: SNR between the trajectory and predicted trajectory from the estimated motion fields.

No. of models	1	2	3	4	5	6	7	8
Ground truth	1.37	4.57	4.09	6.23	5.62	5.4	5.48	3.58
Proposed method	1.15	4.42	4.13	6.02	5.52	3.45	5.05	2.96

ground truth trajectories (corresponding to all activities) and those obtained using the proposed approach for different numbers of fields estimated by the EM algorithm. It can be seen that the maximum SNR obtained from the extracted trajectories is close to that obtained from the ground truth data.

6 CONCLUSIONS

We have proposed a method for automatically computing the trajectories and velocity fields of multiple moving objects in a video sequence, using optical flow. The trajectories obtained were found to be close to the manually edited ground truth trajectories, for a large set of activities occurring in the video sequences. The motion fields estimated from these trajectories using the EM method led to an SNR close to that obtained with the ground truth trajectories. Hence the proposed method allows fully automatic extraction of multiple motion fields. Current and future work includes extending the method to denser environments such as crowds of moving people.

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