A ROBUST TECHNIQUE FOR MULTIUSER DETECTION IN THE PRESENCE OF SIGNATURE UNCERTAINTIES

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Abstract: This paper presents a robust multiuser detection technique to combat multiple access interference (MAI) and impulsive noise for synchronous code-division multiple-access (CDMA) communication systems in the presence of signature uncertainties. A new *M*-estimator (modified Hampel) proposed to robustify the detector is studied and analyzed. The approach is corroborated with simulation results to evaluate the performance of the proposed robust multiuser detector in comparison with the linear decorrelating detector, Huber and Hampel estimator based detectors. Simulation results show that the new *M*-estimator based detector offers significant performance gain over the linear decorrelating detector, the Huber, and the Hampel estimator based detectors with little attendant increase in the computational complexity.

1 INTRODUCTION

Recent research has explored the potential benefits of multiuser detection for code division multiple access (CDMA) communication systems with present multiple access interference (MAI) (Verdu, 1998). These optimal multiuser detectors have led to the developments of the various linear multiuser detectors with Gaussian noise though various experimental measurements confirmed that many realistic channels are impulsive in nature. Lately, the problem of robust multiuser detection in non-Gaussian channels has been addressed in the literature (Wang and Poor, 1999), (Anil Kumar et al., 2004), and (Anil Kumar and Deergha Rao, 2006), which were developed based on the Huber, Hampel, and a new *M*-estimator (modified Hampel), respectively. Recently, robust multiuser techniques, that take into design consideration the effect of signature mismatch at the receiver, have attracted a great interest which includes the robust minimum output energy (MOE) linear detector (Luo et al., 2001), and a worst case performance optimization of the MOE multiuser detector (Vorobyov et al., 2003; Gershman and Shahbazpanahi, 2003). A new robust nonlinear multiuser detector which minimizes the worst-case (WC) probability of error across all possible channel parameters in the region of uncertainty is presented in (Salhov et al., 2004). A robust CDMA multiuser detection technique based on the probability-constrained optimization approach is developed in (Sergiy, 2008).

Hence, this paper considers robust multiuser detection in the presence of signature uncertainties in non-Gaussian channels. A new *M*-estimator proposed to robustify the multiuser detector is presented. Performance gains offered by the proposed approach are demonstrated through simulation results. Simulation results show that the new robust multiuser detector outperforms the linear decorrelating detector, the Huber, and the Hampel estimator based detectors.

The remaining portion of the paper is organized as follows. Section **2** discusses the synchronous CDMA system model with signature uncertainties under non-Gaussian impulsive noise. Section **3** presents an *M*-estimator based regression and influence functions of *M*-estimators. Section **4** discusses the simulation results and finally, conclusion is drawn in section **5**.

2 SYSTEM MODEL

An *L*-user synchronous CDMA system, where each

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user transmits information by modulating a signature sequence, is considered in this paper. The received signal over one symbol duration can be modelled as:

$$r(t) = \sum_{l=1}^{L} A_l b_l s_l + w(t), \quad t \in \tau \tag{1}$$

where b_l is the information symbol transmitted by the l^{th} user, $s_l(t)$ is the l^{th} user spreading-code waveform, $A_l \geq 0$ is the received amplitude of the l^{th} user's signal, L is the number of users, τ is the observation interval, and $w_l(i)$ is assumed as a sequence of independent and identically distributed (i.i.d.) random variables with a non-Gaussian distribution. The probability density function of this noise model has the form

$$f = (1-\varepsilon)\aleph(0,v^2) + \varepsilon\aleph(0,\kappa v^2) \tag{2}$$

with $v > 0$, $0 \leq \varepsilon \leq 1$, and $\kappa \geq 1$. Here $\aleph(0, v^2)$ represents the nominal background noise and the $\aleph(0, \kappa v^2)$ represents an impulsive component, with ε representing the probability that impulses occur. For concreteness, we assume that $b_l \in [-1, 1]$. At the receiver, the resulting discrete-time signal in matrix form is given by

$$\mathbf{r} = \mathbf{SAb} + \mathbf{w}, \tag{3}$$

where $\mathbf{S}$ is the $N \times L$ matrix of columns $\mathbf{s}_l$, where the vector $\mathbf{s}_l$ contains the corresponding samples, $\mathbf{A}$ is the diagonal matrix with diagonal elements $A_l > 0$, $\mathbf{b}$ is the data vector with components b_l, and the vector $\mathbf{w}$ contains the corresponding samples of the noise process.

The purpose of multiuser detection is to detect the symbols $\{b_l\}$ given the observed signal $\mathbf{r}$ assuming that the diagonal matrix $\mathbf{A}$ and the signature matrix $\mathbf{S}$ are known precisely at the receiver. In practice, the signature vectors $\mathbf{s}_l$ and the diagonal matrix $\mathbf{A}$ may not be known exactly because of channel distortion. Since the distorted $\mathbf{A}$ can be directly translated to an appropriate signature distortion, without loss of generality, signature mismatch in the presence of non-Gaussian impulsive noise is addressed in this paper. In the case of signature matrix uncertainty, the received signal can be written as (Salhov et al., 2004)

$$\begin{aligned} \mathbf{r} &= \left[\mathbf{S}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{S}_i\right]\mathbf{Ab} + \mathbf{w} \\ &= \left[\mathbf{H}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{H}_i\right]\mathbf{b} + \mathbf{w}, \end{aligned} \tag{4}$$

where $\mathbf{H}_i$=$\mathbf{S}_i\mathbf{A}$, and δ_i are perturbations that lie in some perturbation set D. Since δ_i are not known precisely, the error probability cannot be directly minimized. The ML detector that minimizes the worst-case error probability over all possible values of δ_i is considered in (Salhov et al., 2004). Thus, we seek the symbols that are solutions to the problem given by (Salhov et al., 2004)

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}\in\{1,-1\}^N} \max_{\delta\in D} \Delta\left(\mathbf{H}(\delta), \mathbf{b}\right), \tag{5}$$

where

$$\Delta\left(\mathbf{H}(\delta), \mathbf{b}\right) = \left\|\mathbf{r} - \mathbf{H}(\delta)\mathbf{b}\right\|^2. \tag{6}$$

3 *M*-ESTIMATION BASED REGRESSION

In *M*-estimates, unknown parameters $\theta_1, \theta_2, \ldots \theta_L$ (where $\theta = Ab$) are solved by minimizing a sum of function $\rho(\cdot)$ of the residuals

$$\hat{\theta} = \arg\min_{\theta\in\Re^K} \sum_{j=1}^{N} \rho\left(r_j - \sum_{l=1}^{L} s_j^l \theta_l\right), \tag{7}$$

where ρ is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square. Suppose that ρ has a derivative with respect to the unknown parameters $\theta(\psi = \rho')$, called the influence function, since it describes the influence of measurement errors on solutions. The solution to (7) satisfies the implicit equation (8), and its vector form is given in (9)

$$\sum_{j=1}^{N} \psi\left(r_j - \sum_{k=1}^{L} s_j^k \theta_k\right) s_j^l = 0, \qquad l = 1 \ldots L \tag{8}$$

$$\mathbf{S}^T \psi\left(\mathbf{r} - \mathbf{S\theta}\right) = \mathbf{0}_L, \tag{9}$$

where $\mathbf{S}^T$ is the transpose of $\mathbf{S}$ and $\mathbf{0}_L$ is an all zero vector of length L. Equation (8) is called an *M*-estimator. Different influence functions yield solutions with different robustness properties. Therefore, an influence function $\psi(\cdot)$ should be chosen such that it yields a solution that is not sensitive to outlying measurements.

3.1 Influence Functions

M-estimators are generalizations of the usual maximum likelihood estimates. In this subsection, the influence functions of *M*-estimators proposed in the literature (Wang and Poor, 1999; Anil Kumar et al., 2004; Anil Kumar and Deergha Rao, 2006) are listed (see Figure 1). Huber's, Hampel's and the proposed *M*-estimators are presented.

3.1.1 Huber's Estimator

The Huber's *M*-estimator is determined by the Huber penalty function $\rho_{\mathrm{HU}}(\cdot)$ and its derivative given by

$$\psi_{\mathrm{HU}}(x)=\begin{cases}\dfrac{x}{\nu^2} & for \quad |x|\le k\nu^2\\ k\,\mathrm{sgn}(x) & for \quad |x|>k\nu^2\end{cases}. \tag{10}$$

where *k* is any constant.

3.1.2 Hampel's Estimator

Similarly, the Hampel's *M*-estimator is determined by the Hampel's penalty function $\rho_{\mathrm{HU}}(\cdot)$ and its derivative given by

$$\psi_{\mathrm{HA}}(x)=\begin{cases} x & for \quad |x|\le a\\ a\,\mathrm{sgn}(x) & for \quad a<|x|\le b\\ a\,\mathrm{sgn}(x)\dfrac{c-|x|}{c-b} & for \quad b<|x|\le c\\ 0 & for \quad |x|>c\end{cases}, \tag{11}$$

where *c* is a constant.

3.1.3 Proposed Estimator

The proposed *M*-estimator (modified Hampel) is determined by the penalty function and its derivative, which is given, respectively, by (Anil Kumar and Deergha Rao, 2006)

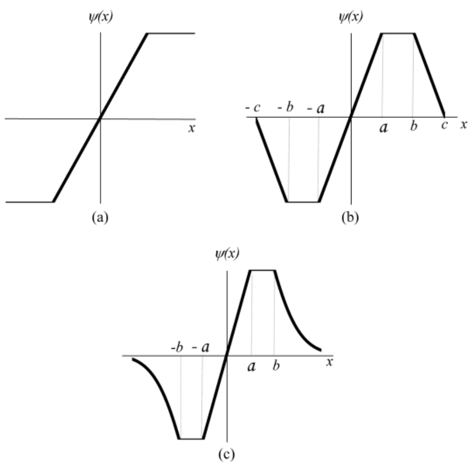

Figure 1: Influence function of (a) Huber estimator, (b) Hampel estimator, and (c) the proposed estimator.

$$\rho_{\mathrm{PRO}}(x)=\begin{cases}\dfrac{x^2}{2} & for\ |x|\le a\\ \dfrac{a^2}{2}-a|x| & for\ a<|x|\le b,\\ \dfrac{ab}{2}\exp\left(1-\dfrac{|x|^2}{b^2}\right)+d & for\ |x|>b\end{cases} \tag{12}$$

where *d* is a constant, and

$$\psi_{\mathrm{PRO}}(x)=\begin{cases} x & for\ |x|\le a\\ a\,\mathrm{sgn}(x) & for\ a<|x|\le b\\ \dfrac{a}{b}x\exp\left(1-\dfrac{|x|^2}{b^2}\right) & for\ |x|>b\end{cases}. \tag{13}$$

The choice of the constants *a* (=$\kappa\nu^2$) and *b* (= $2\kappa\nu^2$) depends on the robustness measures derived from the influence function. A robust estimator should possess a finite value of *c* (see Figure 1). The proposed *M*-estimator is three-part with no sharp rejection point *c* as in Hampel's three-part re-descending estimator. The proposed influence function $\psi_{\mathrm{PRO}}(x)$ is bounded and has continuous derivatives. The proposed detector has bounded influence function, and hence is robust.

Asymptotic probability of error for the class of decorrelating detectors described by (9) for large processing gain *N*, is given by (Wang and Poor, 1999)

$$P_e^l \equiv \Pr\left(\hat{\theta}_l<0 \middle| \theta_l>0\right) = Q\left(\frac{W_l}{\upsilon\sqrt{\left[\mathbf{R}^{*-1}\right]_{ll}}}\right), \tag{14}$$

where $Q(\cdot)$ is the Gaussian *Q*-function defined by $Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}} exp\left(-\xi^2/2\right)d\xi \ , x\ge 0$, and

$$\upsilon^2=\frac{\int\psi^2(u)f(u)du}{\left[\int\psi'(u)f(u)du\right]^2}. \tag{15}$$

4 SIMULATION RESULTS

In Figure 2 and Figure 3, the performance of four decorrelating detectors is studied by plotting the probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of a synchronous system with six users (L = 6) and a processing gain of 31 (N = 31). The noise distribution parameters are ε = 0.01 & κ = 100 and ε = 0.1 & κ = 100 for Figure 2 and Figure 3 respectively. These simulation results demonstrate the performance gains achieved by the minimax decorrelating detector with the proposed influence function over the linear decorrelating detector and minimax decorrelating detector (both with Huber and Hampel estimators), in impulsive noise. Moreover, this performance gain increases as the SNR increases.

Further, an asynchronous system with six users (L =6), processing gain of 31(N = 31), and the noise distribution parameters ε = 0.01 & κ = 100 and ε = 0.1 & κ = 100 (for Figure 4 and Figure 5 respectively) is considered. The delays of the 6 users are randomly generated. The proposed robust multiuser detector, minimax decorrelating detector (both with the Huber and Hampel M-estimators) and the linear decorrelating detector are implemented.

It is seen from the simulation results that the proposed multiuser detector offers substantial gains over the minimax decorrelating detector (both with the Huber and Hampel estimators) and the linear decorrelating detector.

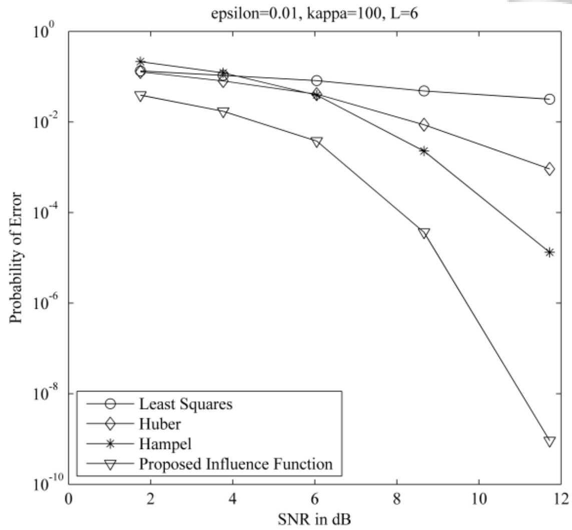

Figure 2: Probability of Error versus SNR for user 1 for linear multiuser detector, minimax detector with Huber, Hampel and proposed M-estimator in synchronous CDMA channel with impulse noise, N=31, ε = 0.01.

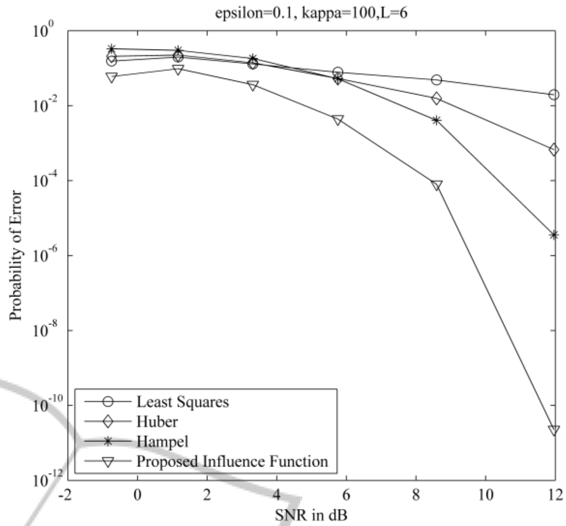

Figure 3: Probability of Error versus SNR for user 1 for linear multiuser detector, minimax detector with Huber, Hampel and proposed M-estimator in synchronous CDMA channel with impulse noise, N=31, ε = 0.1.

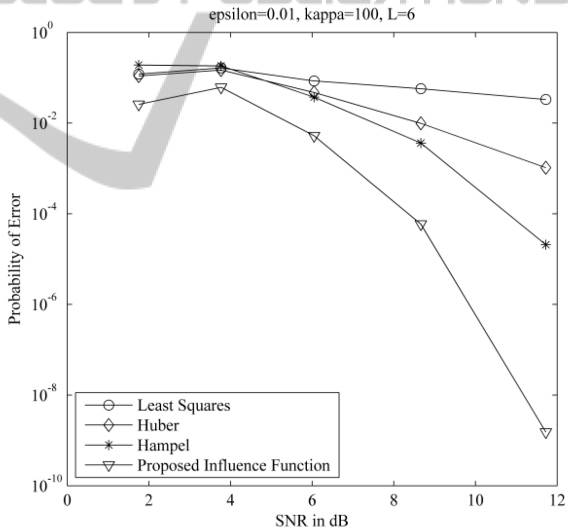

Figure 4: Probability of Error versus SNR for user 1 for linear multiuser detector, minimax detector with Huber, Hampel and proposed M-estimator in asynchronous CDMA channel with impulse noise, N=31, ε = 0.01.

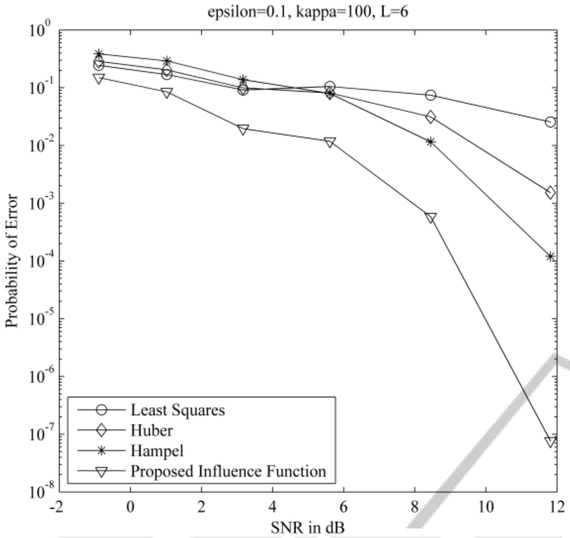

Figure 5: Probability of Error versus SNR for user 1 for linear multiuser detector, minimax detector with Huber, Hampel and proposed *M*-estimator in asynchronous CDMA channel with impulse noise, N=31, $\varepsilon = 0.1$.

5 CONCLUDING REMARKS

In this paper, a new *M*-estimator based robust multiuser detection technique in the presence of signature uncertainties is proposed, which significantly outperforms the linear decorrelating detector and minimax robust multiuser detector (with Huber and Hampel *M*-estimators) in non-Gaussian ambient noise. Simulation results show that the proposed robust multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detectors with Huber and Hampel *M*-estimator, in non-Gaussian noise with little attendant increase in the computational complexity.

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