MODEL OF AGGREGATION
A Topological Approach

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Keywords: Collective Behaviour, Aggregation, Topology, SPP Model, Flock.

Abstract: The aggregate motion of flocks of birds, a herd of land animals, Mexican wave forming in stadia are beautiful and nice examples of collective behaviour. The aggregation is constructed by the action of each individual, each action solely on basis of its local perception of the world. Scientists from different backgrounds have tried to model collective behaviour. Most of the models are strictly metric (based on Euclidian distance among individuals) but flocks of birds do not act on metric perception. In this paper we proposed a model based on topological perspective to construct a flock of birds with large number of individuals and checked flock's density independent behaviour.

1 INTRODUCTION

Collective behaviour could be stated as "*the way in which an individual unit's activity is dominated by its neighbours so that all units simultaneously alter their behaviour to a common pattern*" (Vicsek, 2001). By acting collectively, individuals (both organisms and non-living objects are considerable) synchronize their signals or motion. The main features of collective behaviour are that an individual unit's action is dominated by the influence of its neighbours – the unit behaves differently from the way it would behave on its own; and that such systems show interesting ordering phenomena as the units simultaneously change their behaviour to a common pattern.

The aggregate motion of flock of birds, a herd of land animals, a school of fish are beautiful and nice examples of collective behaviour. People clapping in phase during rhythmic applause, Mexican wave forming in stadia (Farkas, Helbing and Vicsek, 2002) also demonstrates collective behaviour. Even non-living objects like ferromagnets show collective behaviour. These materials can undergo spontaneous magnetization, in effect because they are made up of a host 'tiny magnets' (Vicsek, 2001).

Collective behaviour of animals exhibits many contrasts. In case of flock of birds, flocks are made of discrete birds yet the overall motion seems fluid; it is simple in concept yet is so visually complex, it seems randomly arrayed and yet is magnificently synchronized. The aggregation is constructed by the action of each individual, each action solely on the basis of its local perception of the world (Reynolds, 1987).

Scientists from different backgrounds have tried to understand and model different aggregations: school of fish (Inada and Kawachi, 2002), flock of birds (Reynold, 1987, Bhattacharya and Vicsek, 2010), pedestrian behavior (Moussaid, Helbing and Theraulaz, 2011). Reynolds (1987) first introduced a flock of birds model in computer graphics (Reynolds, 1987). He named the individual units 'boids' related to 'bird-like' or 'bird-oid'. To simulate a flock, he used three simple rules: (1) collision avoidance, (2) velocity matching and, (3) flock centering. Their simulation was confined to some tens to some hundreds of individuals. These three rules seem reasonable, but they are unable to reproduce a flock once the boids separate a little far away. Again, global consideration is not realistic.

Another simple model (SPP model) (Vicsek and Czirok, 1995); (Gonci et al., 2008); (Vicsek, 2008) showed that an individual need not to consider the whole flock to produce collective behaviour. Only interactions with local neighbours and directional averaging with neighbours, while some environmental noise exists, is enough to produce collective motion. In their model, the individuals which exist around a certain radius circle to a reference individual, are considered the neighbours

Rana M. and Cai D..
MODEL OF AGGREGATION - A Topological Approach.
DOI: 10.5220/0003818603550360
In *Proceedings of the International Conference on Computer Graphics Theory and Applications* (GRAPP-2012), pages 355-360
ISBN: 978-989-8565-02-0

of that reference individual. Therefore, collective behaviours created in this model greatly depend on density of the aggregation. However, recent field study from European scientists (Ballerini et al., 2008) confirmed that the starling flock's behaviour is density independent. They argued that birds' behaviour depends on topological distance rather than metric one.

In this paper, we tried to construct a bird flock of large numbers. We take the basic *SPP model* for its simplicity (Vicsek and Czirok, 1995), but include cohesion and collision avoidance. Though the *SPP model* is strictly metric, we would exclude the metric perspective, instead, include the topological perspective for the topological idea is supported from empirical study (Ballerini et al., 2008). Finally, we would check flock's density independent behaviour.

2 SPP MODEL

The particles that make action or motion without the influence or action of any external force are called *self-propelled particles* (Simha and Ramaswamy, 2002). In this sense, animals that produce collective behavior in different sorts of aggregations, can be pointed as self-propelled particles. Instead of the three rules model of Reynolds (Reynolds, 1987), the SPP model (Vicsek and Czirok, 1995) is based on only one rule: *at each time step, a given particle driven with a constant absolute velocity assumes the average direction of motion of the particles in its neighborhood of radius r with some random perturbation added*. The analogy can be formulated as follows: The rule corresponding to the ferromagnetic interaction tending to align the spins in the same direction is replaced by the rule of aligning the *direction of motion* of particles. Random perturbations are applied in analogy with the temperature. Biological subjects have the tendency to move as other subjects do in their neighborhood (Brien, 1989). Therefore, the SPP model can be useful to model the flock of birds and other living organisms.

The simulations were carried out in a square shaped cell of linear size *L* with periodic boundary conditions. Interaction radius *r* was used as the unit to measure distances (*r* = 1), while the time unit, Δt was the time interval between two updating of direction and positions. The initial condition: (1) at time, $t = 0, N$ particles were randomly distributed in the cell, (2) had the same absolute velocity, v_0 and (3) randomly distributed directions. The velocities of particles $\{\vec{v}_i\}$ were determined at each time step, and the position of *i*th particle is updated according to-

$$\vec{v}_i(t+\Delta t) = v_0 \frac{<\vec{v}_j(t)>_r}{|<\vec{v}_j(t)>_r|} + pertubation \quad (1)$$

$$\vec{x}_i(t+\Delta t) = \vec{x}_i(t) + \vec{v}_i(t+\Delta t)\Delta t \quad (2)$$

Here $<..>_r$ denotes averaging of the velocities within a circle of radius *r* surrounding particle *i*. $<\vec{v}_j(t)>_r/|<\vec{v}_j(t)>_r|$ provides a unit vector pointing in the average direction of motion. Perturbation is taken account by adding a random angle corresponding to the average direction of motion in the neighbourhood particle of *i*. Perturbations are random values taken from a uniform distribution in the interval of $[-\eta\pi, \eta\pi]$. The only parameters of the model is the density -- the number of particles in unit square (for 2 dimensions) or unit volume (for 3 dimensions) – the velocity, v_0 and the level of perturbation,$\eta < 1$. In two dimensional simulation, Vicsek showed that, for a wide velocity range $(0.003 < v_0 < 0.3)$, and higher density ($\rho = 12.0$) and smaller level of noise or perturbation ($\eta = 0.1$), after some time steps, all particles move in the almost same direction i.e. synchronize themselves by locally interacting with each others.

In the *SPP Model*, Vicsek introduced an order parameter which denotes the level or ordered motion of the aggregation. The ordered parameter, φ, is determined as follows:

$$\varphi = \frac{1}{N v_0} \left| \sum_{i=1}^{N} \vec{v}_i \right| \quad (3)$$

Where *N* is the number of particles, $\vec{v}_i$ is the velocity of the *i* th particles. φ goes near to 1 when the aggregation is ordered and equal to 1 for fully ordered. In contrast, when φ is near to zero; it means that the particles are randomly walking and showing no collective behaviour.

3 METRIC OR TOPOLOGY

Topological distance: The word 'topology' is derived from Greek word 'topos' which means place or space, and 'logos' which means study or idea or theory (http://en.wikipedia.org/wiki/Topology, http://www.nn.iij4u.or.jp/~hsat/techterm/topos.html) . Therefore topology can be understood as the study of place or space. "Topology" the English form, was first used in 1883 in Listing's obituary in the journal

Nature to distinguish "qualitative geometry from the ordinary geometry in which quantitative relations chiefly treated". In this paper, when we would talk about ‘metric distance’, we would mean the quantitative distance i.e. real distance. And when we use ‘topological distance’, we would rank the surrounding particles to a reference. The rank would be 1, for the most nearest neighbour, 2 for the second nearest neighbour and so on. These ranks would be the topological distances. Therefore topological distances would be discrete: 1, 2, 3,.. The important distinction is that topological distance does not change with the density of aggregation i.e. the first nearest neighbour’s rank would be 1 (topological distance = 1) no matter how far or how near it is. In economics, for example, the relevant quantity is not how many kilometers separate two countries (metric distance), but rather than the number of intermediate countries between them (topological distance) (Henrikson, 2002).

4 BALLERINI’S FIELD STUDY

Ballerini et al., (2008), by reconstructing three-dimensional positions of individual birds of few thousand members showed that the interactions among the birds do not depend on metric distance rather than depend on topological one. Moreover, each bird interacts with a fixed number of birds (6-7 birds). They tried to show that the topological interaction can achieve more cohesion than the metric one while robust cohesion is needed for complex density and shape changes of flock not breaking cohesion among birds.

The main goal of the interaction among individuals is to maintain cohesion of the aggregation. This is very strong biological requirement, shaped by the evolutionary pressure for survivor: stragglers and small groups are significantly more prone to predation than animals belonging to large and highly cohesive aggregation (Vine, 1971). In topological model, cohesion among individuals does no vary with density changes, therefore more suitable to keep cohesion.

Ballerini et al., (2008) discussed about the characterization of structure of birds within flock by showing the spatial distribution of nearest neighbours. Given a reference bird, they measured the angular orientation of its nearest neighbours with respect to the flock’s direction of motion. The measurement shows an anisotropic characteristic and the anisotropic characteristic tends to fade out as the rank of the nearest neighbours increases. This means that the anisotropic characteristic of flock is the result of individual interaction.

5 RESULTS AND DISCUSSIONS

Ballerini et al., (2008) made a simple two dimensional predator-prey model based on *SPP model* to emphasize that the topological interaction should show strong cohesion. However, we reproduced the same results in two dimensional case and extended it to three dimensional predator-prey model. We have been successful to show that the three dimensional model exhibits the same type of cohesion as the two dimensional model does (Figure 1b and 1e).

5.1 Predator-prey Model

In the predator-prey model (two dimensional), we used equation (1) and (2) to update prey’s velocity and position. However, the perturbation or noise part is replaced by the impulsive force from the predator to prey. Predator’s velocity and direction remain unchanged and does not have effect from preys. The impulsive force from predator to prey is determined as equation (4).

$$\vec{F}_i = f_0 \frac{\vec{r}_{predato-i}}{|\vec{r}_{predato-i}|^2} \quad (4)$$

$\vec{F}_i$ is the impulsive force to i th bird, f_0 is the magnitude of the impulsive force posed by the predator and $\vec{r}_{predator-i}$ is the distance vector from predator to prey. For metric case, we used interaction radius as 0.15 and in case of topological situation, we assume that a bird interact with three nearest neighbour -- for two dimensional case individuals show optimum interaction when they interact with three nearest neighbours (Inada and Kawachi, 2002). For both metric and topological case, we calculated the isolated individuals separated by predator attack. Figure 1a shows that in metric case, maximum probability is for three isolated individuals while in topological case, Figure 1b shows that the maximum probability is for zero isolated individual. Again, the probability bars of separated individual decay very quickly in contrast with the metric interaction. Therefore, it shows that metric interaction is prone to predator attack and topological interaction produces more cohesion among individuals in aggregation.

We can assume that the birds may have preference while aligning with the neighbours. We ran another simulation taking weighted average of neighbour's velocity. We modified equation (1) to equation (5) to update velocity, and found that cohesion increased (Figure 1d). We cannot say for sure, but point out to birds may have preferences among nearest neighbours. Same sort of characteristic has been achieved for three dimensional predator-prey simulation (Figure 1e).

$$\vec{v}_i(t+\Delta t) = v_0 \frac{< \vec{v}_j(t) >_r}{| < \vec{v}_j(t) >_r |} + pertubation \qquad (5a)$$

$$\text{where, } < \vec{v}_j(t) >_r \frac{\vec{v}_i(t) + \sum_{j=1}^{N} \vec{v}_j(t)/(1+j)}{1 + \sum_{j=1}^{N} 1/(1+j)} \qquad (5b)$$

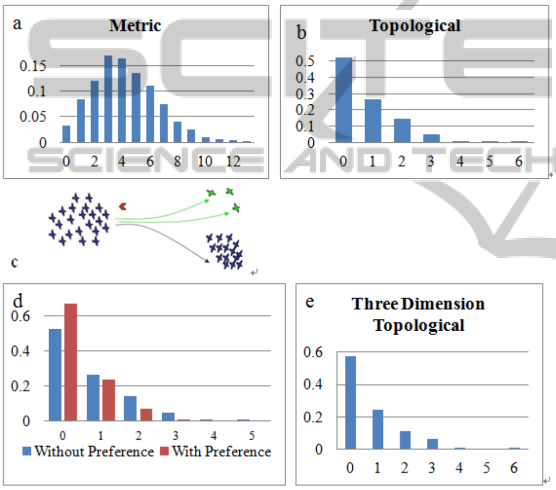

Figure 1: The horizontal axis (in a, b, d and e) shows the number isolated bird after attack and vertical axis shows the probability of that number isolated bird(s). In the model, we valued $v_0 = 0.25, f_0 = 0.05$. At $t = 0$, all birds are initialized with the same direction and the predator is at the opposite direction. (a) shows the probability of isolated bird in metric case (maximum probability is 16.5% for 3 isolated birds), (b) shows the probability in topological case, and the maximum probability is 52.4% for zero isolated bird. (c) shows the image of the simulation; (d) Comparison between non-preferred and preferred velocity alignment. Preferred alignment shows better cohesion. (e) shows the simulation result for three dimensional topology case. Time step is 1000, number of simulation is 1000. 1000 individuals, initially, are distributed in 1 unit radius sphere. The parameter values are, $v_0 = 0.50, f_0 = 0.05$, and isolation determination distance is 1.15.

In the simulations, the number of individuals is 200. Data is measured after 2000 time steps for each simulation, and probability is taken after 2000 simulations done for both metric and topological case. The prey, initially are distributed a radius 1 circle and predator's vertical position is 0.9 from the flock's centre. Interaction range for metric case, i.e. metric range is 0.15 and topological range is 3. We considered a bird is isolated if no other bird is present in 0.45 radius with respect to the reference bird. In 3D simulation, this radius would be 1.15.

5.2 Density Independence

In topological interaction, interactions among individuals should be density independent, i.e. they should show the same sort of interaction results for different densities in aggregation. We have run simulations (the above two dimensional predator-prey model) for different densities and demonstrate that the characteristic of interaction vary negligibly (Figure 2).

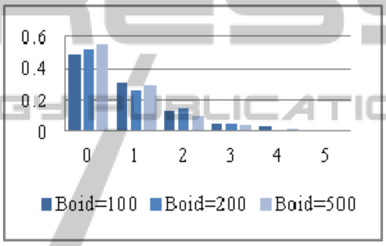

Figure 2: Predator-prey model has been tested for different densities (different numbers of individuals are distributed within the same area). Other parameters coincide with the two dimensional topological model in section 5.1. We used the same parameter values as section 5.1.

5.3 Compatibility of SPP Model

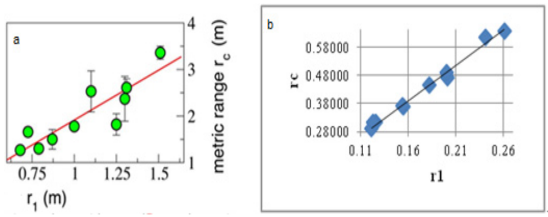

Figure 3: (a) Linear correlation between sparseness and metric range (Ballerini), Pearson correlation = 0.78. (b) Linear correlation between sparseness and metric range (simulation), Pearson correlation = 0.98. We used the same parameter values as section 5.1.

Is the SPP model is compatible to model bird flock? To test this, we have considered one of Ballerini's field study's result (Ballerini et al., 2008). They defined a parameter called sparseness (r_1) – the average first nearest neighbor distance of a flock – which is inverse proportion to the density of the flock; and metric range for topological interaction

(r_c) – the average $n_c(=7)$ th nearest neighbour distance of a flock – and found a strong linear correlation (Figure 3a) between them. We will take this as a test-stone to test the compatibility of SPP model. For ten different initial sparseness of our predator-prey model, we found that our simulations showed that there remains strong linear correlation between sparseness and metric range (Figure 3b).

5.4 Our Model

In SPP model, we could produce some trend of flock's behaviour (staying together under perturbation and linear correlation between sparseness and metric range). But as only directional alignment has been considered, as time passes cohesion will break down (Chate, Gregoire, Peruani and Raynaud, 2008). In our predator-prey simulation, we found that even though boids have strong relation in alignment, the flock tends to get sparser as time passes even when there is negligible perturbation (Figure 4). Therefore, to model a flock consisting large number of individuals we have to consider some other interactive forces that are presented among individuals. Gruler et al., (1999), and Kemkemer et al., (2000) described that human melanocytes - pigment cells of the skin – are also act collectively without external force. That is why, melanocytes can be said as SPPs. But melanocytes do not show directional properties rather show apolar characteristics. Melanocytes show nematic arrangements (Figure 5) and their net motion is zero. They interact with each other nematically. This can be a vital interaction in different SPPs (Simha and Ramaswamy, 2002). Vicsek model (1995) assumes objects as point like while melanocyes are rod like. Therefore, to model bird flock, we can consider birds as a rod like objects that consider nematic forces for cohesion and also tend to make directional alignment. With this hypothesis, we will introduce a topological model where both nematic forces and tenderness for directional alignment would exist. By modifying SPP model with topological essence, we described the velocity update for each bird as equation (6). The main difference of this equation with Chate et. al. is that it only deals with topological range where Chate et. al. considered metric distance.

$$\vec{v}_i(t+\Delta t) = v_0 \upsilon\{s\sum^{N} \vec{v}_j(t) + (1-s)\sum^{N} f_0\vec{e}_{ij} + \eta\vec{z}\} \quad (6)$$

Here, v_0 is the speed, N is the number of neighbours for interaction, f_0 represents the nematic or cohesive force to each other, $\vec{e}_{ij}$ is the unit vector to from i th bird to j th neighbor. η is the system's noise level, $\vec{z}$ represents the random unit vector. $\vec{v}_j$ is the velocity of j neighbour. s represents a strategy parameter, where, $0 \le s \le 1$. It determines to what extent, a bird is going to evaluate directional alignment and cohesion. Vicsek's (1995) *SPP model* does not consider the prevention of collision among the individuals. We introduced collision prevention by imposing an infinite value to f_0 and, setting $\vec{e}_{ij} = -\vec{e}_{ij}$ when the nearest neighbor(s) are too close. υ makes a vector to a unit vector, i.e. $\upsilon = \vec{a}/\|\vec{a}\|$.

In large flocks, some characteristics can be found: density fluctuation, wave flow and complex patterns. *SPP model* for large number of particles shows density variance in the system both in two and three dimensions (Chate et al. 2008). By simulating a large number of individuals with our proposed topological cohesive-directional alignment model, we were able to produce real like flock (Figure. 6). The simulated flock mainly showed two properties of real flock: visual complexity and density variations through flock. Though the flock shows visual similarities, we must test the internal structures of simulated flock. At this point, we could argue that the proposed model is able to create visual complexity and density variations in flocks.

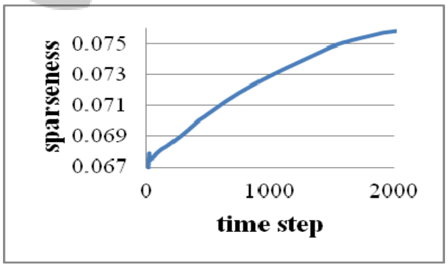

Figure 4: Sparseness increases with time steps.

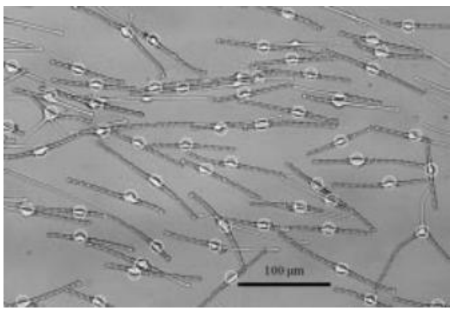

Figure 5: Human melanocytes on a glass surface. We can see that these cells have nematic arrangements (Simha and Ramaswamy, 2002).

We think that velocity alignment is responsible for density variation and nematic cohesive force is responsible for complex pattern. However, yet, we have not been able to include wave flow in flock of birds. We are working on this.

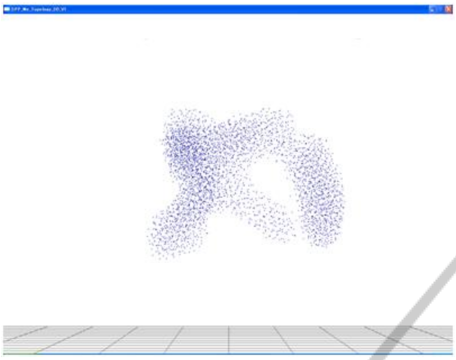

Figure 6: A snapshot of flock of birds in our simulation. Number of individuals is 4096. Initially we distributed the individual randomly in a box of length 7 and initial directions were randomly taken. Individuals were updated according to equation (6) and equation (2). Time step was 1500. Other parameters are: $f_0 = 0.5, \eta = 0.001, s = 0.94$, $v_0 = 0.05, \Delta t = 1.0$, and collision prevention distance = 0.25.

6 CONCLUSIONS

Though interactions among birds in a flock depend on topological range and birds interact only local perception of the world, previous models for bird flock lacks these properties of birds' behaviour. We presented a model of bird flocks from topological perspective. We took two important behaviours of self-propelled particles to model the bird flock: alignment and cohesion with neighbours. The simulation result presents two important properties of bird flocks: complexity in shapes and density variations through flocks. We were also able test the density independence characteristics of flock of birds and bird's preferential behaviour that might be true. Still we need to check flocks' internal structure of flocks to compare simulated flocks with real flocks. Again, we are unable to create wave passing through flock. We are working on this topic.

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