MARGINING COMPONENT OF THE STOCK MARKET CRASH OF OCTOBER 2008

A Lesson of the Struggle with Combinatorial Complexity

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Abstract: In July 2005 the US stock market started using the risk-based approach to margining customer accounts gradually excluding from margining practice the strategy-based approach, which has been used for more than four decades. In this paper we argue that this change has a direct link to the stock market crash of October 2008. We also show that among the main reasons of this change are high strategy-based margin requirements in comparison with much lower risk-based, the combinatorial complexity of the strategy-based approach, and the failure of the brokerage industry to adopt the achievements of combinatorial optimization.

1 INTRODUCTION

> "We still have a 1930s regulatory system in place. We've got to update our institutions, our regulatory frameworks, … the banking system has been 'dealt a heavy blow,' the result of lax regulation, massive overleverage, huge systematic risks taken by unregulated institutions, as well as regulated institutions." – *Barack Obama*[1]

In the *margin accounts* of investors, margin payments are based on established minimum *margin requirements* according to the margin rules enforced by the *margin regulations*.

Deducting the minimum margin requirement for an account from its market value we obtain its *loan value*, which is the maximum portion of the account's market value that the broker can lend. The amount that is actually lent to a customer by the broker constitute the *margin credit*. The total margin credit provided by all brokers in a market constitutes the market's *margin debt*.

High margin requirements can reduce investors' activity, lead to underpricing of securities, and cause economic slowdowns. Low margin requirements, in turn, lead to overpricing of securities, high levels of speculation, cash deficits, market crashes, and, again, economic slowdowns. The challenge for margin regulators is to find a proper approach to margining, a "golden mean" that keeps the growth of margin debt within tolerable limits.

Current margining practice uses two approaches, *strategy-based* and *risk-based*.[2] In comparison with the strategy-based, the risk-based approach uses *substantially* lower margin rates, therefore the final approval of the latter for margining customer accounts by the SEC[3] in July 2008, at the time of the global financial crisis, appeared to be one of the most radical steps in the history of margin regulations.

The goal of this paper is to briefly discuss the difference between the strategy-based and risk-based approaches and to illustrate the influence of risk-based margin calculations on the stock market crash of October 2008 by considering the margin debt levels in the period 2005-2008.

2 STRATEGY-BASED APPROACH

The combinatorial essence of the strategy-based approach arises from the ability to partition a margin account in different ways in accordance with a large variety of offsets in the margin rule book. The challenge is to find a partition with a minimum total margin of the offsets and naked positions left uncovered by the offsets. Each securities market follows its own margin rule book, for example, NYSE Rule 431 in the U.S. or Regulation 100 in Canada. The strategy-based offsets are of fixed size and imitate trading strategies. The account margin minimization (AMM) problem reduces to a *mixed integer program* (MIP) as follows.

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Let an account have m positions $i = 1,2,\ldots,m$ in securities with quantities q_i and n prime offsets[4] $j = 1,2,\ldots,n$. Prime offset j can be represented by the column vector $\mathbf{o}_j$ whose ith component o_{ij} is the quantity of the ith component of prime offset j.[5]

Note that $o_{ij} = 0$ if and only if position i is not involved in prime offset j. We assume that $m \leq n$ because we consider that the first m prime offsets are trivial, i.e., they represent positions in security units. Thus, each of the first m column vectors has only one nonzero element $o_{jj} = 1$.

Let m_j be the margin rate for prime offset j. Then m_j is the margin rate for a security unit in position j if $1 \leq j \leq m$. Let x_j denote the multiplicity of prime offset j. If we introduce the vectors[6]

$$\begin{array}{rcccccc} \mathbf{m}^\top & = & (& m_1 & m_2 & \ldots & m_n &) \\ \mathbf{q}^\top & = & (& q_1 & q_2 & \ldots & q_m &) \\ \mathbf{x}^\top & = & (& x_1 & x_2 & \ldots & x_n &) \end{array}$$

which are the *margin*, *quantity* and *variable vectors*, respectively, and the *offset matrix* $\mathbf{O} = [\mathbf{o}_1\ \mathbf{o}_2\ \ldots\ \mathbf{o}_n]$ of size $m \times n$, then the AMM problem can be posed as finding an $\mathbf{x}$ which solves

$$\min\left\{\mathbf{m}^\top\mathbf{x} : \mathbf{O}\mathbf{x} = \mathbf{q}, \mathbf{x} \geq \mathbf{0}\right\} \qquad (1)$$

where $\mathbf{0}$ is a zero column vector.

Note that $\mathbf{O} = [\mathbf{I}\ \mathbf{P}]$, where $\mathbf{I}$ and $\mathbf{P}$ are the *identity matrix*, i.e., the matrix of trivial prime offsets, and the matrix of *proper prime offsets*, i.e., the matrix of prime offsets that involve at least two positions, respectively.

In this MIP, components of the variable vector $\mathbf{x}$ from $m+1$ to n must be integers because they define offset multiplicities, while some of the components from 1 to m can be real. They are integers if they are quantities of positions in stocks, options, warrants, etc., but they can be real if they are, for example, bond quantities. The quantity vector $\mathbf{q}$ is also integer but the margin vector $\mathbf{m}$ is real.

It is important to mention that the components of the vector $\mathbf{m}$ are calculated on the basis of the *current* security market prices.

MIP (1) can be viewed as an extension of the transportation model introduced in (Rudd and Schroeder, 1982) for the calculation of the minimum margin by pairing, i.e., in the case where the matrix of proper prime offsets is a matrix of prime pairs.

Surprisingly, current strategy-based margin calculation practice still uses heuristics designed in the mid seventies based on brokers' intuition and taste, even for margining by pairing. In the general case, the most advanced heuristics take advantage of the result from (Rudd and Schroeder, 1982). There exist, however, examples showing that these heuristics can raise minimum margin requirements to catastrophic margin calls. Therefore, margin accounts maintained by prime brokers up to 2005 with the use of the strategy-based approach were substantially over-margined, just because there has been no serious attempt to find efficient optimization algorithms.

Despite the fact that the AMM problem was posed (originally as a problem of margining accounts with options only) more than thirty five years ago, it has not been well studied and remains one of the most intractable problems in the brokerage industry. Neither useful theoretical results nor solution techniques with reasonable computing times are known.

The only exception is the already mentioned paper (Rudd and Schroeder, 1982) devoted to a reduction of the AMM problem by pairing to the bipartite minimum-cost network-flow problem and the very recent paper (Matsypura and Timkovsky, 2011) extending this result to the case with more complex offsets.

An analysis of the literature suggests that the AMM problem has never been considered in the MIP form. However, as was shown in (Coffman et al., 2010b; Coffman et al., 2010a), the MIP for the AMM problem can be efficiently used in practice.

3 RISK-BASED APPROACH

The risk-based approach uses *variations* of the *current* underlying security prices in an attempt to catch the worst-case price movements for the entire portfolio. This technique is called *portfolio shocking*.

According to the portfolio shocking technique, the margin requirement for each position is the largest potential loss on this position among the losses calculated for ten *valuation points* surrounding current underlying security price. The lowest (highest) valuation points must be

$$8\%(6\%),\ 10\%(10\%),\ 15\%(15\%)$$

lower (higher) than the current market price for high-capitalization BBIs[7] and ETFs[8] based on it, low-capitalization BBIs and ETFs based on it, and NBIs[9] and a margin eligible securities, respectively.[10]

It is important to observe that, in accordance with this rule, the risk-based margin rate for stocks and margin eligible equities in customer margin accounts is only 15%, the lowest margin rate since 1929. Before the stock market crash of October 1929 it was 10%. The current strategy-based margin rates are 50% (initial) and 25% (maintenance).

In our opinion, this significant margin reduction for stock positions and the faulty hedging mechanism of risk-based offsets for stocks were the main contributors to the stock market crash of October 2008.

Now let us explain how to calculate the loss on a position s. Let c_v, $1 \leq v \leq 11$, be one of the eleven valuation points including the current underlying security price c. If s is a position in the underlying security, then the difference $o_v = c_v - c$ or $c - c_v$ shows the outcome (*gain* if positive, and *loss* if negative) associated with point c_v for long or short position s, respectively, for each security unit.[11]

If s is a position in a derivative, then the outcome o_v associated with the valuation point c_v should be calculated in accordance with the mechanism of the derivative. In most cases, o_v is a function of c_v, e (the exercise price of the derivative) and p_v (the market price of the position s estimated at the valuation point c_v).[12] The estimated price p_v must be calculated using a qualified *theoretical pricing model*.[13]

Unlike the strategy-based approach, the risk-based approach uses much looser offsets whose hedging mechanism is based on the cash settlement only.

A risk-based offset involves all positions with the same underlying instrument, and the margin requirement for this offset is simply the net loss on the involved positions. Thus, for each underlying instrument, a single risk-based offset is the union of all possible strategy-based offsets.

Without portfolio shocking, the risk-based approach represents just a relaxed case of the strategy-based approach with substantially lower rates.

The risk-based approach squeezes the entire rule book of the strategy-based approach into only one rule, therefore it allows to substantially simplify calculations of margin requirements for offsets.

With using strategy-based offsets, the risk-based approach would turn into the extension of the strategy-based approach adopting the portfolio shocking technique. It would have been natural to use such an extension in the pilot program for a "cushioned" transition to the risk-based approach.

4 BRIEF HISTORY PRIOR TO 2005

The strategy-based approach to margining customer accounts has been used in the brokerage industry for more than four decades. By the end of the nineties, it was commonly recognized that this approach yields excessively high margin requirements. One of the main reasons for this phenomenon is that the calculation of the minimum regulatory margin by the strategy-based approach is an intractable combinatorial optimization problem that is neither well studied nor properly understood.

Despite the fact that margin regulations have a 75-year history dating from Regulation T in the Securities Act of 1934, the literature on margin calculations is surprisingly small.[14] We can point to only two books (Geelan and Rittereiser, 1998; Curley, 2008) and two papers (Fortune, 2000; Fortune, 2003) devoted to margining practice, two papers (Moore, 1966; Luckett, 1982) studying the influence of margin requirements on investor's equity ratio, and two papers (Rudd and Schroeder, 1982; Fiterman and Timkovsky, 2001) devoted to margining algorithms. The vast majority of publications on margining prior to 2005 consisted primarily of regulatory circulares.

Consequently, margin calculation systems, developed and used in the brokerage industry up to 2005, ignored highly effective and broadly applicable discrete optimization methods. In particular, the reduction of the AMM problem by pairing to the minimum-cost network-flow problem (Rudd and Schroeder, 1982) was seemingly forgotten for more than 20 years.

As a result, existing margin calculation technology, faced with the combinatorial complexity of the strategy-based approach, failed to take advantage of efficient combinatorial optimization algorithms. The vast majority of margin calculation systems used in the brokerage industry prior to 2005, as our study shows, used outdated heuristics proposed by brokers in the mid seventies, cf. (Cox and Rubinsein, 1985; Geelan and Rittereiser, 1998). But the failure to find an exact solution, as shown in (Coffman et al., 2010a), can increase the margin requirement from zero to several thousands of dollars.

The risk-based approach was proposed in 1989 by the OCC[15] to calculate the net capital requirements for brokers' proprietary portfolios of listed options.[16] It was implemented in 1996 in TIMS[17] and approved by the SEC[18] to be effective as of September 1, 1997. However, the approach was not used for margining customer accounts prior to 2005.[19]

Employing lower margin rates and avoiding any combinatorics in contrast to the strategy-based approach, the risk-based approach produces substantially lower margin requirements. In the examples provided by the CBOE, the requirements for naked options and basic option spreads are at least two or three times lower.[20] (72 times lower for a long straddle!) After two NYSE proposals,[21] the SEC approved the risk-based approach to margining customer accounts under a temporary *pilot program*.[22]

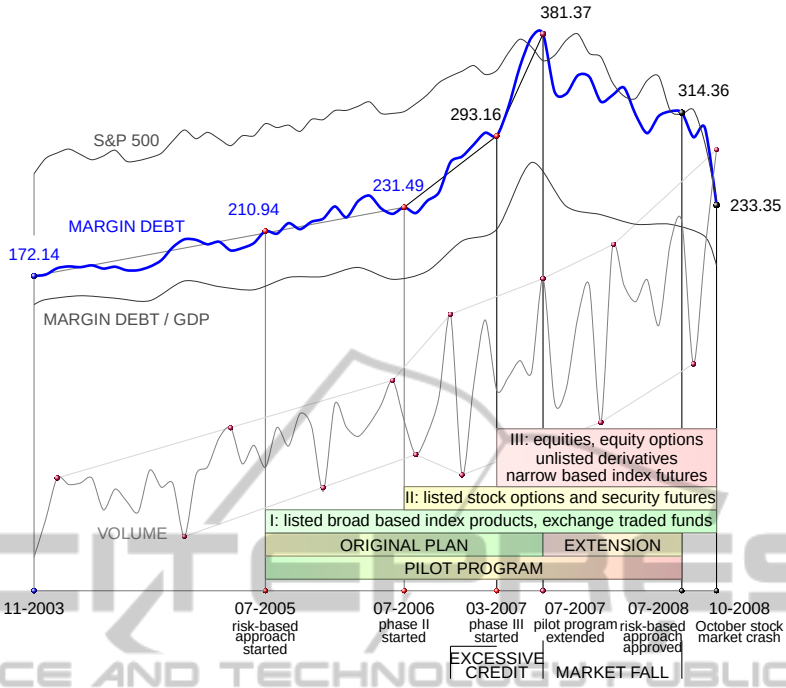

Figure 1: The margin debt in billions of dollars, the margin debt relative to GDP, and the level and trading volume of the S&P 500 index in the period from November 2003 through October 2008. The curves are scaled to fit on one graph and drawn by smoothing the end-of-the-month data from http://www.nyse.com/ and http://finance.yahoo.com/.

5 PILOT PROGRAM OF 2005–2008

The pilot program can be divided into the following three phases; see Fig. 1:

Phase I started on July 14, 2005 and permitted the use of the risk-based approach to margin accounts with only listed BBI and ETF derivatives. Phase II started on July 11, 2006 and included listed stock options and securities futures.[23] Phase III started on April 2, 2007 and included equities, equity options, unlisted derivatives and NBI futures.

In contrast with Phases I and II, Phase III was widely advertised in the media after its approval on December 12, 2006,[24] i.e., more than 3 months before it would become effective. Hence, the substantial decrease of margin requirements for equities, equity options, all unlisted derivatives and NBI futures was widely anticipated by numerous investors who entered the market on April 2, 2007 stimulating demand for stocks and driving the market up.

The pilot program was to expire on July 31, 2007. However, on July 19, 2007 it was extended for one more year, and the risk-based approach was finally approved to be used permanently on July 30, 2008.[25]

Monthly margin debt reports published by the NYSE provide clear evidence of the influence of the pilot program on the stock market; see Fig. 1. During the initial 20-month period of the pilot program (July 2005 - April 2007), the margin debt had increased by $82.22 billion compared to the $38.80 billion increase during the previous 20 months. Although the beginning of this period, between the starting points of Phases I and II, looks ordinary, the ending of this period, between the starting points of Phases II and III, is remarkable owing to the unusually high rate of increase in the margin debt and the trading volume volatility of the S&P 500 index.

During the subsequent four months (April 2007 - July 2007), the period when equities, equity options, unlisted derivatives and NBI futures joined the pilot program, the margin debt increased by another $88.21 billion, i.e., at a rate at least five times higher. Thus, since April 2, 2007, the margin debt increased at a rate of more than $22.05 billion per month.

Historical records show that a fast growth of margin debt can be a sign of an approaching market crash. Such was the case in October 1929 and October 1987. Examining the two most recent examples, consider the assessments of the market crash of October 2000 in (Geelan and Rittereiser, 1998) (p. 87):

> "As of February 2000, total margin debt stood at $265 billion. It had grown 45 percent since the previous October and had more than tripled since the end of 1995. Relative to GDP,[26] margin debt was

> the highest it had been since 1929, and over three times as high as it was in October 1987. It was an unmistakable sign of rampant speculation."

Bringing these assertions more up-to-date, we observe that, as of July 2007, total margin debt stood at $381 billion. It had grown 30 percent since the previous March and had almost tripled since the end of 2002. Relative to GDP, margin debt was the highest it had been since February 2000.

Figure 1 clearly shows that the market credit in the period from April through July 2007 was *excessive* in the extreme: 27% increase in Margin Debt/GDP. Even though the margin debt reached the level of $381.37 billion by the end of July 2007, the pilot program had, nevertheless, been extended for an additional year.

In September 2007, it was clear that the growth in margin debt had lessened because in August 2007 it fell to $331.37 billion as a result, in particular, of numerous margin calls received by investors and associated forced sales from their undermargined accounts.[27]

By July 2008, the time of the final approval of the risk-based approach, the margin debt had plunged to $314.36 billion, signalling an approaching stock market downfall. In September 2008 the margin debt had another plunge from $299.96 to $233.35 billion, and the stock market downfall was evident as indicated by the level of S&P 500.

> "The task of the Board, as I see it, is to formulate regulations with two principal objectives. One is to permit adequate access to credit facilities for security markets to perform their basic economic functions. The other is to prevent the use of stock market credit from becoming excessive. The latter helps to minimize the danger of pyramiding credit in a rising market and also reduces the danger of forced sales of securities from undermargined accounts in a falling market." – *W. McC. Martin, Jr.*[28]

In July 2007, it was clear that the market credit had been excessive during the preceding three months: it had been growing at an unprecedented pace and relative to GDP had reached its highest level since the market crash of October 2000. In July 2008, it was clear that the stock market had been falling for the previous twelve months. Yet the practice of using the risk-based approach, which evidently caused excessive market credit, was continuing.

We argue in this paper that the stock market crash of October 2008 has a direct link to the adoption of the risk-based approach for margining customer accounts in the US stock market. As shown in our computational experiments with both approaches and randomly generated portfolios (Coffman et al., 2010a), this approach produces substantially lower margin requirements, especially for investors playing bear.

6 CONCLUDING REMARKS

The results of our research show that the strategy-based approach is, at this point, the most appropriate one for margining security portfolios in customer margin accounts because it provides exit strategies. Over-margining can be eliminated by offsets of higher sizes and related strategy-based algorithms.

In contrast, the risk-based approach does not provide exit strategies for security portfolios. In addition, it can be misleading with respect to the level of risk exposure of security portfolios.

However, the risk-based approach can work efficiently for margining index portfolios in customer margin accounts and inventory portfolios of brokers, because the liquidations of such portfolios do not involve broker-to-broker security movements. It is possible in the former case because an index itself does not have trading units. The application of the risk-based approach to security (including ETFs) portfolios in customer margin accounts is in fact very risky.

We suggest that Phases II and III of the pilot program of using the risk-based approach for margining security portfolios in customer margin accounts were major contributors to the growth of margin debt in the period from December 2006 through July 2007 and the subsequent stock market crash of October 2008.

Margin minimization problems with complex offsets can be efficiently solved by optimization packages such as CPLEX. In our opinion, the strategy-based approach was unjustly discredited by the belief that the combinatorial problem stemming from the use of complex offsets could not be efficiently solved with the help of standard optimization packages. Naturally, the development of special optimization algorithms can bring much better results.

The practice of margin calculations has a long history. However, the science and art of margin calculations has only just begun to evolve. We hope that this paper will attract the attention of margin regulators and academic researchers who are involved in studying efficient exits from the current economic crisis.

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APPENDIX

Notes

1. From "Obama, Brown call for global changes, say financial regulations need to be revamped" by Roger Runnigen and Robert Hutton, *Bloomberg News*, p. 4A · March 4, 2009 · USA TODAY.
2. The risk-based approach also appears in margin regulations and business-related literature under the names "portfolio margining" and "risk-based portfolio margining" approach or methodology. (We omit the terms "portfolio margining" and "methodology" for compactness.) The argument standing behind the term "portfolio" is based on the *misrepresentation* of the strategy-based approach as a treatment of a margin account by considering individual positions only, while the "portfolio approach" treats a margin account as a whole. The strategy-based approach also treats a margin account as a whole; although it does so in a different way, it is also a portfolio approach. So we consider the term "risk-based approach" to be most appropriate. For the same reason, we do not think that the term "rule-based approach," frequently used in the Internet as synonymous with the strategy-based approach, is suitable because the risk-based approach is also based on certain rules, such as Rule 15c3-1a or rules from Regulation T.
3. The U.S. Security and Exchange Commission.
4. Prime offsets have minimum position quantities.
5. Component quantities of prime offsets are integers, unlike the quantities of convertible securities with non-integer conversion ratios.
6. The upper index $^\top$ denotes the transposition.
7. BBI: Broad Based Index.
8. ETF: Exchange Traded Fund.
9. NBI: Narrow Based Index.
10. These percentages follow Rule 15c3-1a(b)(1)(i)(B).
11. These gains and losses are called "theoretical gains and losses" in SEC Release 34-53577.
12. For example, if s is a position in a call option, then, after calculating $i_v = \max\{c_v - e, 0\}$, i.e., its in-the-money amount for valuation point c_v, and its estimated market price p_v corresponding to c_v, its outcome can be calculated as $\max\{i_v, p_v\} - p$, where p is the purchased price of the call option, multiplied by the option contract size.
13. The model must be approved by the DEA (Designated Examining Authority). By February 2008, only the OCC model implemented in STANS was approved, see Federal Register, Vol 73. No. 29, February 12, 2008.
14. This remark does not refer to the literature devoted to studying the relationship between margin requirements and market volatility; see a survey in (Kupiec, 1998).
15. The Options Clearing Corporation.
16. SEC Release 34-27394, October 26, 1989.
17. Theoretical Intermarket Margining System.
18. SEC Releases 34-38248, February 6, 1997.
19. The SEC published the related NYSE proposal for public comments in SEC Releases 34-46576, October 1, 2002 and 34-50885, December 20, 2004, before approving the approach in July 2005.
20. www.cboe.com/margin, CBOE Rules 12.4, 9.15(c), 13.5 and 15.8A.
21. SEC Releases 34-46576, October 1, 2002, and 34-50885, December 20, 2004.
22. SEC Release 34-52031, July 14, 2005.
23. SEC Release 34-54125, July 11, 2006.
24. SEC Release 34-54918.
25. Exchange Act Release No. 58251, July 30, 2008, 73 FR 45506, August 5, 2008.
26. Gross Domestic Product.
27. See, for example, The Wall Street Journal, July-August 2007, for numerous reports on margin calls and associated forced sales.
28. From the speech of William McC. Martin, Jr., Chairman of the Board of Governors of the Federal Reserve System from April 2, 1951, through January 31, 1970, at the hearing on the study of the stock market before the U.S. Senate Committee on Banking and Currency on Monday, March 14, 1955.