

# OVERVIEW OF MATH LESSONS AUTHORIZING SYSTEM (MLAS)

Samer F. Khasawneh and Paul S. Wang

*Department of Computer Science, Kent State University, Kent, Ohio, U.S.A.*

**Keywords:** (Virtual) Manipulative, Online Mathematics Lesson Authoring.

**Abstract:** Web-based Mathematics Education (WME) provides new and promising way to use the web to improve mathematics education. However, WME lessons still require substantial programming skills to create and modify. MLAS enables teachers, who might not have the required programming know-how, to create and customize web-based mathematics lessons. MLAS is an on-web lesson authoring system with simple, yet powerful, graphical user interface. Dynamic lessons can be generated with little effort and time by using lesson templates. When authoring a lesson, a teacher may include one or more interactive virtual manipulatives from MLAS. The manipulatives can be customized and can interact with one another or questions and comments in the lesson page. MLAS offers a growing library of virtual manipulatives that are fully customizable, editable, and reusable. The MLAS library is easily expandable by adding new manipulatives contributed by teachers and other experts.

## 1 INTRODUCTION

Manipulatives, in general, are any objects, such as coins, tiles, and even a paper that is cut or folded, used to help students understand abstract math concepts such as fractions and percentages in an active, hands-on approach. With the advent of the web and based on its potential effect in enhancing the quality of education in general, and the math subject in particular, a new term has come into existence, “virtual manipulatives”. This term refers to those manipulatives that cannot be “touched” but rather can be “seen” on a computer screen, allowing students to explore them using computer hardware, such as a mouse and keyboard (CITED Research center).

A great deal of research has focused on the usage of manipulatives in education. Researchers largely agree on the benefits manipulatives can achieve. “Manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning”(ETA Cuisenaire). A good manipulative bridges the gap between informal math and formal math. “The most valuable learning happens when students build their own mathematical perception, which is often accomplished through the use of manipulatives” (Boggan, Harper, and Whitmire, 2010). With the advent of virtual manipulatives,

other advantages come into play. “Virtual manipulatives allow users to engage and control their actions in a way that extends real-world manipulatives” (CITED Research center). Virtual manipulatives often overcome the limitations of their original counterparts. For example, the shape, size, color, and movement are not as limited as concrete objects. Other advantages of virtual manipulatives include:

- **Availability:** Most virtual manipulatives are available for public use, and are free of charge.
- **Time Saving:** Teachers do not need to spend time creating their own concrete manipulatives.
- **Motivation:** Students may find virtual manipulatives more stimulating.
- **Immediate feedback:** Virtual manipulatives are often “intelligent” and users can be notified when they give right or wrong answers.

There are current technologies that help teach mathematics. Some of these are web-oriented and can be accessed with a URL, while others come in downloadable software packages. The usability of these applications varies significantly.

Most online systems that give teachers the ability to edit mathematics contents come in worksheet

format. In (Superkids Math Worksheet Creator) and (Math Fact Cafe) for example, a user can print a PDF document containing a number of computer-generated series of mathematical calculations. The output on the worksheets relies on the user's input parameters such as range, minimum, and maximum, or it can be fully generated by the system. Once printed, these worksheets become static and fail to take advantage of technology, serving only as supplements for textbooks.

The (National Library of Virtual Manipulatives) (NLVM) provides online math lessons, activities, and assessments. This NSF-funded project has a library of 200+ applets covering the five NCTM standards (NCTM, 1989). (Arcademic Builders) offers fun and free online multi-player educational math applet-based games for students and teachers. In (E-learning Authoring System), an e-learning authoring system is presented. It provides users with an HTML editor to allow them to write and style text, upload images, and ask questions. However, this software package is not subject-oriented and is not intended to focus on any one discipline. Further, the relatively high cost of the system might be a barrier against its usage.

## 2 MLAS IS DIFFERENT

Throughout our literature review, we have not come across any system that allows teachers to create and upload their own math lessons to the web. MLAS overcomes the problems associated with the above systems and tries to utilize the web to provide a better atmosphere for teaching and learning math online. Most of the above applications are not customizable. Manipulatives have to be used "as-is" with no editing or customization permitted. While an applet renders nicely inside a browser window, it does not interact well with its enclosing web page. Applets cannot easily take advantage of the document object model (DOM) for dynamic interaction. Also, applets are slow to load and require the java browser plug-in to be installed.

MLAS is a contribution towards the Web-based Math Education (WME) project (Chiu, 2005), (Wang et al., 2003-2006). The WME project unifies the efforts of computer science and education members to deliver classroom-ready, interactive, and hands-on mathematics contents using cutting-edge web technologies. With the presence and supervision from educational experts in WME, WME lessons are ensured to offer content-rich material abiding by the NCTM standards (Figure 1).

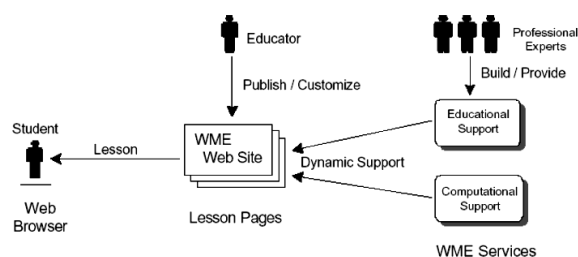


Figure 1: The WME concept model.

MLAS, powered by up-to-date web technologies including HTML, Javascript and jQuery, PHP, AJAX, and MySQL, seeks to be a firsthand assistant to math teachers. It allows them to author their own lessons by offering a growing number of fully-customizable manipulatives, sometimes referred to as "manipulative builders". A manipulative builder allows a manipulative to be customized per teacher's specifications, therefore it can appear in several forms and features for students' use. Before beginning lesson authoring, teachers deploy their lessons under a topic module - a collection of lessons pertaining to a certain grade level. Students can access their lessons directly with a given URL, by browsing the whole list of lessons under a particular module, or by searching by keywords through the search feature MLAS provides.

MLAS was built with the following goals in mind:

- **Ease of Use:** MLAS can be used by anyone and does not require any advanced computer skills. A complete math lesson can be created and shared with a few mouse clicks.
- **Accessibility:** MLAS is fully web-based and can be accessed anytime and from anywhere. Further, there is no software to download, update, or buy.
- **Extensibility:** Users who got some HTML and Javascript knowledge can add their own manipulatives to augment and extend the basic functionalities and features of MLAS.
- **Security:** Depending on the user's type, some features might not be available. Thus, contents remain safe and cannot be altered easily.
- **Content Control:** Manipulatives offered by MLAS are directly supervised by the WME educational members. All contents are to comply with the NCTM standards.

### 3 NATIVELY-SUPPORTED MANIPULATIVE BUILDERS BY MLAS

In this section, we briefly mention the current customizable manipulatives natively supported by our system. The ideas behind some of these come from already-existing WME manipulatives deployed on the WME site. However, complete reimplementations were done to add more flexibility, customization, and functionalities. Other manipulatives below originated after suggestions from the educational experts of WME. These manipulatives can be used to cover several topics such as percentages, fractions, and data visualization. This list is constantly growing and more manipulatives are underway.

#### 3.1 Pie and Bar Graphs

Although two separate entities in MLAS, they are combined in this context due to their inherent relationship in visualizing data.

The bar graph manipulative allows the user to fill out some input fields so a bar graph will be generated. Such fields include: *Graph title*, *Y axis maximum range*, and bar-related info like *bar name*, *value*, and *color*. There is no limit on the number of bars that can be created (Figure 2). Adding or deleting data will reflect instantly on the generated graph as well.

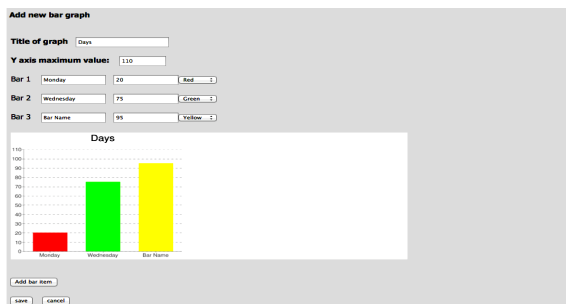


Figure 2: An editable bar graph manipulative.

In MLAS, a pie graph can be created with the same ease as the bar graph. In the pie chart manipulative, a user can fill out pairs of *label-value* fields with inputs. Adding such pairs can simply be done with a mouse click. The pie chart will be automatically generated and updated instantly as the user adds or changes values in the fields above.

Students, on the other hand, will only be able to see the generated pie or bar graph in the lesson page.

They will not be able to add or edit any data (Figure 3).

An underway development on the bar graph manipulative aims to allow it to natively compute values of several statistical operations, such as mean, mode, and median, on the displayed data.

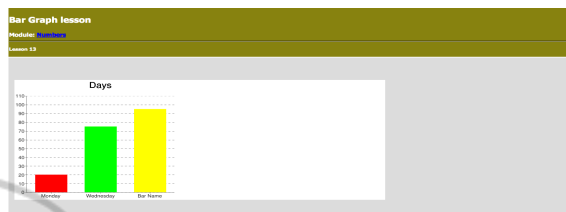


Figure 3: The bar graph in the lesson page.

#### 3.2 Numbered Cubes

This manipulative enables students to virtually roll numbered cubes with a click of a mouse. Surpassing its original WME counterpart, teachers can select up to six cubes for students to explore. Features, including “power roll”, where any number of rolls can be simulated with a mouse click, and “rolls description table”, which indicates the number of times a particular number of a cube appeared, can be added or removed simply by a check or uncheck of an HTML checkbox.

#### 3.3 Calculations

This manipulative tests students’ ability to find out the results of mathematical expressions. It allows teachers to decide the types and values of the mathematical operations and operands, respectively. Teachers are not limited by a certain number of operators and operands. Further, they do not have to calculate the result of any composed expression as it will automatically be calculated. This feature allows for “auto-grading” and the system can independently check the correctness of students’ answers to provide immediate feedback.

#### 3.4 Percentages

This manipulative encompasses three different question formats on percentages namely:

- What is *anyNumber* % of *anyNumber*?
- *anyNumber* is what percent of *anyNumber*?
- *anyNumber* is *anyNumber* % of ?

Above, *anyNumber* is represented by an HTML textbox that can be filled with any integer value. The teacher, through three available HTML radio

buttons, can select the format of the question desired. Students need to answer the question and the system can independently calculate the answer and compare it with the students’ answers.

### 3.5 Question Composer

This manipulative allows teachers to ask questions in different formats such as T/F, fill in the blank, multiple choice, and others (Figure 4).

In the multiple choice format, a teacher can specify an error message for each option available. This message would pop up when that option is selected. This feature can only exist in a virtual environment brought by the technology.

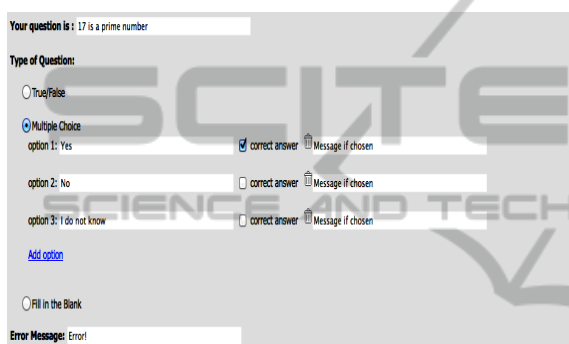


Figure 4: Questions can appear in multiple formats.

### 3.6 Manipulative-complementary Kit

MLAS has a built-in kit geared towards making lesson authoring easier and more efficient. MLAS features an embedded WYSIWYG HTML editor so users can create text, symbols, notations, and upload pictures.

MLAS also has a “Progress Menu” on the right of the lesson authoring interface that keeps track of all user activities such as adding, deleting, or editing a manipulative. Each activity will be noted in the progress menu and clicking on that activity will undo all activities occurred after that activity. Thus, a mistakenly deleted manipulative can easily be restored saving the user time and trouble.

Although lesson contents reflect instantly on the lesson page whenever teachers add or edit contents, teachers can make their own lessons hidden at any time to hide contents from students. Thus, an incomplete lesson will not appear in the “Lessons Page” and a teacher will have the opportunity to finish the lesson at a later time.

Finally, all of the above manipulatives are editable at any time and more than one instance of a manipulative is permitted in the same lesson.

These manipulatives are intended to cover certain mathematical concepts pertaining to certain grade levels and cover good number of modules to be perceived in hands-on approach.

## 4 MLAS SYSTEM ARCHITECTURE

MLAS distinguishes between three types of users: student and general group, teacher, and administrator. To allow easy and open access to MLAS, students and general users do not need login information. Once they access the system, they can go the “students’ page” and start browsing the lessons. Login, however, is needed for a teacher’s account. This allows a teacher to author and deploy dynamic new educational pages. Teachers can keep track of all lessons they authored so they can add, edit, or delete contents any time. The administrator has full control over the site and for instance can add or delete users, lessons, and modules.

MLAS considers manipulative customization as one of its goals. This means that a manipulative can appear to students in different forms and possibly with different inputs. Ultimately, it is the teacher who decides in which form and input an instance of a manipulative would look like. This is usually done when a teacher checks a designated HTML checkbox or radio button. All current manipulatives are HTML/Javascript-based. When a user clicks on a manipulative to appear, a Javascript function is called and will dynamically generate big HTML content to be displayed. This is considered “teacher’s view” and will not be seen by students. When the teacher is done with customization and ready to save his/her work, that will call another Javascript function that will, in most cases, generate minimized HTML content to be saved into the database and thus the manipulative becomes posted so students can see it (student’s view). The teacher’s and student’s views are related but occasionally different. This can be depicted in the following diagram.

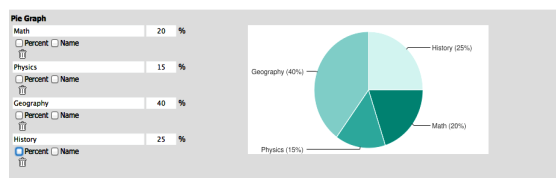


Figure 5: A teacher’s view for the pie graph manipulative after some customization.



Figure 6 below illustrates what the students would see on their screens:

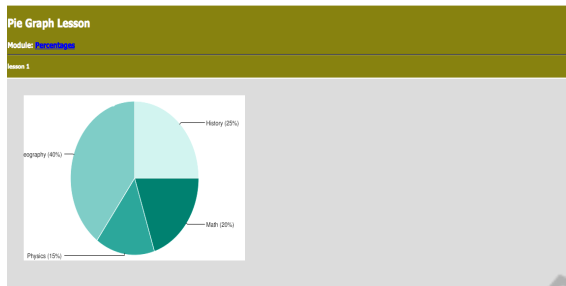


Figure 6: A student's view for the graph in figure 5.

## 5 USER-ADDED MANIPULATIVES

MLAS offers an intuitive Graphical User Interface (GUI) that enables users with HTML and Javascript skills to deploy their own code and add any manipulatives they want to be used by all users.

There are basically little or no limitations using the self-guided interface. Through the available HTML form, users either upload files of HTML, Javascript, and CSS content to the server, or alternatively they can type their codes directly into designated HTML text input fields. Because a manipulative often engages the use of dynamic resources, the interface also allows our users to upload any type of resource they wish (.swf, .class, .jpg, etc.).

\* indicates optional field

Manipulative Name:	<input type="text"/>
Manipulative image: *(if one not uploaded, default one will be created)	<input type="text"/> <input type="button" value="Browse..."/>
CSS file: *	<input type="text"/> <input type="button" value="Browse..."/>
Source one: *	<input type="text"/> <input type="button" value="Browse..."/>
Source two: *	<input type="text"/> <input type="button" value="Browse..."/>
Source three: *	<input type="text"/> <input type="button" value="Browse..."/>
Box 1: HTML Code >> [Teacher's view]	<div style="border: 1px solid gray; height: 40px;"></div> <input type="button" value="Or upload your file: Browse..."/>
Box 2: Javascript Code >> Need to make explicit call to the Javascript "save" function:	<div style="border: 1px solid gray; height: 40px;"></div> <input type="button" value="Or upload your file: Browse..."/>
Box 3: Javascript Code >> Javascript functions needed: *	<div style="border: 1px solid gray; height: 40px;"></div>

Figure 7: The GUI for manipulative embedding.

As appears in figure 7, the user interface has three separate code segments for the user to fill with HTML and/or Javascript. The first box is to be filled with HTML only and can have some Javascript inside any HTML event such as *onmouseover* or *onclick*. This aids in manipulative flexibility and multi-form property. However, this HTML only forms the teacher's view of a manipulative and hence it is not yet ready to be previewed on the lesson page.

The second box should have Javascript code embedded to capture specific behaviour from the HTML in the first box. For instance, if the HTML from the box above had two HTML checkboxes and one is initially checked, like:

```
<input checked="checked" type="checkbox" id="cb1" />First <br /><input type="checkbox" id="cb2" />Second <br />
```

The jQuery statement below can be used in the second box of the GUI and should pop-up the *id* of the selected checkbox.

```
if ($('#cb1').attr('checked')==true) alert('cb1!'); else alert('cb2!');
```

To make the manipulative available for display, a programmer has to make an explicit call to the Javascript function "save", which is defined by MLAS. This allows for a manipulative to be saved in the appropriate database table through some PHP/AJAX implementations. The save function takes three string parameters: The first parameter is the manipulative HTML content to be saved in the database and thus to be previewed in the lesson page. It could be related to the HTML from the first box, but they are not necessarily the same. The second and third string parameters do not affect the functionality and behaviour of a manipulative, but rather needed to describe its overall meaning. The third parameter, in particular, will appear in the progress menu described earlier.

The third box allows the user to define the Javascript functions needed. Very often, HTML events will make explicit calls to Javascript functions which can be defined in this box. To avoid name conflicts, we ask our users to choose unique names for their functions. Since the user might not always need to define his/her own functions, this box can be left unfilled.

Once the user finishes filling out the necessary items in the HTML form and submits it, everything else gets taken care of by MLAS. The resources get saved in a dedicated directory on the server and the user's code gets saved in a secure database. Then

through PHP, we extract that code from the database and encapsulate it, along with other pieces of data, under a couple of Javascript functions.

A typical user cannot distinguish between a user-added and an admin-added manipulative builder. A user-chosen manipulative image and name would appear inside the manipulative builders menu as if they were natively supported by MLAS. MLAS also includes all the necessary code needed to make user-added manipulatives appear and interact seamlessly in their enclosing pages.

Since user's HTML input is allowed, this implies that interactive contents can be inserted to MLAS and will be supported as well. For example Java applets and flash files can be embedded to the system with the use of the appropriate HTML tags. Below is a simple example of how an applet can be embedded to MLAS. In the applet context, only the compiled version of the java program (*.class* extension) needs to be uploaded to the server so the browser can display the applet (assuming the browser has the java plug-in installed).

### 5.1 Example

In this section we present a simple example to show how to embed an applet in MLAS. The applet that we will be showing is very simple. It is a single button labelled as "This button doesn't do anything". For this example, the only resources we need are the applet image and the applet *.class* file. We begin by giving our manipulative a name (say, *Applet Example*), and then we upload the manipulative image and the *.class* file (Assume named, *ExampleApplet.class*) through the user interface.

To embed an applet in a web page, the HTML `<applet>` tag needs to be used, with the *codebase* attribute indicating the directory on the server where the *.class* file exists, and the *code* attribute to denote the name of the *.class* file itself. Other attributes, such as *width* and *height*, might be used to control the size of the applet. Therefore, the first HTML box can be filled with the following HTML segment.

```
<applet codebase="uploaded_files/"
code="ExampleApplet.class" width="400"
height="50"></applet>
```

The GUI clearly indicates that all user uploads go inside "uploaded\_files" directory. Therefore that directory is referenced in the *codebase* attribute above.

In order to properly display the applet in the lesson page, we need to have the above *applet* tag to be the first parameter of the *save* function. Since, in

this case, the teacher's and student's views are similar, nothing else needs to be added to the second box. That can be something like:

```
var applet = '<applet
codebase="uploaded_files/"
code="ExampleApplet.class" width="400"
height="50"></applet>';
save(applet,"Applet","Applet added!");
```

All other fields of the HTML form can be left blank, and the user can now proceed. The applet image will now be available in the manipulative builders menu together with all other builders and a click on that image will show the applet (Figure 8).



Figure 8: User-added applet to MLAS.

## 6 CONCLUSIONS AND FUTURE WORK

As presented earlier, there is a great need in our society to address the issue of creating new and innovative ways of educating children, particularly in math. This paper presents a fully web-oriented system that allows teachers to create and deploy online mathematics lessons through a set of customizable manipulative builders. The authored lessons can be utilized to better engage students and stimulate their way of thinking. Our continuous work seeks to add more manipulative builders to widen the topic areas of instruction. We also intend to add import and export capabilities to our lessons to allow teachers to share lessons with each other.

## REFERENCES

- CITED Research Center, Learning Mathematics with virtual manipulatives, [http://www.cited.org/index.asp?page\\_id=151](http://www.cited.org/index.asp?page_id=151). Accessed Nov 3.
- ETA Cuisenaire, Research on the Benefits of Manipulatives, [http://www.etaquisenaire.com/pdf/Benefits\\_of\\_Manipulatives.pdf](http://www.etaquisenaire.com/pdf/Benefits_of_Manipulatives.pdf). Accessed on Nov. 3.
- Smith, S. S., (2009). Early Childhood Mathematics (4th ed.) Boston: Pearson education using manipulatives (2009). <http://www.teachervision.fen.com/pro-dev/teaching-methods/48934.html>. Retrieved Nov 5.
- Boggan, M., Harper, S. and Whitmire, A., 2010. Using manipulatives to teach elementary mathematics. *Journal of Instructional Pedagogies*, 3(1), 1-10. National Library of Virtual Manipulatives, Virtual Manipulatives, Utah State University <http://>

- nlvm.usu.edu/en/nav/index.html
- National Council of Teachers of Mathematics. 1989. Curriculum and evaluation standards for school mathematics. Reston, VA.
- Superkids Math Worksheet Creator. <http://www.superkids.com/aweb/math/>. Accessed on Sep. 20.
- Math fact cafe. <http://www.mathfactcafe.com/>. Accessed Oct 4.
- Arcademic Builders <http://www.arcademicskillbuilders.com/>. Accessed Nov 6
- E-learning Authoring System, [www.elearningconsulting.com](http://www.elearningconsulting.com). Accessed Oct 16.
- D. Chiu, 2005. Customization and Interoperability in WME. *Proceedings of the IEEE Southeast Conference, IEEE*, pp. 636-640.
- P. Wang, M. Mikusa, S. Al-shomrani, X. Lai, X. Zou, D. Zeller, 2006. WME: a Web-based Mathematics Education System for Teaching and Learning. *ICME 11 – TSG 22 Theme 3 the 11th International Congress on Mathematical Education*.
- P. S. Wang, N. Kajler, Y. Zhou, and X. Zou, 2003. WME: Towards a Web for Mathematics Education. *Proceedings of ISSAC*, ACM Press, pp. 258-265.
- P. S. Wang, M. Mikusa, S. Al-shomrani, D. Chiu, X. Lai, and X. Zou., 2005. Features and Advantages of WME: a Web-based Mathematics Education System, *Proceedings of the IEEE Southeast Conference, IEEE*, pp. 621-629.

