Reconstruction-based Set-valued Observer
A New Perspective for Fault Detection within Uncertain Systems

Letellier Clément, Chafouk Houcine and Hoblos Ghaleb
Institut de Recherche en Systèmes Electroniques Embarqués, Rouen, France

Keywords: Uncertain Systems, Set-valued Observer, Sensor Reconstruction, Fault Detection.

Abstract: This paper presents an extension of a particular type of observer called the Set-valued Observer; this kind of observer is very well suited for uncertain fault detection. But some limitations restrict its use. Indeed, all the sensors are needed to observe the state and as a consequence this method does not allow fault detection when some sensor information is not available. Other work has focused on the well-known Luenberger Observer applied to uncertain systems; but once again, this option is limited. Indeed, it is difficult to converge the algorithm because of the wrapping effect induced by recursivity. Here a new approach is proposed combining the power of the two algorithms. The Luenberger Observer coupled with the Set-Valued Observer allows us to reconstruct the states without divergence. This combination is a substantial contribution for fault detection within uncertain systems.

1 INTRODUCTION

Industrial processes appeared many years ago. They facilitated the improvement of the quality and quantity of production. However, these processes are not infallible. Failures can damage the functional units of the system such as measurement, action and control systems which results in a decrease of productivity.

In order to overcome this problem, monitoring methods have emerged to detect, isolate and identify the faults. These methods are known under the generic name FDI (Fault Detection & Isolation). The functioning depends on sensor feedback information. Accompanied with a model, this information makes it possible to recreate the state and by extension detect the appearance of faults. This state reconstruction is of major importance as it allows us to create virtual sensors which decrease the system's cost or the space requirement. Furthermore, sometimes some of the sensors cannot be implemented because of measurement accessibility.

For decades, these systems have brought substantial advances by estimating the state values and comparing them to the reference values, making it possible to obtain residuals and fault indicators. Many diagnosis methods, such as observers, have been inspired by this approach. Indeed, a traditional way to estimate the state relies on observers such as the Luenberger observer (Luenberger, 1964). An extension of this method—called the Kalman filter (Kalman, 1960)—has been developed to deal with measurement noises. When the latter are white and Gaussian, the Kalman filter provides the state's optimal filter in the sense of minimum variance. Another approach called "parity space" is based on the analytical redundancy of state equations (Chow and Willsky, 1984). The principle is to choose an orthonormal solution cancelling the observability matrix in order to obtain fault-sensitive residuals. Finally, less common methods such as direct filter synthesis exist. They can be found in two forms: those based on H∞ robust estimators (Mangoubi, 1998) and those based on the common synthesis of a dynamic filter and two structure matrices (Henry and Zolghadri, 2005).

In a general manner all these methods are called "model-based" or "analytical". The major problem of models is they do not represent reality accurately. Indeed, for instance, it is well known that resistances in electrical circuits change according to the surrounding temperature. A fault detection method relying on such a model will provide false alarms.

From this observation, conventional methods of diagnosis have been redefined to accommodate the uncertain framework. Different approaches have been used to address this problem.

Clément L., Houcine C. and Ghaleb H..
Reconstruction-based Set-valued Observer - A New Perspective for Fault Detection within Uncertain Systems.
DOI: 10.5220/0003972301110117
In *Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2012), pages 111-117
ISBN: 978-989-8565-21-1

First, the active approach attempts to cancel the uncertainties to overcome their effect at the fault detection step. Residuals are calculated to be insensitive to uncertainties while being sensitive to faults. Several approaches have been developed in this direction in recent years: the unknown input observer, the eigenstructure assignment (Chen and Patton, 1999) and structured parity equations (Gertler, 1998).

Secondly, the passive approach (Puig et al., 2002) is based on the propagation of uncertainties in the estimated values to obtain the enclosures in which all possible trajectories are included. A fault is detected when the measurement goes beyond the enclosure.

Uncertainty modeling is not straightforward. The primary idea was to model uncertainties in a statistical manner using confidence intervals. Later, interval analysis allowed a natural modeling of uncertainties (Jaulin et al., 2001). In the literature, interval analysis can be found under the names "set-membership approach" or "bounding approach".

Taking the uncertainties into account brings a new dimension to the diagnosis but is not without drawbacks.

Based on a recursive computation, observers face the recurrent phenomenon with uncertain systems called the wrapping effect. To prevent this phenomenon— causing the exponential expansion of the bounds of the state—several methods have been developed. Some methods are more suitable than others. Among them, there is the parity space approach using the bounding approach (Ploix and Adrot, 2006). Other methods based on interval observers for fault detection have been presented by Gouzé et al. (Gouzé et al., 2000) and more recently by Raïssi et al. (Raïssi et al., 2010). The idea of this method within uncertain systems is to use two Luenberger-like observers. In this manner, the bounds of the states are computed separately: one observer for the upper value and the other one for the lower value. On the other hand, another approach based on LPV and qLPV models have been developed (Darengosse and Chevrel, 2002). Finally, the last approach is a particular type of observer developed for set-membership systems. This prediction–correction-based observer has been introduced by Shamma et al. (Shamma and Tu, 1995) and more recently, used by Haimovich et al. and Benothman et al. (Haimovich et al., 2004; Benothman et al., 2007). Called "Set-Valued Observer", this observer overlaps two pieces of information: one coming from the model and the other one from the sensor (Letellier et al., 2011).

In this paper, an extension of this observer will be presented in order to bypass some limitations by reconstructing the sensor value when the measurement is not available.

The paper is organized in the following manner. Section 2 introduces the problem statement. Section 3 presents the background material used in the proposed algorithm. The main contribution of the paper is presented in section 4. Section 5 provides a numerical example and the simulation results. Finally, section VI draws the conclusion.

2 PROBLEM STATEMENT

Let us consider an uncertain linear system described by the following discrete-time dynamic equations:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + w_k \end{cases} \quad (1)$$

where $x \in \mathbb{D} \subseteq \mathbb{R}^{n_x}$ is the state vector of the system, w is the measurement noise, $u \in \mathbb{R}^{n_u}$ is the input vector of the system and $y \in \mathbb{R}^{n_y}$ is the output vector of the system. *A, B, C* are respectively the state, the input and the output matrices and are considered uncertain. They are modeled by intervals: $[Z] \Leftrightarrow Z(\theta)$ with $\theta = \left\{\theta \in \mathbb{R}^{n_\theta} \mid \underline{\theta} \le \theta \le \overline{\theta}\right\}$.

In this paper, the Set-Valued Observer is extended in order to reconstruct the state when the measurement is not available.

The conventional Set-Valued Observer is defined as follows:

$$\begin{cases} X_k^p = \left\{Ax_{k-1} + Bu_{k-1} \mid x_{k-1} \in X_{k-1}\right\} \\ X_k^e = \left\{C^{-1}y_k \mid y_k \in Y_k^m\right\} \\ X_k = \left(X_k^p \cap X_k^e\right) \\ Y_k^m = y_k + [w_k] \end{cases} \quad (2)$$

where X_k^p, X_k^e and X_k are respectively the predicted state set, the estimated state set and the corrected state set. The matrices *A, B, C* are bounded within intervals. The w measurement noise is bounded within intervals and added to the y measurement.

This observer has numerous advantages for estimating the state within uncertain systems; the correction step avoids the wrapping effect. The major limitation of this observer is the estimation step where the state is deduced from the sensor.

Indeed, the observation matrix inversion is not always achievable. Moreover, when measurements are not available, the states cannot be deduced from the sensor.

From this observation, we propose an improvement on the conventional Set-Valued Observer. The limitations are bypassed by reconstructing the state from measurements instead of deducing it directly. Section 4 will introduce the proposed method.

3 BACKGROUND MATERIAL

3.1 Interval Tools

The central idea of the interval analysis is to replace real numbers by intervals $[x]=\{x \in \mathbb{R} \mid \underline{x} \leq x \leq \overline{x}\}$; in this manner, calculation algorithms can be used to obtain guaranteed numerical results (Jaulin *et al.*, 2001).

An interval is defined as a connected subset of $\mathbb{R}$ noted $\mathbb{IR}$. For instance: $[1,3]$ and $[-\infty,-2]$ are intervals even though the use of bounded intervals is recommended.

An interval can be defined in two ways: directly by the bounds $[\underline{inf}, \overline{sup}]$ or by the couple (*Midpoint, Radius*).

The operations are redefined: let us consider an operator $\circ \in \{+;-;*;/\}$ and $[a],[b]$ two intervals, then $[a]\circ[b]=\{x\circ y \mid x\in[a], y\in[b]\}$.

The width of an interval $[x]$ is defined by $w[x]=\overline{x}-\underline{x}$, its midpoint by $mid[x]=(\overline{x}+\underline{x})/2$ and its radius by $rad[x]=(\overline{x}-\underline{x})/2$.

3.2 Inclusion Functions

Consider $\mathbf{f}: \mathbb{R}^{n_x} \rightarrow \mathbb{R}^m$. The range of the function $\mathbf{f}$ over an interval vector $[x]$ is given by:

$$\mathbf{f}([x])=\{\mathbf{f}(x) \mid x\in[x]\} \tag{3}$$

An interval function $[\mathbf{f}]: \mathbb{IR}^{n_x} \rightarrow \mathbb{IR}^m$ is an inclusion function of $\mathbf{f}$ if:

$$\forall [x]\in \mathbb{IR}^{n_x}, \mathbf{f}([x]) \subseteq [\mathbf{f}]([x]) \tag{4}$$

where $\mathbf{f}([x])$ denotes the set-theoretical image of $[x]$ by $\mathbf{f}$.

4 OBSERVER DESIGN

In this section, an observer architecture, to some extent analogous to that of the Set-Valued Observer, will be proposed. Actually, this extension combines the power of both the SVO and the well-known Luenberger observer.

The idea here is to use the SVO architecture but instead of deducing the state directly from the sensor, we propose reconstructing the state from the sensor. In this manner, we can implement virtual sensors and we do not have the limitation of the observation matrix inversion. To do this, the Luenberger-like estimation equation is used under observability conditions. As the SVO has a correction step, the estimation equation will not suffer from the wrapping effect due to recursivity.

This method involves three steps as for the conventional SVO:

1) The **Prediction** of the state set according to the model and its uncertainties.
2) The **Estimation** of the state set according to the uncertain measurements available and the model.
3) The **Correction** of the state set by computing the intersection of both previous sets

In the rest of this paper we will call the proposed architecture Set-Valued Luenberger Observer (SVLO) to distinguish it from the conventional SVO.

4.1 Observer's Architecture & Methodology

The architecture of the observer is nearly the same as that of the SVO except for the estimation step. In order to make a correlation with the SVO architecture, the equations will be written in the form of prediction/update as in the Kalman filter. Figure 1 represents the architecture of this observer and equation (5) represents the strategy based on the two observers allowing the estimation of the state in presence of model and sensor uncertainties.

Considering the above description, the Set-Valued Luenberger Observer is defined as follows:

$$\begin{cases} X_k^p=\{Ax_{k-1}+Bu_{k-1} \mid x_{k-1}\in X_{k-1}\} \\ X_k^e=\{x_k^p+L(y_k-Cx_k^p) \mid x_k^p=\text{mid}(X_k^p), y\in Y_k^m\} \\ X_k=(X_k^p\cap X_k^e) \\ Y_k^m=y_k+[w_k] \end{cases} \tag{5}$$

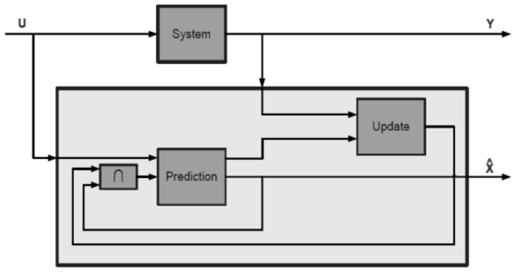

Figure 1: Diagram of the proposed observer architecture.

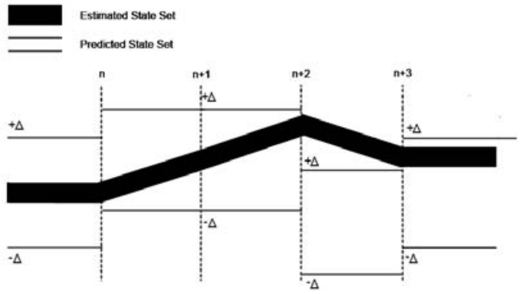

Figure 2: Set representation of the proposed observer.

where X_k^p, X_k^e and X_k are respectively the predicted state set, the estimated state set and the corrected state set. The matrices *A, B, C* are considered uncertain and consequently are bounded within intervals and once again the *w* measurement noise is bounded within intervals and is added to the y measurement. Finally L is the Luenberger gain which is defined as usual with certain systems.

The key point of this observer is the separation of the model uncertainties and the measurement uncertainty as in the conventional SVO; the major difference is the Luenberger-like reconstruction of the state from the sensor.

In order to do this, the optimal value—that is to say the middle value—of the predicted state set is considered in the estimation step. In this manner we obtain, as it is the case in the SVO, a state set considering model uncertainties and a state set considering the sensor uncertainty. The intersection of the two sets of data gives the correct state set.

In order to explain how this observer operates, let us consider a model uncertainty ranging between $\pm\delta$ on all parameters of the state matrix, inducing a $\pm\Delta$ enclosure on the predicted state. Figure 2 gives a discrete-time representation of the method; this diagram shows different cases.

At iteration *n*, the prediction and the estimation are perfectly consistent and the $n+1$ prediction and estimation are computed.

At iteration $n+1$, the prediction and the estimation are again totally consistent; the observation of the state is perfect. The prediction and the estimation continue to iteration n+2.

At iteration n+2, the set of admissible trajectories—the predicted state set—equals the n+1 predicted state set. The estimated state set should equal the n+1 estimated state set. But, the estimated state set deviates as it is no longer centered on the prediction state set. This phenomenon occurs when the parameters of the real system deviate. Indeed, as the system is influenced by its environment, the measurement varies and so does the state estimation.

Fortunately, this case was predicted by taking into account the uncertainties on parameters in the prediction step. As a consequence, both the predicted and estimated state sets are still consistent. The next state is predicted and estimated to iteration $n+3$.

At iteration $n+3$, the estimation deviates totally from the prediction. The prediction and the estimation are not consistent. This case appears when the measurement deviates abnormally, that is to say, out of the range admitted by the prediction—defined according to the model uncertainties. This simulated case corresponds to a fault. Finally, the observation of the state continues on this manner.

To sum up, the prediction propagates model uncertainties on the state and the estimation computes the trajectory of the state from the sensor measurement. If the estimation is consistent with the prediction, the state observation continues. If the estimation deviates beyond the frontiers predicted by model uncertainties, a fault has appeared. This property will be used to set up the fault detection procedure.

As illustrated above, this observer tends to enlarge the SVO strategy (Shamma and Tu, 1995) to systems with missing sensors. Rather than deduce the state directly from the sensor—supposing the observation matrix to be invertible—the sensor is estimated with the Luenberger approach. The convergence of the SVLO is supported by the correction step.

4.2 Interval Observer Convergence

Even though the correction step allows the observer to avoid the wrapping effect, the Luenberger gain L needs to fulfill requirements in order to ensure the convergence of the state. As the system is uncertain, the convergence will be studied around a box and not around a point.

The convergence of the interval observer is studied by considering the total error (Raïssi et al., 2010), that is to say the error between the lower and the upper bounds of the state:

$$We(t) = \left[e(t)\right] = \overline{x}(t) - \underline{x}(t) \qquad (6)$$

If $We(t)$ converges exponentially toward zero, then the lower and the upper trajectories converge toward the current state of the system. The dynamic equation of the total error $We(t)$ is described by:

$$\begin{aligned}\dot{We}(t) &= \left(\overline{A} - L\overline{C}\right)\overline{x}(t) + \left(\overline{B} - L\overline{D}\right)u(t) + L\left(y(t) + \overline{e}\right) \\ &\quad - \left(\underline{A} - L\underline{C}\right)\underline{x}(t) - \left(\underline{B} - L\underline{D}\right)u(t) - L\left(y(t) + \underline{e}\right)\end{aligned} \qquad (7)$$

Considering $\hat{x}(t)$ the midpoint of the set $\left[x(t)\right]$:

$$\hat{x}(t) = mid\left[x(t)\right] = \left(\overline{x}(t) + \underline{x}(t)\right)/2 \qquad (8)$$

The dynamic equation (7) can be expressed as:

$$\dot{We}(t) = \left(mid\left[A\right] - Lmid\left[C\right]\right)We(t) + \lambda_e(t) \qquad (9)$$

with

$$\lambda e(t) = \left(w\left[A\right] - Lw\left[C\right]\right)\hat{x}(t) + w\left[B\right]u(t) \qquad (10)$$

If the gain L is chosen such that $\left(mid\left[A\right] - Lmid\left[C\right]\right)$ is asymptotically stable and that $\lambda e(t)$ is a positive vector Λ_e then the total error converges asymptotically toward:

$$W_{e_{\max}} = -\left(mid\left[A\right] - Lmid\left[C\right]\right)^{-1}\Lambda_e \qquad (11)$$

Consequently, the enclosure converges toward a box $W_{e_{\max}}$. But in order to meet this requirement, $\left(mid\left[A\right] - Lmid\left[C\right]\right)$ needs to be stable. Therefore, the Luenberger gain L is determined as follows:

$$L = \left\{ L \in \mathbb{R}^{n_x \times n_y} \left| \begin{array}{c} \left(mid\left[A\right] - Lmid\left[C\right]\right)\ stable \\ \left(\lambda_e(t)\right)_i \geq 0 \end{array} \right. \right\} \qquad (12)$$

4.3 Fault Detection Algorithm

The SVLO strategy has been defined and it has been demonstrated how the state observation can be implemented with missing sensors.

Here, we will present this observer for a fault detection purpose. Table 1 shows the fault detection algorithm associated with the proposed observer.

The algorithm starts by initializing the state set X_k to enable the beginning of the recursive algorithm. Once the initialization has been done, a loop is generated to compute every state set and detect the presence of faults throughout the simulation time.

Table 1: SVLO Algorithm.

0. $X_k \Leftarrow X_0$

For $k = 1$ to N

1. Compute Y_k^m

2. Compute X_k^p

3. Compute X_k^e

If $X_k^p \cap X_k^e \neq \emptyset$ **then**

4. $X_k \Leftarrow X_k^p \cap X_k^e$

Else

$X_k \Leftarrow \mathrm{mid}\left(X_k^p\right)$

End if

5. Compute $Y_k^p = CX_k^p$

If $Y_k^p \cap Y_k^m = \emptyset$ **then**

6. Fault detected

End if

End for

For every loop, the following steps are repeated: 1) The measurement set is computed according to the measurement itself and the w measurement uncertainty. 2) The predicted state set is computed in function of model uncertainties and the previous corrected state set. 3) The estimated state set is computed in function of measurement uncertainty. 4) If the intersection between the predicted state set and the estimated state set is not empty then the set is considered as valid. The set is assigned to the corrected state set in order to be used at the next iteration. Otherwise, the measurement is not considered and is ignored. The midpoint of the predicted state set is assigned to the corrected state set; this prevents the algorithm from stopping when the intersection is empty. 5) The predicted output set is computed; it represents the image of the predicted state set through the observation matrix. 6) If the intersection between the predicted output set and the measurement set is empty, a fault is detected. Finally, the loop is finished and the next loop can be performed.

5 NUMERICAL EXAMPLE

In order to validate the proposed method, the following numerical example is studied.

Let us consider the linear continuous-time state representation of a mass-spring-damper:

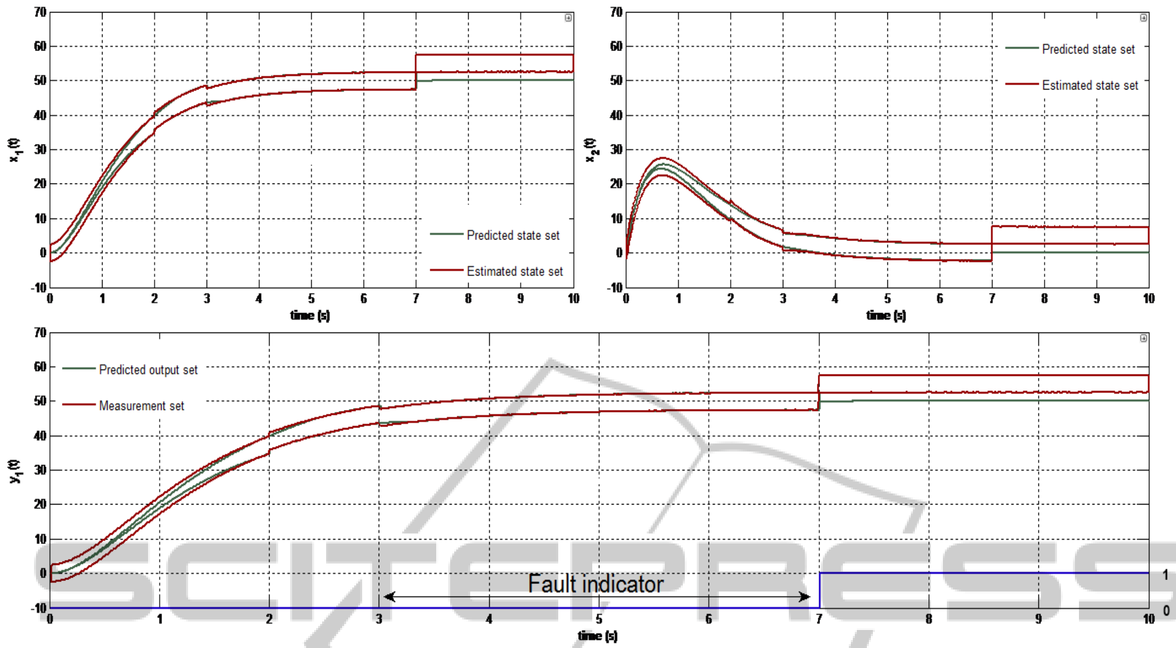

Figure 3: Observation and fault detection results for a mass-spring-damper system.

$$\begin{cases} \dot{x}_1(t) = a_{12} x_2(t) \\ \dot{x}_2(t) = a_{21} x_1(t) + a_{22} x_2(t) + u(t) \\ y(t) = x_1(t) + w(t) \end{cases} \quad (13)$$

where $\mathbf{x} = (x_1 \quad x_2)^T = (p \quad v)^T$ is the state vector representing respectively the position and the velocity of the damper; $\mathbf{u} = F$ is the input value representing the force applied to the damper; $\mathbf{w} = \{w(t) \,|\, |w| \le 0.05\}$ is the $\pm 5\%$ measurement noise. a_{12}, a_{21} *and* a_{22} are parameters of the states, whose values are respectively $1, -2$ *and* -3. These parameters are considered uncertain in what follows.

The model is discretized, the Set-Valued Luenberger Observer is implemented and finally the fault detection is performed.

Considering an academic example, a model uncertainty of $\pm 2\%$ is added to all the state parameters. The measurement uncertainty is supposed to be $\pm 5\%$.

In order to test the effectiveness of the proposed fault detection method, two offsets are added to the measurement to simulate faults. As presented in equation (13), only the position is measured by a sensor. Thus, the faults will be introduced on the position measurement.

The first fault occurs between the 2-3s interval, whereas, the second fault occurs from 7-10s. The amplitude of the fault is around 2% of the maximum value for the first fault and around 10% for the second fault.

Figure 3 depicts the results obtained from the simulations. At the top of the figure, the predicted and estimated state sets for both the position and the velocity states can be seen. It can be noted that the velocity is well estimated—as intended in Luenberger theory—even with missing sensors and model uncertainties.

The fault detection is represented at the bottom of Figure 3. The predicted output set and the measurement set are computed. As previously shown in Table I, if the intersection of both sets is empty then a fault is detected.

If we look closely at the fault detection result, we can report that the second fault is perfectly detected between the 7-10s interval. But the first fault is not detected from 2-3s. Indeed, the 2% bias fault is very low compared to the 5% uncertainty admitted on the sensor measurement. Therefore, the behavior of the fault detection is totally in accordance with what was expected. Indeed, a low bias fault is included within the uncertainty enclosures and thus not considered as a fault. This is why it is important at the design stage to take into account reasonable uncertainties to detect reasonable faults.

The SVLO gives the expected results. Its real benefits compared to the traditional SVO are the use of analytical redundancy to reconstruct the states. Another interest of this approach is, this observer does not require a matrix inversion, what allows us to use it in a broader context than the mere SVO.

6 CONCLUSIONS

In this paper, the Set-Valued Observer has been studied. An extension of the Set-Valued Observer has been proposed in order to reconstruct the state when sensors are missing. The idea is to bring together two methods developed in different contexts in order to make them work in synergy. With uncertain systems, the implementation of observer is difficult because of the wrapping effect. This is where the Set-Valued Observer is interesting; it can avoid this phenomenon. But the SVO is not without drawbacks; the deduction principle of the state implies that all measurements are available which is not always true.

From this observation, the use of a Luenberger-like reconstruction of the state within the SVO seems to be a good solution. The computation of the predicted state with model uncertainties makes it possible to determine the set of all possible trajectories. Then, the computation of the estimated state with the measurement uncertainty allows the algorithm to determine trajectories consistent with the measurement. The intersection of the two sets corrects the state set throughout the simulation.

Through the numerical example of the mass-spring-damper, results have demonstrated that the state in presence of model uncertainties can easily be reconstructed. Moreover, the fault detection algorithm based on the proposed observer has demonstrated its efficacy; the observer yields the expected results.

The Set-Valued Luenberger Observer gives encouraging results and brings new perspectives to the field of uncertain systems. A real-time implementation of the observer is planned. The Luenberger-like reconstruction of the state will permit future work to extend fault detection to fault isolation by implementing this observer in the form of benches.

REFERENCES

Benothman, K., Maquin, D., Ragot, J., Benrejeb, M., others, 2007. Diagnosis of uncertain linear systems: an interval approach. *International Journal of Sciences and Techniques of Automatic control & computer engineering 1*, 136–154.

Chen, J., Patton, R. J., 1999*. Robust model-based fault diagnosis for dynamic systems.*

Chow, E., Willsky, A., 1984. Analytical redundancy and the design of robust failure detection systems. Automatic Control, *IEEE Transactions on* 29, 603–614.

Darengosse, C., Chevrel, P., 2002. Expérimentation d'un observateur H∞ LPV pour la machine asynchrone. *Journal européen des systèmes automatisées* 36, 641–655.

Gertler, J., 1998. *Fault Detection and Diagnosis in Engineering Systems*, 1st ed. CRC Press.

Gouzé, J., Rapaport, A., Hadj-Sadok, M., 2000. Interval observers for uncertain biological systems. *Ecological modelling* 133, 45–56.

Haimovich, H., Goodwin, G.C., Welsh, J.S., 2004. Set-valued observers for constrained state estimation of discrete-time systems with quantized measurements, in: *Control Conference,* 2004. 5th Asian. pp. 1937–1945.

Henry, D., Zolghadri, A., 2005. Design and analysis of robust residual generators for systems under feedback control. *Automatica* 41, 251–264.

Jaulin, L., Kieffer, M., Didrit, O., Walter, E., 2001. Applied Interval Analysis, 1st ed. Springer.

Kalman, R. E., 1960. A new approach to linear filtering and prediction problems. *Journal of basic Engineering 82*, 35–45.

Letellier, C., Hoblos, G., Chafouk, H., 2011. Robust Fault Detection based on Multimodel and Interval Approach. Application to a Throttle Valve, in: *IEEE Med*. Corfu, Greece

Luenberger, D. G., 1964. Observing the state of a linear system. Military Electronics, *IEEE Transactions on 8*, 74–80.

Mangoubi, R. S., 1998. Robust estimation and failure detection: A concise treatment. Springer-Verlag NY, Inc.

Ploix, S., Adrot, O., 2006. Parity relations for linear uncertain dynamic systems. *Automatica* 42, 1553–1562.

Puig, V., Quevedo, J., Escobet, T., De las Heras, S., 2002. Passive robust fault detection approaches using interval models, in: P*roceeding of the 15th IFAC World Congress*, Barcelona.

Raïssi, T., Videau, G., Zolghadri, A., 2010. Interval observer design for consistency checks of nonlinear continuous-time systems. *Automatica* 46, 518–527.

Shamma, J. S., Tu, E., 1995. Optimality of set-valued observers for linear systems, in: Decision and Control, 1995., *Proceedings of the 34th IEEE Conference On.* pp. 2081–2086.