Developing A New Variables Sampling Scheme for Product Acceptance Determination

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Abstract: Acceptance sampling is a useful tool for determining whether submitted lots should be accepted or rejected. With the current increase in outsourcing production processes and the high quality levels required, it is very desirable to have an efficient and economic sampling scheme. This paper develops a variables repetitive group sampling (RGS) plan based on the third generation of process capability index. The plan parameters are determined by minimizing the average sample number (ASN) for inspection and fulfilling the classical two-point-condition on the operating characteristic (OC) curve. Besides, the efficiency of the proposed plan is investigated and compared with the existing variables single sampling plan. Tables of the plan parameters are also provided.

1 INTRODUCTION

Acceptance sampling is one of the most practical tools in classical quality control and assurance applications, which deal with quality contracts for product orders between factories and their customers. Acceptance sampling plans provide the producer and the consumer with a general criterion for lot sentencing. A well-designed sampling plan can substantially reduce the difference between the required and the actual supplied product quality (Pearn and Wu, 2006; Pearn and Wu, 2007). Unfortunately, it cannot avoid the risk of accepting unwanted poor product lots, nor can it avoid the risk of rejecting good product lots without implementing 100% inspection (e.g., Montgomery, D. C., 2009). The criteria used to measure the performance in an acceptance sampling plan are usually based on the operating characteristic (OC) curve, which quantifies the risks of producers and consumers. The OC curve plots the probability of accepting a lot against the actual quality level of the submitted lots. In other words, the OC curve shows the discriminatory power of the sampling plan, which provides the producer and the buyer with a common base for judging whether the sampling plan is appropriate.

Sherman (1965) developed a new type of sampling plan, called the repetitive group sampling (RGS) plan, for attributes. The operating procedure of this RGS plan is similar to that of the sequential sampling plan. Balamurali and Jun (2006) extended the RGS concept to variables inspection for a normally distributed quality characteristic. They also compared the efficiency of the variables RGS plan with the variables single and double sampling plans. These results indicate that the variables RGS plan give the desired protection with the minimum average sample number (ASN).

It is highly desirable to have an efficient and economic acceptance sampling scheme, especially when the required quality level is very high. Therefore, the main purpose of this paper is to develop a new variables sampling scheme for product acceptance determination.

2 PROCESS CAPABILITY INDICES

Process capability indices (PCIs), including C_p, C_{pk}, C_{pm} and C_{pmk}, are convenient and powerful tools for measuring process performance from different perspectives. These indices establish the relationship between actual performance and the specification limits, and convey critical information regarding whether a process is capable of reproducing items satisfying customer requirements. For thorough

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discussions on PCIs and the reviews for the development of PCIs, refer to Kotz and Lovelace (1998), Kotz and Johnson (2002), and Wu, Pearn and Kotz (2009). In addition, Yum and Kim (2011) summarized the related literature of process capability analysis from 2000-2009.

In particular, the C_{pmk} index is appropriate for capability measure due to high standard and stringent requirement on product quality and reliability.

For a normally distributed process that is demonstrably stable (under statistical control), Pearn et al. (1992) suggested using the following estimator:

$$\hat{C}_{pmk} = \min\left\{ \frac{USL-\bar{X}}{3\sqrt{S_n^2+(\bar{X}-T)^2}}, \frac{\bar{X}-LSL}{3\sqrt{S_n^2+(\bar{X}-T)^2}} \right\} = \frac{d-|\bar{X}-M|}{3\sqrt{S_n^2+(\bar{X}-T)^2}},$$

where USL and LSL are the upper and lower specification limits, T is the target value, $d=(USL-LSL)/2$ is the half-length of the specification interval and $M=(USL+LSL)/2$ is the midpoint of the specification limits, $\bar{X}=\sum_{i=1}^{n} X_i/n$ and $S_n^2=\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2/n$ are the maximum likelihood estimators of μ and σ^2, respectively. Note that $S_n^2+(\bar{X}-T)^2 = \sum_{i=1}^{n}(X_i-T)^2/n$ in the denominator of $\hat{C}_{pmk}$ is the uniformly minimum variance unbiased estimator (UMVUE) of $\sigma^2+(\mu-T)^2=E[(X-T)^2]$, which appears in the denominator of C_{pmk} (Pearn, Kotz and Johnson (1992) and Pearn and Lin (2002)).

Wright (1998) developed an explicit but rather complicated expression for the probability density function (PDF) of $\hat{C}_{pmk}$. More recently, Pearn and Lin (2002) rewrote the cumulative distribution function (CDF) of $\hat{C}_{pmk}$ by taking variables transformation and the integration techniques similar to that presented in Vännman (1997). The CDF can be expressed as

$$F_{\hat{C}_{pmk}}(y) = 1-\int_0^{b\sqrt{n}/(1+3y)} G\left(\frac{(b\sqrt{n}-w)^2}{9y^2}-w^2\right) \times\left[\phi(w+\xi\sqrt{n})+\phi(w-\xi\sqrt{n})\right]\mathrm{d}w,$$

for $y>0$, where $b=d/\sigma$, $\xi=(\mu-T)/\sigma$, $G(\cdot)$ is the CDF of the chi-square distribution with degrees of freedom $n-1$, and $\phi(\cdot)$ is the PDF of the standard normal distribution.

3 DEVELOPING A NEW VARIABLES RGS SAMPLING SCHEME

If the quality characteristic of interest follows a normal distribution and has two-sided specification limits (*LSL* and *USL*). It is common to use the AQL (acceptable quality level) and LQL (limiting quality level) points on the OC curve to designing an acceptance sampling plan. This implies that the probability of acceptance must be greater than $1-\alpha$ if the quality level of the submitted lot is at $C_{pmk}=C_{\mathrm{AQL}}$ (in high quality). The probability of acceptance is no more than β if the quality level of the submitted lot is only at $C_{pmk}=C_{\mathrm{LQL}}$ (in low quality), where α and β are commonly called the producer's risk and the consumer's risk, respectively.

The operating procedure of the proposed variables RGS plan based on the C_{pmk} index can be stated as follows.

Step 1. Decide the capability requirements and the risks for the consumer and the producer (i.e., determine the values of C_{AQL}, C_{LQL}, α, and β).

Step 2. Take a random sample of size n from the lot, and calculate the estimated C_{pmk} value, $\hat{C}_{pmk}$, based on these inspected samples.

Step 3. Make a decision based on the following rules.

(i) Accept the entire lot if $\hat{C}_{pmk}$ is greater than the critical value for acceptance k_a.

(ii) Reject the entire lot if $\hat{C}_{pmk}$ is smaller than the critical value for rejection k_r.

(iii) Otherwise, we do not have sufficient information to determine if the submitted lot meets the present capability requirement. In this case, we should take a new sample for further judgment (i.e., repeat Step 2).

The definition of the C_{pmk} index can be rewritten as $C_{pmk}=(d/\sigma-|\xi|)/[3(1+\xi^2)^{1/2}]$, where $\xi=(\mu-T)/\sigma$. Further, given $C_{pmk}=C$, $b=d/\sigma$ can be rewritten as $b=3C(1+\xi^2)^{1/2}+|\xi|$. The probability of accepting the lot based on the C_{pmk} index can be expressed as

$$P_a(C_{pmk}) = P(\hat{C}_{pmk} \geq k_a)$$
$$= \int_0^{b\sqrt{n}/(1+3k_a)} G\left(\frac{(b\sqrt{n}-w)^2}{9(k_a)^2} - w^2\right)\left[\phi(w+\xi\sqrt{n}) + \phi(w-\xi\sqrt{n})\right]\mathrm{d}w.$$

Similarly, the probability of rejecting the lot based on the C_{pmk} index, $P_r(C_{pmk})$, can be expressed as

$$P_r(C_{pmk}) = P(\hat{C}_{pmk} < k_r)$$
$$= 1 - \int_0^{b\sqrt{n}/(1+3k_r)} G\left(\frac{(b\sqrt{n}-w)^2}{9(k_r)^2} - w^2\right)\left[\phi(w+\xi\sqrt{n}) + \phi(w-\xi\sqrt{n})\right]\mathrm{d}w.$$

So, the OC function of the designed variables RGS plan based on the C_{pmk} index, $\pi_A(C_{pmk})$, can be obtained as

$$\pi_A(C_{pmk}) = \frac{P_a(C_{pmk})}{P_a(C_{pmk}) + P_r(C_{pmk})}.$$

As noted before, the parameters of the designed variables RGS plan should simultaneously satisfy the following two conditions specified by the producer and the consumer:

$$\pi_A(C_{\mathrm{AQL}}) = \frac{P_a(C_{\mathrm{AQL}})}{P_a(C_{\mathrm{AQL}}) + P_r(C_{\mathrm{AQL}})} \geq 1-\alpha$$

and

$$\pi_A(C_{\mathrm{LQL}}) = \frac{P_a(C_{\mathrm{LQL}})}{P_a(C_{\mathrm{LTPD}}) + P_r(C_{\mathrm{LQL}})} \leq \beta,$$

where C_{AQL} and C_{LQL} denote the quality levels of AQL and LQL based on the C_{pmk} index, respectively.

Three plan parameters (n, k_a, k_r) must be determined for the designed variables RGS plan. There may be several combinations of the plan parameters that satisfy the above two equations.

The ASN for the proposed variables RGS plan can be calculated by

$$\mathrm{ASN}(C_{pmk}) = \frac{n}{P_a(C_{pmk}) + P_r(C_{pmk})}.$$

It is usual to determine the plan parameters by minimizing the ASN evaluated at AQL or LQL. Therefore, the plan parameters (n, k_a, k_r) of the proposed VRGS plan based on the C_{pmk} index could be determined simultaneously by solving the following optimization problem while the ASN is the objective function.

$$\text{Min } \mathrm{ASN}(C_{\mathrm{AQL}})$$

subject to

$$\pi_A(C_{\mathrm{AQL}}) \geq 1-\alpha,$$
$$\pi_A(C_{\mathrm{LQL}}) \leq \beta,$$
$$C_{\mathrm{AQL}} > C_{\mathrm{LQL}},\ k_a \geq k_r \geq 0,$$

where $b_A = 3C_{\mathrm{AQL}}(1+\xi^2)^{1/2} + |\xi|$ and $b_L = 3C_{\mathrm{LQL}}(1+\xi^2)^{1/2} + |\xi|$. If $k_a = k_r$, the developed variables RGS plan will reduce to the existing variables single sampling plan based on the C_{pmk} index by Wu and Pearn (2008).

4 DETERMINATION OF PLAN PARAMETERS AND DISCUSSIONS

Given the producer's α-risk, the consumer's β-risk and two benchmarking quality levels $(C_{\mathrm{AQL}}, C_{\mathrm{LQL}})$, the plan parameters ($n$, k_a, k_r), the corresponding ASN value of the proposed variables RGS plan can be obtained by solving the above optimization model.

Tables 1-2 summarize the plan parameters (n, k_a, k_r) and the corresponding ASN value under various α-risks and β-risks = 0.01, 0.05 and 0.10, with several selected values of $(C_{\mathrm{AQL}}, C_{\mathrm{LQL}})$ = (1.33, 1.00) and (1.50, 1.00), respectively.

Based on the given tables, the practitioner can know how large a sample size is required for inspection and the associated critical values for acceptance and rejection (k_a, k_r). For instance, if the benchmarking quality levels $(C_{\mathrm{AQL}}, C_{\mathrm{LQL}})$ are set to (1.33, 1.00) with (α, β) = (0.05, 0.10) then the plan parameters will be (n, k_a, k_r) = (34, 1.297, 1.031). This implies that the lot will be accepted if the 34 inspected product items yield measurements with $\hat{C}_{pmk} >$ 1.297, and the lot will be rejected if $\hat{C}_{pmk} <$ 1.031. Otherwise, a new sample must be taken for further judgment.

Table 1: The values of n, k_a, k_r, and the corresponding ASN for various α and β with quality levels $(C_{\text{AQL}}, C_{\text{LQL}}) = (1.33, 1.00)$.

α	β	n	k_a	k_r	ASN
0.010	0.010	87	1.278	1.074	124.4
	0.050	56	1.275	1.021	82.1
	0.100	44	1.271	0.989	65.2
0.050	0.010	78	1.289	1.258	114.2
	0.050	45	1.295	1.068	74.2
	0.100	34	1.297	1.031	52.8
0.100	0.010	74	1.295	1.159	106.3
	0.050	40	1.311	1.099	62.8
	0.100	30	1.319	1.059	45.6

Table 2: The values of n, k_a, k_r, and the corresponding ASN for various α and β with quality levels $(C_{\text{AQL}}, C_{\text{LQL}}) = (1.50, 1.00)$.

α	β	n	k_a	k_r	ASN
0.010	0.010	44	1.418	1.127	62.1
	0.050	28	1.416	1.050	40.5
	0.100	22	1.413	1.003	31.8
0.050	0.010	41	1.433	1.203	57.5
	0.050	23	1.447	1.120	35.0
	0.100	17	1.455	1.066	26.0
0.100	0.010	39	1.441	1.252	53.7
	0.050	21	1.470	1.166	31.5
	0.100	15	1.486	1.096	22.6

Figure 1 displays OC curves of the variables single sampling plan and the variables RGS plan with $n = 100$. It can be seen that the OC curve for the proposed variables RGS plan is more discriminating than the variables single sampling plan. This is because a greater slope in the OC curve represents greater discriminatory power. It provides a better OC curve than the variables single sampling plan at good quality levels and protects against the consumer point of view at poor quality levels.

This implies that the same OC curve can be achieved by the proposed variables RGS plan with smaller sample size than required by the existing variables single sampling plan. Thus, the proposed variables RGS plan is economically superior to the variables single sampling plan in terms of sample size required for inspection. Thus, the proposed plan will give the desired protection with minimum inspection, and reduce the cost of inspection greatly.

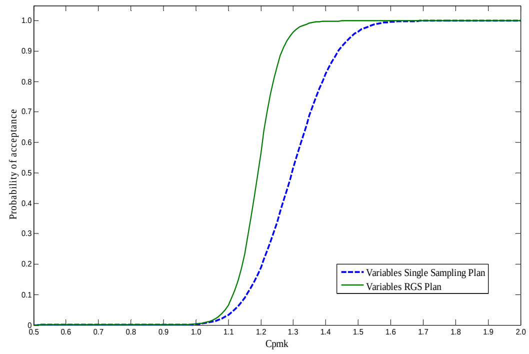

Figure 1: OC curves of a variables single sampling plan and a variables RGS plan with $n = 100$.

5 CONCLUSIONS

This paper develops a variables RGS plan based on the C_{pmk} index. The OC curve of the proposed variables RGS plan is based on an exact sampling distribution rather than approximation. The sample size required for inspection and the corresponding acceptance and rejection criteria are determined by minimizing the ASN such that two critical constraints required by the producer and the consumer can be satisfied. This paper also compares the efficiency of the proposed variables RGS plan with the existing variables single sampling plan in terms of the ASN required for inspection. Results indicate that the proposed variables RGS plan requires less sampling for product acceptance determination than the variables single sampling plan under the same conditions. It would be useful when inspection or testing of the product quality characteristic is costly or destructive.

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