Sphere Decoding Complexity Reduction using an Adaptive SD-OSIC Algorithm

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Abstract: Sphere decoding is a technique able to achieve the optimal performance of the maximum likelihood decoder, but its high and variable complexity can make the practical implementation infeasible. In this paper, we present an adaptive system, called adaptive SD-OSIC, as a way of reducing the decoding complexity while maintaining the error performance of conventional sphere decoding.

1 INTRODUCTION

OPTIMAL maximum likelihood (ML) decoding measures the distance from the received vector y to all possible codewords in the lattice, but if we use a multiple-input multiple-output (MIMO) system with N_t transmit antennas, and modulation of 2^bconstellation points, where b is the number of bits per symbol, the number of possibilities for vector x that should be tested becomes 2^{bN_t}. In 1999, Viterbo and Boutros (Viterbos and Boutros, 1999) proposed a universal lattice decoding technique, now known as sphere decoding (SD), which achieves the error performance of ML, but simplifies the search by restricting it to codewords that lie inside a sphere centered at the received signal vector. SD algorithm is less complex than the conventional ML, but the number of operations that must be performed varies with the SNR and channel conditions; therefore, suboptimal decoding techniques, such as zero forcing (ZF), minimum mean-square error (MMSE), and ordered successive interference cancelation (OSIC) are usually preferred as they are easier to implement in hardware.

Among the recent attempts to reduce SD complexity are hybrid SD-ZF (Lee and Kim, 2006), K-best (Viterbos and Boutros, 1999) and fixed SD (FSD) (Guo and Nilsson, 2006); (Barbero and Thompson); the last two techniques achieve a constant number of iterations independent of the SNR or channel conditions, but exhibit a tradeoff between the bit error rate (BER) performance and computational cost; while the former one proposes reducing the search operations performed by SD by decoding the symbols with high SNR using ZF.

In this paper, we propose to reduce the number of iterations needed by conventional SD and improve the concept presented in (Lee and Kim, 2006) by combining SD, OSIC, and the principle of link adaptation. The resulting system achieves a very low, quasi-constant complexity over the entire SNR range, exhibits better error performance than SD and OSIC at a low SNR, and makes a minimum sacrifice of BER at a high SNR.

This paper is divided as follows: Sections II and III explain the SD algorithm and the Adaptive SD-OSIC system, respectively; Section IV presents and discusses the simulation results; and, in Section V, we present our conclusions.

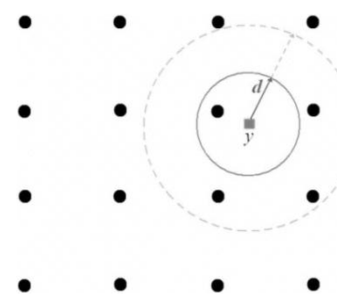

Figure 1: Graphic representation of the SD concept.

2 SD ALGORITHM

In a MIMO system with N_t transmit antennas and N_r

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receive antennas, the $N_r \times 1$ received signal vector can be expressed using equation (1):

$$y = Hx + n \quad (1)$$

where H is the $N_r \times N_t$ channel matrix, x represents the $N_t \times 1$ transmitted vector, and n represents the $N_r \times 1$ noise matrix.

To perform SD, the complex matrices and vectors are decomposed into real values:

$$s = [Re\{x\} \quad Im\{x\}]^T \quad (2)$$

$$\mathrm{M} = \begin{bmatrix} Re\{H\} & -Im\{H\} \\ Im\{H\} & Re\{H\} \end{bmatrix} \quad (3)$$

$$v = [Re\{n\} \quad Im\{n\}]^T \quad (4)$$

$$r = [Re\{y\} \quad Im\{y\}]^T \quad (5)$$

When searching for a candidate to be the transmitted vector, SD selects the codeword that minimizes the metric (6).

$$\|y - Hx\|^2 \quad (6)$$

The SD algorithm requires applying (6) only over the codewords or lattice points that are inside a sphere of initial radius $d = \sqrt{C}$, centered at the received signal vector y. Each time a valid point is found, the radius of the sphere is reduced further until there is only one lattice point inside the sphere. After the real decomposition, equation (6) can be expressed as:

$$\|r - Ms\|^2 \quad (7)$$

To perform the SD algorithm, it is necessary to estimate the values of the received signal vector by applying a simple decoding technique such as ZF or MMSE. In our research, we decided to work exclusively with MMSE, since it offers the same BER performance as ZF, with an additional advantage in complexity (Zimmermann et al.). The MMSE estimation is given by equation (8):

$$\rho = Gr \quad (8)$$

where

$$G = [M^T M + \sigma^2 I]^{-1} M^T$$

Now, equation (7) can be expressed in terms of the estimated vector ρ:

$$\|r - Ms\|^2 = \|M(s-\rho)\|^2 + \|r\|^2 - \|M\rho\|^2 \quad (9)$$

Based on (9), we deduce that one necessary condition for any codeword or lattice point to lie inside a sphere of radius d is:

$$\mathrm{d}^2 > \|M(s-\rho)\|^2 > (s-\rho)^T M^T M (s-\rho) \quad (10)$$

If we apply the Cholesky factorization to the matrix $M^T M$, we obtain the $2\mathrm{N_t} \times 2\mathrm{N_t}$ upper triangular matrix R; and then, we can express equation (10) as:

$$d^2 > (s-\rho)^T R^T R (s-\rho) > \sum_{(i=1)}^{m} R^2_{(m,m)} \left[(s_i - \rho_i) + \sum_{(j=i+1)}^{m} \frac{R_{(i,j)}}{R_{(i,i)}} (s_j - \rho_j) \right]^2 > R^2_{(m,m)} (s_m - \rho_m)^2 + R^2_{(m-1,m-1)} \left[s_{(m-1)} - \rho_{(m-1)} + \frac{R_{(m-1,m)}}{R_{(m-1,m-1)}} (s_m - \rho_m) \right]^2 + \cdots$$

where $m = 2N_t$.

In SD we search for the most accurate value of each of the m components of vector s. The algorithm starts searching the points in descending order, from $i = m$ to $i = 1$. The number of possible values of s_i is determined by the calculation of the lower and upper bounds:

$$\left\lceil u_i - \sqrt{\frac{d_i^2}{R^2_{(i,i)}}} \right\rceil \le \hat{s}_i \le \left\lfloor u_i + \sqrt{\frac{d_i^2}{R^2_{(i,i)}}} \right\rfloor \quad (11)$$

where ⌈ ⌉ rounds to the largest possible integer and ⌊ ⌋ rounds to the smallest possible integer.

For $\mathrm{i} = m$ the radius d_i is set to the initial value $\sqrt{C}$ and we center the search around the m^{th} MMSE estimated value ρ_m; therefore, we set $u_i = \rho_m$. For the other values of i, the radius d_i and u_i are calculated with the equations:

$$d^2_{(i-1)} > d_i^2 - R^2_{(i,i)} (s_i - u_i)^2 \quad (12)$$

$$u_{(i-1)} = \rho_{(i-1)} - \sum_{j=1}^{m} \frac{R_{(i-1,j)}}{R_{(i-1,i-1)}} (s_j - \rho_j) \quad (13)$$

After calculating the upper and lower bounds, the options for $\hat{s}_i$ are sorted in ascending order, according to the Euclidean distance to u_i, which increases the possibility of finding the accurate answer during the first iterations (Chan and Lee, 2002); (Noording et al.).

When $i = 1$, we estimate a new radius,

$$\hat{d}^2 = d_m^2 - d_1^2 + R_{1,1} (s_1 - u_1)^2 \quad (14)$$

If this new radius is smaller than the previous initial radius, i.e., if it fulfills the condition $\hat{d}_m < d_m$, then, the estimated vector $\hat{s}$ is stored and becomes a strong candidate for possessing the actual transmitted values. If the condition above is not satisfied, the algorithm will try other options for the members of $\hat{s}$, starting with $\hat{s}_1$, until there are no more points to evaluate inside the sphere. Then, it will output the last stored vector $\hat{s}$.

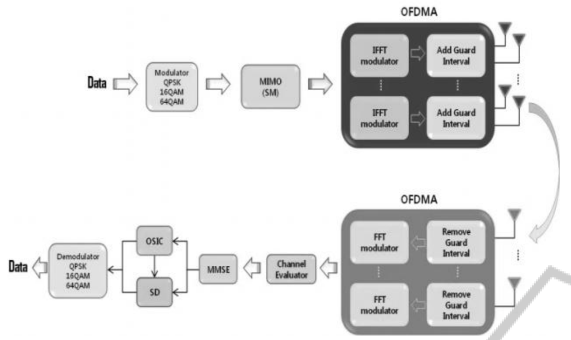

Figure 2: Architecture of the adaptive SD-OSIC system.

3 ADAPTIVE SD-OSIC SYSTEM

The system we propose aims to improve the complexity of SD by combining it with OSIC and the link adaptation principle. The hybrid SD-ZF system proposed in (Lee and Kim, 2006) uses a simple noise estimation based on the ZF pre-decoded symbols. In our case, we use the OSIC algorithm to estimate noise considering the channel interference and canceling the interference from the decoded symbols, yielding better results in terms of BER. Below, we show the OSIC algorithm, where the components of the received vector with the best channel conditions are decoded with priority and subtracted from the vector *y* until all symbols are decoded.

In a low SNR region, where the SD performs the highest number of iterations, our system uses OSIC to decode the symbols influenced by a favorable channel response and decodes the rest using the modified SD algorithm based on the work of Chan & Lee (Chan and Lee, 2002). The nature of OSIC guarantees an enhancement in the BER performance while reducing complexity. On the other hand, as the SNR increases, the number of computations needed by SD decreases and tends to be constant. Therefore, a combination of SD and OSIC for a very high SNR would degrade the BER performance of conventional SD and would not offer a great advantage in complexity. Thus, we decided to use the link adaptation principle, which allows us to combine SD and OSIC only for low and middle SNR, and apply SD exclusively for high SNR, which yields very low, quasi-constant complexity over the entire SNR range.

OSIC algorithm.

$$INPUT : y, H, \sigma$$

$$\downarrow$$

$$G = [H^H H + \sigma^2 I]^{-1} H^H$$

$$\downarrow$$

for $i = 1 : N_t$

$k_i = arg\,min\|(G_i)_{N_t}\|^2$

$\rho_{k_i} = G_i(k_i, :)y$

$\hat{s}_{k_i} = Q(\rho_{k_i}) \rightarrow (symb.decision)$

$y = y - \hat{s}_{k_i} H(:, k_i)$

$H(:, k_i) = 0$

$G = [H^H H + \sigma^2 I]^{-1} H^H$

end

4 SIMULATION RESULTS

In this section, we analyze the performance of MATLAB simulations based on the parameters shown in Table 1. The performance is measured in terms of BER and elapsed time (complexity). Prior to MATLAB version 6, the function 'flops' was used to count the number of floating point operations executed by an algorithm. The latest versions of MATLAB, we use the commands ‘tic’ and ‘toc’ to measure the complexity of an algorithm. These commands measure the time it takes the MATLAB software to execute one or more lines of MATLAB code.

Figure 3 and 4 shows the BER and complexity performance of hybrid SD-OSIC vs. hybrid SD-ZF systems, respectively; the number in parenthesis indicates the number of symbols decoded by OSIC or ZF. Because hybrid SD-OSIC uses MMSE as pre-decoding technique and no matrix decomposition is required for OSIC, the complexity performance between hybrid SD-OSIC and hybrid SD-ZF is very similar; however, as explained in section 3, better results in error performance are obtained with hybrid SD-OSIC.

Table 1: Simulation parameters.

MIMO technique	SM
No. of antennas	4x4
Modulation	16 QAM
Coding rate	none
Pre-detection	MMSE
Detection	SD and OSIC

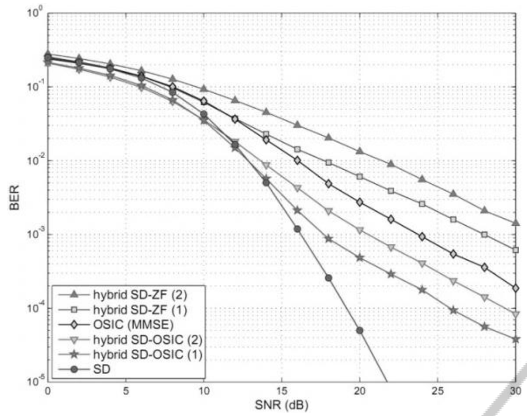

Figure 3: BER performance of hybrid SD-OSIC vs. SD, OSIC, and hybrid SD-ZF.

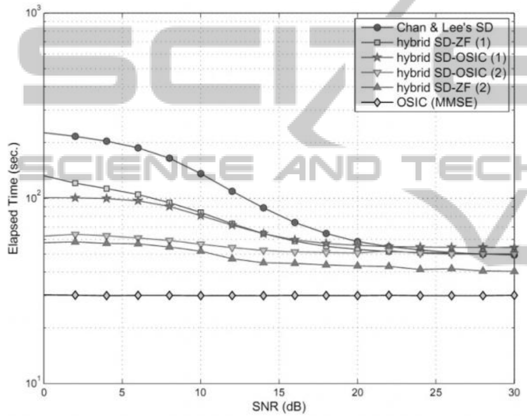

Figure 4: Complexity performance of hybrid SD-OSIC vs. SD, OSIC, and hybrid SD-ZF.

In Figure 3, we can also observe the BER performance of conventional SD (the original and Chan & Lee's SD produce exactly the same BER), vs. the BER performance of hybrid SD-OSIC. At a low SNR, the hybrid systems exhibit better BER performance than SD and OSIC. The reason for this is that in SD, the last levels of the tree search have more points to be compared with, this make those levels less susceptible to noise. In the conventional SD algorithm the elements of the received vector are not ordered according to noise or channel condition; hence, the elements with more noise can be found at the beginning or at the end of the "tree branches". But by using OSIC, we guarantee that the elements that are most affected by noise are decoded with SD; this can be similar to moving them forward in the tree levels. In addition, because OSIC is also a very efficient algorithm, the symbols decoded by OSIC contribute to enhance the performance at low SNR. When SNR reaches values higher than 12 or 14 dB using hybrids SD-OSIC (2) and (1), respectively, the hybrid scheme exhibits BER performance inferior to that of conventional SD. At high SNR, hybrids SD-OSIC (1) and (2) do not achieve the BER performance of ML, but they are approximately 3 and 6 dB better than conventional OSIC (MMSE), respectively.

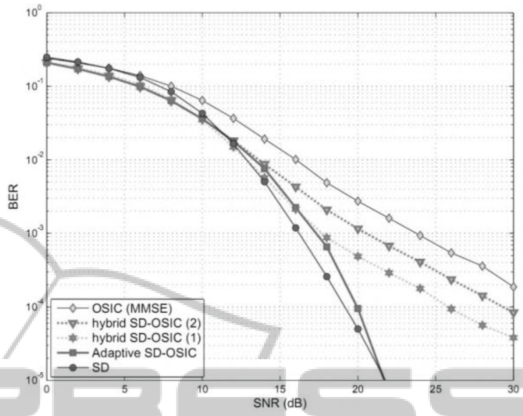

Figure 5: BER performance of the adaptive SD-OSIC system vs. SD, OSIC, and hybrid SD-OSIC schemes.

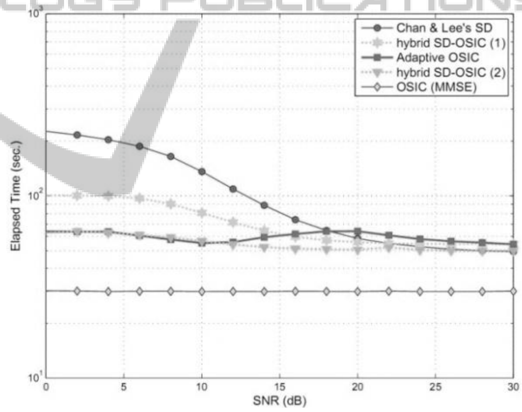

Figure 6: Complexity performance of the adaptive SD-OSIC system vs. SD, OSIC, and hybrid SD-OSIC schemes.

In Figure 4, we can verify that for hybrid SD-OSIC (2) the number of iterations is significantly reduced in the low SNR region. Hybrid SD-OSIC (1) shows a slightly higher complexity compared to hybrid SD-OSIC (2); however, the performance is better if we compare it to Chan & Lee's SD. We also observe that for very good SNR the number of iterations needed by SD and the hybrid algorithms become constant. Based on the simulation results shown in Figure 3 and 4 and considering the complexity-BER tradeoff, we set the SNR thresholds for the adaptive SD-OSIC system. For very low SNR ($SNR \leq 14dB$), the system chooses hybrid SD-OSIC (2), which allows fast symbol decoding; for midrange SNR ($14\,dB < SNR < 18\,dB$), the system selects hybrid SD-OSIC (1); and finally, for high SNR ($SNR \geq 18\ dB$), it chooses only Chan & Lee's SD.

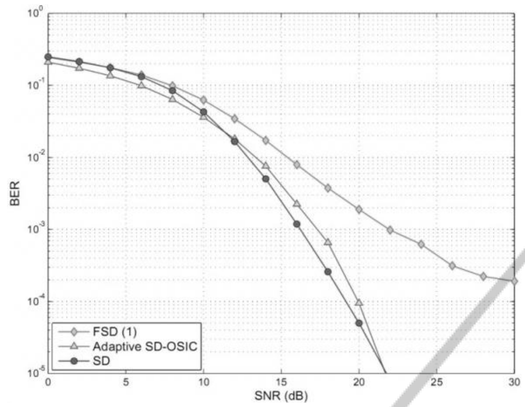

Figure 7: BER performance of the adaptive SD-OSIC system vs. FSD (1)

Figure 5 shows the BER performance of the proposed adaptive SD-OSIC system. As in the MIMO closed loop, the SNR is estimated at the receiver; however, this information is not sent to the transmitter; instead, it is used to choose the scheme according to the SNR threshold.

Figure 6 shows that the adaptive SD-OSIC system achieves a significant improvement in average complexity compared to conventional SD algorithms. Currently, we are working on optimizing the SD-OSIC system to achieve a fixed number of iterations.

Figure 7 and 8 show the performance of the adaptive SD-OSIC system vs. FSD. The scheme proposed in (Lee and Kim, 2006) and (Guo and Nilsson, 2006) was originally simulated using the complex version of SD.

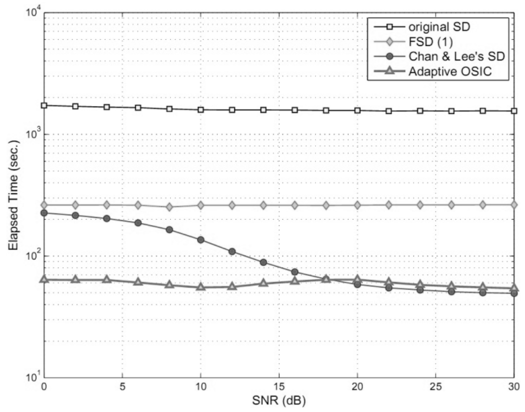

Figure 8: Complexity performance of the adaptive SD-OSIC system vs. FSD (1).

However, the same algorithm is applicable when we use matrix decomposition. In our simulation, the symbol with worst SNR is allocated in the last level of the tree search which contains all the possible options according to the modulation scheme; the other levels contain only the symbol decoded using OSIC. This way, it is possible to fix the iterations using by SD. In other words, FSD is a SD algorithm where the radius of the sphere does not shrink and the same amount of codewords is evaluated regardless of the SNR. As we can see in Figure 8, FSD exhibits constant complexity; however such complexity can be even greater than that of conventional SD.

5 CONCLUSIONS

In this paper, we proposed an adaptive SD-OSIC system that achieves better performance than conventional SD algorithms. One important feature of the proposed system is that it exhibits quasi-constant complexity through the entire SNR range, which provides better throughput performance. Unlike other proposed algorithms, our system reduces and improves the complexity of the conventional SD, with little sacrifice of BER in high SNR regions. Furthermore, for low SNR, our system achieves better BER performance than conventional SD and OSIC algorithms.

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