From PID to Extended Learning Control

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Abstract: It has been recently shown in Marino, Tomei, and Verrelli (2011) that the output error feedback regulation problem with (unknown) periodic reference and/or disturbance signals of known common period can be effectively solved for the class of single-input, single-output, minimum phase, nonlinear, time-invariant systems in output feedback form (of known relative degree one or two) which are affected by unknown parameters and unknown output-dependent nonlinearities. The resulting nonlinear control, which relies on advanced learning control techniques, can be interpreted as a generalization of the classical PID control which solves the problem when both reference and disturbance signals are constant. In this paper, we present sophisticated analytical arguments which prove that the learning control designed in Marino, Tomei, and Verrelli (2011) can be endowed with a period identifier when the output reference signal is periodic of uncertain period but available at each time instant. The generalized resulting control preserves the achievement of the closed loop properties obtained in Marino, Tomei, and Verrelli (2011) while maintaining an overall simple structure. The application of the presented control techniques to the position synchronization problem for current-fed permanent magnet step motors with non-sinusoidal flux distribution and uncertain position-dependent load torque allows us to provide a solution to a yet unsolved problem.

1 INTRODUCTION

Learning control design relies on the common observation that human beings are able to improve task executions through repeated trials. In contrast to general non-learning ones, learning controllers suitably use, in a repetitive framework, the richness of information owned by error signals from previous executions (Ahn, Chen, and Moore (2007), Bristow, Tharayil, and Alleyne (2006)). Learning controls thus iteratively extract from the past the sufficient experience to improve the closed loop performances and to guarantee the output tracking even for nonlinear systems affected by large uncertainties and disturbances. In particular, it has been recently shown in Marino, Tomei, and Verrelli (2011) that for the classes of:

- single-input, single-output, minimum phase, uncertain, nonlinear, time-invariant n-dimensional systems in output feedback form[1]

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} x + \psi(y) \\ &+ \begin{bmatrix} 0_{(\rho-1)\times 1} \\ b_\rho \\ \vdots \\ b_n \end{bmatrix} u + w \qquad (1)\\ y &= [1, 0, \cdots, 0]x \end{aligned}$$

  with known relative degree $\rho \in \{1,2\}$ and $b_\rho, \ldots, b_n \in \mathbb{R}^+$ unknown positive reals such that the zeroes of the polynomial $p(s) = b_\rho s^{n-\rho} + \ldots + b_n$ all belong to $\mathbb{C}^-$;

- single-input, single-output, observable, minimum phase, uncertain, linear, time-invariant systems[2]

$$\begin{aligned}\dot{\zeta} &= F\zeta + gu + w \qquad (2)\\ y &= h\zeta\end{aligned}$$

  with known relative degree $\rho \in \{1,2\}$ and input-

[1] $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$, $w \in \mathbb{R}^n$; $\psi(\cdot)$ is an unknown smooth vector-valued function; $0_{(\rho-1)\times 1}$ is the column vector with $(\rho-1)$ zero-components.

[2] $\zeta \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$, $w \in \mathbb{R}^n$; $F \in \mathcal{M}(n,\mathbb{R})$, $g \in \mathbb{R}^n$, $h^{\mathrm{T}} \in \mathbb{R}^n$ are unknown.

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output transfer function[3]

$$W(s) = \frac{\mathcal{L}(y(t))[s]}{\mathcal{L}(u(t))[s]} = \frac{b_\rho s^{n-\rho} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n}$$

whose zeroes all belong to $\mathbb{C}^-$,

the PD-type learning control[4]

$$\begin{aligned}
u(t) &= -(\rho-1)k_D\dot{\tilde{y}}(t) - k_P\tilde{y}(t) + \hat{u}_*(t) \\
\hat{u}_*(t) &= \mathrm{sat}_{M_u}\left(\hat{u}_*(t-T)\right) - k_I T \varphi_T(t)\Big[(\rho-1)\Big(\dot{\tilde{y}}(t) \\
&\quad + k_P/k_D\tilde{y}(t)\Big) + (2-\rho)\tilde{y}(t)\Big] \qquad (3) \\
\hat{u}_*(t) &= 0, \quad \forall\, t \leq 0 \\
\tilde{y}(t) &= y(t) - y_*(t)
\end{aligned}$$

is able to guarantee (with a proper choice of the control gains) the asymptotic output tracking

$$\lim_{t\to+\infty} [y(t) - y_*(t)] = 0$$

of (unknown) periodic output reference signals $y_* \in \mathcal{C}^{p_y}$ ($p_y \in \mathbb{N}_{\geq 1+\rho}$) of known period T despite the presence of the (unknown) periodic disturbance vector signal $w \in \mathcal{C}^{p_w}$ ($p_w \in \mathbb{N}_{\geq \rho}$) of the same period T.

Control (3) can be interestingly interpreted as a generalization of the classical PID control

$$u(t) = -(\rho-1)k_D\dot{\tilde{y}}(t) - k_P\tilde{y}(t) - k_I\int_0^t \tilde{y}(\tau)\mathrm{d}\tau \quad (4)$$

whose development started about one hundred years ago (see Sperry (1922) and Minorsky (1922)) and which, with a proper choice of the control gains, guarantees for the above classes of systems asymptotic output regulation in the case of constant output reference signals y_* and disturbances w. If the classical PID control is equivalently rewritten as

$$\begin{aligned}
u(t) &= -(\rho-1)k_D\dot{\tilde{y}}(t) - k_P\tilde{y}(t) + \hat{u}_*(t) \\
\dot{\hat{u}}_*(t) &= -k_I\tilde{y}(t), \quad \hat{u}_*(0) = 0,
\end{aligned}$$

it can be clearly noticed that k_D and k_P in (3) play the roles of the derivative and proportional gains in (4) while the learning estimation scheme

$$\begin{aligned}
\hat{u}_*(t) &= \mathrm{sat}_{M_u}\left(\hat{u}_*(t-T)\right) - k_I T \varphi_T(t)\Big[(\rho-1)\Big(\dot{\tilde{y}}(t) \\
&\quad + k_P/k_D\tilde{y}(t)\Big) + (2-\rho)\tilde{y}(t)\Big] \qquad (5) \\
\hat{u}_*(t) &= 0, \quad \forall\, t \leq 0
\end{aligned}$$

apparently extends the integral action $-k_I\int_0^t \tilde{y}(\tau)$ to the case of periodic references/disturbances. By neglecting the saturation action and by taking the limit for $T \to 0$ (if it exists), we can in fact obtain

$$\begin{aligned}
\hat{u}_*(t) &= -k_I\int_0^t \Big[(\rho-1)\Big(\dot{\tilde{y}}(\tau) + k_P/k_D\tilde{y}(\tau)\Big) \\
&\quad + (2-\rho)\tilde{y}(\tau)\Big]\mathrm{d}\tau \\
&\doteq \hat{u}_*(0) - k_I(\rho-1)\tilde{y}(t) \qquad (6) \\
&\quad - k_I\int_0^t \left[\frac{(\rho-1)k_P}{k_D}\tilde{y}(\tau) + (2-\rho)\tilde{y}(\tau)\right]\mathrm{d}\tau.
\end{aligned}$$

The signal $\hat{u}_*$, in both the above controls, plays the role of estimating the unknown reference input u_* which guarantees perfect output tracking for compatible initial conditions with u_* being restricted to be periodic of period T for (3) or simply constant (periodic of any period T) for the classical PID control.

2 THE CASE OF THE UNCERTAIN PERIOD

Control (3) feeds back only the regulation error $\tilde{y}(t)$ (and its time derivative $\dot{\tilde{y}}(t)$ if $\rho = 2$). However it relies on the knowledge of the period T characterizing the periodic reference signal $y_*(t)$ and disturbance vector signal $w(t)$. The aim of this section is to provide a generalization of the learning control presented in Marino, Tomei, and Verrelli (2011) to the case in which the output reference signal $y_*(t)$ is periodic of uncertain period T but measurable at each time instant t.

2.1 Control Design

Assume that the uncertain period T belongs to a certain compact set and that a known upper bound T_M on T is available ($T \leq T_M$) along with a nominal value

[3] $\mathcal{L}[f(t)](s)$ denotes the Laplace transform of the time function $f(t) : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}$, $s \in \mathbb{C}$, $b_i \in \mathbb{R}^+$ and $a_j \in \mathbb{R}$ are unknown reals, $\rho \leq i \leq n$, $1 \leq j \leq n$.

[4] The reals k_D, k_P, k_I, M_u are suitable positive control parameters; $\mathrm{sat}_{M_u}(\cdot) : \mathbb{R} \to [-M_u, M_u]$ is a continuous odd increasing function satisfying $\mathrm{sat}_{M_u}(q) = q$ for any $q \in (0, M_u]$ and $\mathrm{sat}_{M_u}(q) = M_u$ for any $q > M_u$; $\varphi_x(\cdot) : \mathbb{R}^+ \cup \{0\} \to [0,1]$ ($x > 0$) is a continuous increasing function for $t \in [0,x]$ with $\varphi_x(0) = 0$ and $\varphi_x(t) = 1$ for any $t \geq x$.

$T_N \leq T_M$ of T which is not necessarily equal to it. By setting T_N in place of the uncertain T in (3), we obtain $[\mu = k_I T_N]$

$$\begin{aligned} u(t) &= -(\rho-1)k_D\dot{\tilde{y}}(t) - k_P\tilde{y}(t) + \hat{u}_*(t) \\ \hat{u}_*(t) &= \mathrm{sat}_{M_u}\left(\hat{u}_*(t-T_N)\right) - \mu\varphi_{T_N}(t)\Big[(\rho-1)\Big(\dot{\tilde{y}}(t) \\ &\quad + k_P/k_D\tilde{y}(t)\Big) + (2-\rho)\tilde{y}(t)\Big] \qquad (7) \\ \hat{u}_*(t) &= 0, \quad \forall\, t \leq 0. \end{aligned}$$

If $T_N \neq T$, then asymptotic output regulation

$$\lim_{t\to+\infty} [y(t) - y_*(t)] = 0$$

my be in general not achieved. With the aim of incorporating a suitable period identifier, we first rewrite the input reference as $u_*(t) = u_{*\alpha}(t) + u_{*\beta}(t)$ where:

- $u_{*\alpha}(t)$ and $u_{*\beta}(t)$ are periodic time functions of period T_N and T, respectively;
- each $u_{*j}(\cdot)$, $j = \alpha, \beta$, is restricted to be either $u_*(\cdot)$ or the identically null function $\mathcal{N}(\cdot) \equiv 0$.

If $T_N = T$ then, without loss of generality, $u_{*\alpha}(\cdot) = u_*(\cdot)$ and $u_{*\beta}(\cdot) = \mathcal{N}(\cdot)$; if $T_N \neq T$ then $u_{*\beta}(\cdot) = u_*(\cdot)$ and $u_{*\alpha}(\cdot) = \mathcal{N}(\cdot)$. In any case, both $u_{*j}(\cdot)$, $j = \alpha, \beta$, satisfy $|u_{*j}(t)| \leq M_u$. We then modify (7) as (μ and ν are the new positive learning gains)

$$\begin{aligned} u(t) &= -(\rho-1)k_D\dot{\tilde{y}}(t) - k_P\tilde{y}(t) + \hat{u}_{*\alpha}(t) + \hat{u}_{*\beta}(t) \\ \hat{u}_{*\alpha}(t) &= \mathrm{sat}_{M_u}\left(\hat{u}_{*\alpha}(t-T_N)\right) - \mu\varphi_{T_N}(t)\Big[(\rho-1)\Big(\dot{\tilde{y}}(t) \\ &\quad + k_P/k_D\tilde{y}(t)\Big) + (2-\rho)\tilde{y}(t)\Big] \\ \hat{u}_{*\alpha}(t) &= 0, \quad \forall\, t \leq 0 \\ \hat{u}_{*\beta}(t) &= \mathrm{sat}_{M_u}(\hat{u}_{*\beta}(t-\hat{T}(t))) - \nu\varphi_{T_M}(t)\Big[(\rho-1)\Big(\dot{\tilde{y}}(t) \\ &\quad + k_P/k_D\tilde{y}(t)\Big) + (2-\rho)\tilde{y}(t)\Big] \\ \hat{u}_{*\beta}(t) &= 0, \quad \forall\, t \leq 0 \end{aligned}$$

where $\hat{u}_{*j}(\cdot)$, $j = \alpha, \beta$, play the role of estimating the unknown periodic functions $u_{*j}(\cdot)$, $j = \alpha, \beta$. According to the recent advances in Verrelli (2011a), the estimate of the uncertain period T results from an explorative search in the domain of the admissible values for T (see also Tyukin (2011) for similar explorative approaches). We accordingly introduce the estimate $\hat{T}$ of the uncertain period T defined as

$$\hat{T}(t) = T_M \qquad (8)$$

for $0 \leq t < 2T_M$ and satisfying $[M \in \mathbb{N} \cup \{+\infty\}]$

$$\dot{\hat{T}}(t) = \begin{cases} 0 & \text{if } \sum_{i=0}^{M} |y_*(\pi_i) - y_*(\pi_i + \hat{T}(t))| = 0 \\ -1 & \text{otherwise,} \end{cases} \qquad (9)$$

for $t \geq 2T_M$. If the set $\Sigma = \{\pi_i, 0 \leq i \leq M\}$ is chosen as the (countable) set of all rational numbers in $[0, T_M]$ with the field $\mathbb{Q}$ being a dense subset of $\mathbb{R}$, then $\hat{T}$ belongs to the compact set $[T, T_M]$ and converges to the uncertain T in finite time $3T_M - T$. In fact, by contradiction, suppose that $\hat{T}(3T_M - p) = p$ satisfying $\sum_{i=0}^{M} |y_*(\pi_i) - y_*(\pi_i + p)| = 0$ is different from T or equivalently that there exists a certain $\varpi \in \mathbb{R} / \mathbb{Q} \cap [0, T_M]$ such that $|y_*(\varpi) - y_*(\varpi + p)| \neq 0$; then, since $y_*(t)$ is a continuous function, there will exist a neighbourhood $\Upsilon \subset \mathbb{R}$ of ϖ such that $|y_*(t) - y_*(t+p)| \neq 0$ for any $t \in \Upsilon$, with Υ containing at least one element of $\mathbb{Q} \cap [0, T_M]$.

Remark 1: The estimate $\hat{T}$ may converge in finite time to T *via* a finite number M of evaluations when y_* is a band-limited periodic signal with zero spectrum outside the compact set $[-\omega_y/2, \omega_y/2]$ with T_m a positive lower bound on T. In this case, it suffices:

- to set, according to the sampling theorem 6.4 in Kalouptsidis (1997), $\pi_i = iT_M/M \doteq iT_s$, $i = 0, 2, \ldots, M$ with $T_s \leq 2\pi/\omega_y$ provided that T lies on the sequence of π_i;
- to design Σ such that $\mathrm{card}\,(\Sigma \cap [0, T_m]) > 2\bar{M}$ with $\bar{M}$ denoting the order of the trigonometric polynomial $y_*(t) = \sum_{k=-\bar{M}}^{\bar{M}} c_k e^{ik2\pi t/T}$.

2.2 Mathematical Details

Instead of using the Lyapunov function in Marino, Tomei, and Verrelli (2011) $V(t) = W(t) + \mathcal{T}(t)$ with

$$\mathcal{T}(t) = (2\mu)^{-1} \int_{t-T}^{t} [u_*(\tau) - \mathrm{sat}_{M_u}(\hat{u}_*(\tau))]^2 \, d\tau$$

consider the function

$$\begin{aligned} \mathcal{V}(t) = W(t) &+ (2\mu)^{-1} \int_{t-T_N}^{t} [u_{*\alpha}(\tau) - \mathrm{sat}_{M_u}(\hat{u}_{*\alpha}(\tau))]^2 \, d\tau \\ &+ (2\nu)^{-1} \int_{t-\hat{T}_*(t)}^{t} \left[u_{*\beta}(\tau) - \mathrm{sat}_{M_u}\left(\hat{u}_{*\beta}(\tau)\right)\right]^2 d\tau \end{aligned}$$

in which $\hat{T}_*(t)$ is a suitable class $\mathcal{C}^1$ cubic spline-based approximation of $\hat{T}(t)$ (with non-negative $\dot{\hat{T}}_*(t)$) which differs from $\hat{T}(t)$ only on an arbitrarily small compact set. The same closed loop convergence properties established in Marino, Tomei, and Verrelli (2011), that is:

- $x(t)$ and $u(t)$ are bounded on $[0, +\infty)$;

- $\tilde{y}(t)$ - and $\dot{\tilde{y}}(t)$ if $\rho = 2$ - asymptotically tend to zero as $t \to +\infty$ and are exponentially attracted into residual connected compact sets containing the origin whose diameters decrease as the learning gains increase;
- the asymptotic input learning

$$\lim_{t \to +\infty} [u(t) - u_*(t)] \quad = \quad 0$$

  is achieved under certain mild condition involving the speed of convergence of the output tracking errors $\tilde{y}$ (and $\dot{\tilde{y}}$ if $\rho = 2$),

are obtained by recognizing, in the computation of $\dot{\mathcal{V}}$, the following crucial facts:

- $\dot{\hat{T}}_*$ multiplies, in $\dot{\mathcal{V}}$, the non-negative term

$$\mathcal{L}(t) \quad = \quad \frac{1}{2\nu}[u_{*\beta}(t - \hat{T}_*(t)) - \mathrm{sat}_{M_u}(\hat{u}_{*\beta}(t - \hat{T}_*(t)))]^2;$$

- $u_{*\beta}(t - \hat{T}_*(t)) - u_{*\beta}(t - T) = 0$ if $T = T_N$ (i.e. if $u_{*\beta}(\cdot) = \mathcal{N}(\cdot)$);
- $|u_{*\beta}(t - \hat{T}_*(t)) - u_{*\beta}(t - T)| \le c_u |T - \hat{T}_*(t)|$ if $T \neq T_N$ (i.e. if $u_{*\alpha}(\cdot) = \mathcal{N}(\cdot)$) with c_u being a bound on $|\dot{u}_*(t)|$;
- $\hat{T}$ belongs to the compact set $[T, T_M]$ and converges in finite time to T.

3 SIMULATION RESULTS

We apply the control techniques proposed in the previous section to solve the tracking control problem addressed in Bifaretti, Tomei, and Verrelli (2011) (see also Bifaretti, Iacovone, Rocchi, Tomei, and Verrelli (2011) for experimental results) for uncertain current-fed permanent magnet step motors with non-sinusoidal flux distribution and uncertain position-dependent load torque. In constrast to Bifaretti, Tomei, and Verrelli (2011), a reference signal for the rotor position [$\theta_*(t) = \frac{3}{\pi}[1 - \cos(\pi/3t)]$ rad in Figure 1] - periodic of uncertain period $T = 6$ s and available at each time instant t (along with $\dot{\theta}_*(t)$) - is required to be tracked in this case. In other terms, a *master-slave* synchronization problem (see Verrelli (2011a) and Verrelli (2012)) is considered, in which the measured position $\theta_*(t)$ - periodic of uncertain period - of the *master* drumming human arm constitutes the reference signal to be tracked by the position $\theta(t)$ of the *slave* drumming robotic arm connected to the PMSM motor(see Figure 2): the periodic human arm movement is to be imitated by the robotic arm one (Andry, Gaussier, Moga, Banquet, and Nadel (2001)). The problem of synchronizing robots with external signals has been largely studied in the field of humanoid robotics. Musical performances - drumming in particular - in fact exemplifies the kind of synchronization challenge in which humans excel and at which robots typically fail. The results of this section, in the case of current-fed motor operations, will extend the ones presented in Verrelli (2011a) to the case of uncertainties in all motor parameters excepting N_r (recall that the model for the surface-mounted permanent magnet synchronous motor in Verrelli (2011a) is a particular case of the model considered in this section). On the other hand, current-fed motor operations are related to the use of high gains in the current loop which the presence of severe model uncertainties lead to (see Marino, Tomei, and Verrelli (2012) and Verrelli (2011b)). The learning control algorithm proposed in Bifaretti, Tomei, and Verrelli (2011) is a slight modification of the generalized PID control (3) with $\rho = 2$. It si designed for the current-fed permanent magnet step motor with two phases in the (d,q) reference frame rotating at speed $N_r\omega$ and identified by the angle $N_r\theta$ in the fixed (a,b) reference frame attached to the stator [θ is the rotor position, ω is the rotor speed and N_r is the number of rotor teeth, $m \ge 4$ is an uncertain integer]

$$\begin{aligned}
\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} &= \omega(t) \\
\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} &= -\frac{D}{J}\omega(t) + 2N_r L_1 i_d(t) i_q(t) \\
&\quad + \frac{i_f N_r}{J} \sum_{j=1}^{m} j L_{mj} \cos[(1-j)N_r\theta(t)] i_q(t) \\
&\quad + \frac{i_f N_r}{J} \sum_{j=2}^{m} j L_{mj} \sin[(1-j)N_r\theta(t)] i_d(t) \\
&\quad - \frac{N_r i_f^2}{2J} \sum_{j=4}^{m} j L_{fj} \sin[jN_r\theta(t)] - \frac{T_L(\theta(t))}{J}
\end{aligned}$$

where: (i_d, i_q) are the stator current vector (d,q) components [which constitute the control inputs], D is the friction coefficient, J is the rotor inertia, $T_L(\cdot)$ is the load torque, i_f is the fictitious constant rotor current provided by the permanent magnet, L_1 is a non-negative parameter, the harmonics $\sum_{j=1}^{m} L_{mj} \cos[jN_r\theta]$ and $\sum_{j=1}^{m} L_{mj} \cos\left[jN_r\theta - \frac{\pi}{2}\right]$ model the non-sinusoidal flux distribution in the air-gap with the parameters L_{mj}, $2 \le j \le m$ (which are zero under the standard assumption of sinusoidal flux distribution) being much smaller than L_{m1}, the term $\frac{N_r i_f^2}{2} \sum_{j=4}^{m} j L_{fj} \sin[jN_r\theta]$ represents the disturbance torque due to cogging.

The simulation is carried out with reference to the permanent magnet step motor in Bifaretti, Tomei,

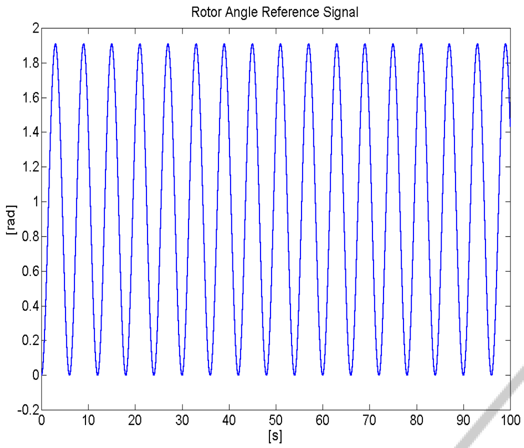

Figure 1: Rotor angle reference signal $\theta_*(t)$.

and Verrelli (2011) and Krishnamurthy and Khorrami (2003) with load torque $T_L(\theta) = N_T \sin(\theta)$ and parameters: $J = 0.0733$ Kgm^2, $L_1 = 0$, $m = 4$, $L_{m1} = 5$ mH, $L_{m2} = 0.5$ mH, $L_{m3} = 0.166$ mH, $L_{m4} = 0.0625$ mH, $L_{f4} = 1.766$ mH, $N_r = 50$, $i_f = 1$ A, $D = 0.002$ kgm^2/s, $N_T = 1.7201$ kgm^2/s^2. The considered application is the robotic application described in Dawson, Hu, and Burg (1998) (see Figure 2) in which the load torque $T_L = N_T \sin(\theta)$ models the position-dependent single link robotic load represented by a metal bar link attached to the rotor shaft and a brass ball attached to the free end. All the motor initial conditions are set to zero as in a realistic setting in which the motor is initially at rest and the initial position is compatible with the corresponding reference signal. According to Section 2, the learning control algorithm reads

$$
\begin{aligned}
i_d(t) &\equiv 0 \\
i_q(t) &= -k_\omega \tilde{\omega}(t) - k_v \tilde{\theta}(t) + \hat{u}_{*\alpha}(t) + \hat{u}_{*\beta}(t) \\
\hat{u}_{*\alpha}(t) &= \mathrm{sat}_{M_u}(\hat{u}_{*\alpha}(t - T_N)) - \mu \varphi_{T_N}(t)\tilde{\omega}(t) \\
\hat{u}_{*\alpha}(t) &= 0, \quad \forall\, t \le 0 \\
\hat{u}_{*\beta}(t) &= \mathrm{sat}_{M_u}(\hat{u}_{*\beta}(t - \hat{T}(t))) - \nu \varphi_{T_M}(t)\tilde{\omega}(t) \\
\hat{u}_{*\beta}(t) &= 0, \quad \forall\, t \le 0 \\
\tilde{\theta}(t) &= \theta(t) - \theta_*(t) \\
\tilde{\omega}(t) &= \omega(t) + k_\theta \tilde{\theta}(t) - \dot{\theta}_*(t)
\end{aligned}
$$

with $\hat{T}(t)$ given by (8)-(9) and control parameters (all values are in SI units) $k_\theta = \mu = \nu = 72$, $k_\omega = 12$, $k_v = 1$, $T_N = 3.5$ s, $T_m = 2$ s, $T_M = 7$ s and $\varphi_x(t) = t^2/x^2$ for $t \in [0, x]$. The choice: $\pi_i = 0.6 + 0.6(i-1)$, $1 \le i \le 4$ guarantees, according to Remark 1, the convergence of $\hat{T}(t)$ to $T = 6$ s in 1 s (see Figure 3). Figure 4 shows the time histories of the rotor angle $\theta(t)$, the rotor speed $\omega(t)$ and the rotor angle and speed tracking errors $\tilde{\theta}(t)$, $\tilde{\omega}(t)$, while Figure 5 shows the time histories of the uncertain function $u_*(t)$ with its estimate

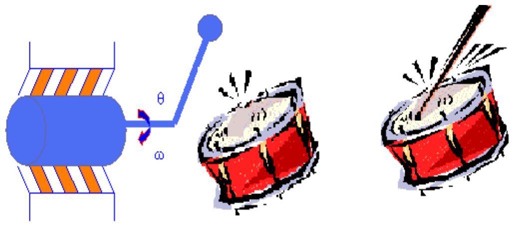

Figure 2: *Slave* PMSM motor and *master* drummer.

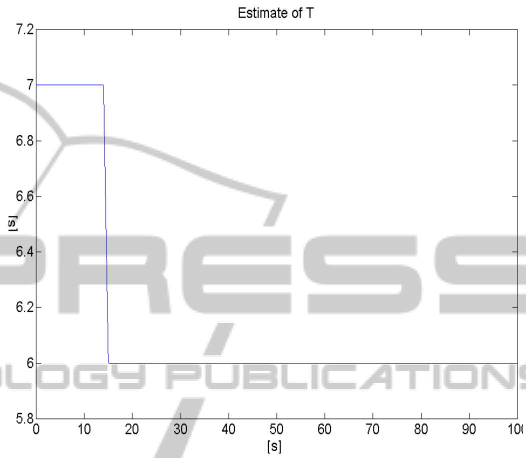

Figure 3: Estimate $\hat{T}(t)$.

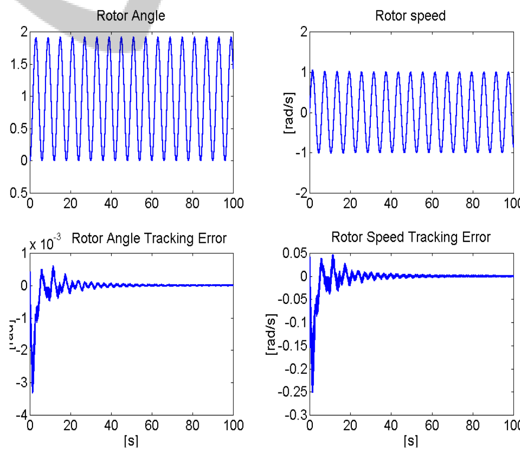

Figure 4: Rotor angle $\theta(t)$, rotor speed $\omega(t)$ and rotor angle and speed tracking errors $\tilde{\theta}(t)$, $\tilde{\omega}(t)$.

$\hat{u}_*(t) = \hat{u}_{*\alpha}(t) + \hat{u}_{*\beta}(t)$. Satisfactory position tracking is achieved despite system uncertainties along with satisfactory estimation of the uncertain periodic input reference signal $u_*(t)$.

4 CONCLUSIONS

The mathematical details concerning a generalization

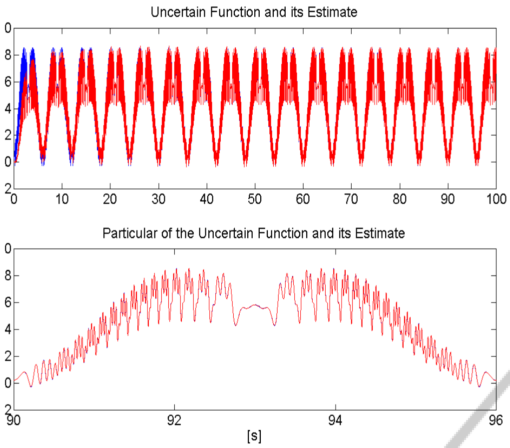

Figure 5: Reference input $u_*(t)$ and its estimate $\hat{u}_*(t)$.

of the learning control design presented in Marino, Tomei, and Verrelli (2011) are provided. The derived solution is able to solve the output regulation problem when the output reference signal (available at each time instant) is periodic of uncertain period. Sophisticated analytical arguments show that the resulting learning control, which incorporates the generalized PID learning control of Marino, Tomei, and Verrelli (2011) as a special case, preserves the achievement of the closed loop properties obtained in Marino, Tomei, and Verrelli (2011). The effectiveness of the result presented in the paper is demonstrated by its successful application to the yet unsolved position synchronization problem for current-fed permanent magnet step motors with non-sinusoidal flux distribution and uncertain position-dependent load torque.

REFERENCES

H.-S. Ahn, Y. Chen, and K. L. Moore (2007). Iterative learning control: brief survey and categorization. *IEEE Transactions on Systems, Man, and Cybernetics, C*, 37, 1099-1121.

P. Andry, P. Gaussier, S. Moga, J. P. Banquet, and J. Nadel (2001). Learning and communication via imitation: an autonomous robot perspective. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans* 31, 431-442.

S. Bifaretti, V. Iacovone, A. Rocchi, P. Tomei, and C. M. Verrelli (2011). Global learning position controls for permanent magnet step motors. *IEEE Transactions on Industrial Electronics*, 58, 4654-4663.

S. Bifaretti, P. Tomei, and C. M. Verrelli (2011). A global robust iterative learning position control for current-fed permanent magnet step motors. *Automatica*, 47, 227-234.

D. A. Bristow, M. Tharayil, and A. G. Alleyne (2006). A survey of iterative learning control. *IEEE Control Systems Magazine*, 96-114.

D. M. Dawson, J. Hu, and T. C. Burg (1998). *Nonlinear Control of Electric Machinery*, Marcel Dekker.

N. Kalouptsidis (1997). *Signal Processing Systems*, John Wiley & Sons.

P. Krishnamurthy and F. Khorrami (2003). Robust adaptive voltage-fed permanent magnet step motor control without current measurements. *IEEE Transactions on Control Systems Technology*, 11, 415-425.

R. Marino, P. Tomei, and C. M. Verrelli (2012). Robust adaptive learning control for nonlinear systems with extended matching unstructured uncertainties. *International Journal of Robust and Nonlinear Control*, 22, 645-675.

R. Marino, P. Tomei, and C. M. Verrelli (2011). Generalized PID learning control. *18th IFAC World Congress*, 18, 28 August-2 September.

N. Minorsky (1922). Directional stability of automatically steered bodies. *American Society of Naval Engineers*.

E. Sperry (1922). Automatic steering. *Transactions of ASME*.

I. Tyukin (2011). *Adaptation in Dynamical Systems*, Cambridge University Press.

C. M. Verrelli (2011a). Fourier series expansion for synchronization of permanent magnet electric motors. *Applied Mathematics and Computation*, 217, 4502-4515.

C. M. Verrelli (2012). Synchronization of permanent magnet electric motors: new nonlinear advanced results. *Nonlinear Analysis: Real World Applications*, 13, 395-409.

C. M. Verrelli (2011b). Adaptive learning control for robotic manipulators driven by permanent magnet synchronous motors. *International Journal of Control*, 84, 1024-1030.