An Integrated Replemishment Model under Dynamic Demand Conditions

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Abstract: This research develops an integrated replenishment model considering supplier selection, procurement lot-sizing, quantity discounts and safety stocks under dynamic demand conditions. The objectives of the model are to minimize total costs, which include ordering cost, purchase cost, transportation cost, shortage cost and holding cost, and to maximize service level of the system over the planning horizon. First, a multi-objective programming (MOP) model is proposed in the paper. Next, the model is transformed into a mixed integer programming (MIP) model based on the ε-constraint method. Then, the genetic algorithm (GA) model is constructed to solve a large-scale optimization problem by finding a near-optimal solution. An example of a bike manufacturer is used to illustrate the practicality of the proposal model. The results demonstrate that the proposed model is an effective and accurate tool for the integrated replenishment and logistics management.

1 INTRODUCTION

Good inventory management is essential for a firm to be cost competitive and to acquire reasonable profit in the market. How to achieve an outstanding inventory management has already been a popular topic in both the academic field and in real practice. There are two major categories of inventory models: deterministic and stochastic. In deterministic models, all input data are assumed to be deterministic, and a mathematical programming model is usually sufficient to obtain the optimal solution. For example, Su and Wong (2008) studied a stochastic dynamic lost-sizing problem under the bullwhip effect. A framework of two-stage ant colony optimization (TACO) was proposed, and a mutation operation was added in the second stage to determine the replenishment policy. Stochastic models, on the other hand, are often limited to highly restricted assumptions, and most current literature is a variation of the deterministic lot sizing problem (Şenyiğit and Erol, 2010).

The contribution of this research can be summarized as follows. First, a general formulation of the lot-sizing problem by mixed integer programming (MIP) is proposed. The model considers various costs such as ordering cost, purchase cost, transportation cost, shortage cost and holding cost. It aims to minimize the total cost in the system with safety stock while maximizing the service level for each planning period. Second, a genetic algorithm (GA) model is constructed to solve the problem when it becomes too complicated. We find that the GA model can find solutions that are very close to the optimal ones.

The remaining of this paper is organized as follows. Section 2 reviews some related methodologies and works. In section 3, the problem under consideration and the assumptions are described. The formulation of the lot-sizing problem by MIP and the construction of the GA model are presented. Case study is carried out in section 4. In the last section, some conclusion remarks are made.

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2 RELATED METHODOLOGY AND RESEARCH

Dynamic lot-sizing can be referred back to Wagner and Whitin (1958), and diverse lot-sizing heuristics have been adopted in many operations management works. For example, Teunter, Bayindir and Van Den Heuvel (2006) studied the dynamic lot sizing problem for systems with product returns and remanufacturing, and proposed modifications of the Silver Meal (SM), least unit cost and part period balancing heuristics.

Decision makers may want to optimize two or more objectives simultaneously under various constraints, and a MOP can then be applied. A complete optimal solution seldom exists, and a Pareto-optimal solution is used then (Wee et al., 2009). There are a few methods to derive a compromise solution (Rosenthal, 1985). For example, the weighting method assigns priorities to the objectives and sets aspiration levels for the objectives. The ε-constraint method is a modified weight method. One of the objective functions is optimized while the other objective functions are incorporated in the constraint part of the model.

GA, a heuristic search process for optimization, was first developed by Holland (1975). Based on Darwin's survival of the fittest principle, GA mimics the process of natural selection (Maiti et al., 2006). It has been widely applied to solve production and operations management problems (Aytug et al., 2003). The fundamental concept of GA is to code the decision variables of the problem as a finite length array, which is called chromosome, and to calculate the fitness, the objective function, of each string (Yang, Chan and Kumar, 2012).

3 PROBLEM DESCRIPTION AND ASSUMPTIONS

The following assumptions and notations are defined with the modification of those used in the models of Kang (2008) and Kang and Lee (2010). The assumptions are summarized as follows:

- The demand of each period is independent and follows a normal distribution with a constant coefficient of variation (θ).
- At most one order can be placed from each supplier in each period.
- The replenishment lead time is of known duration, and the entire order quantity is delivered at once in the beginning of a period.
- All-units discount schedule is considered. The price of each unit is dependent on the order quantity.
- The inventory holding cost for each unit is known and constant, independent of the price of each unit.
- Planning horizon is finite and known. There are T periods in the planning horizon, and the duration of each period is the same.
- The expected ending inventory level in period t (i.e., the expected beginning inventory level in period t+1) is the safety stock level in period t.
- The initial inventory level (X_1) is zero.

All the required notations in this paper are defined below.

Notations

Indices:

i Supplier ($i = 1,2,\ldots, I$).

k Price break ($k = 1,2,\ldots, K$).

t Planning period ($t = 1,2,\ldots, T$).

v Integer number for calculating the quantity purchased ($v = 1,2,\ldots, V$).

w Integer number for calculating the time transported ($w = 1,2,\ldots, W$).

Parameters:

$E(d_t)$ Expected demand in period t.

$\hat{\sigma}_t$ Standard deviation of demand in period t.

σ_t Pool standard deviation of demand in period t.

h Inventory holding cost, per unit per period.

r_i Transportation cost per time from supplier i.

s Shortage cost, per unit per period.

z_α Standard normal value of service level α.

$L(z_\alpha)$ Standardized number of units short with service level α.

M A large number.

o_i Ordering cost per replenishment from supplier i.

p_{ik} Unit purchase cost from supplier i with price break k.

q_{ik} The upper bound quantity of supplier i with price break k.

Decision variables:

$P(Q_{it})$ Purchase cost for one unit based on the discount schedule of supplier i with order quantity Q_{it} in period t.

Q_{it} Purchase quantity from supplier i in period t.

$\lceil Q_{it}/b_i \rceil$ The smallest integer greater than or equal to Q_{it}/b_i.
N_{it} Number of transportations from supplier i in period t.
F_{it} A binary variable, set equal to 1 if a purchase is made from supplier i in period t, and 0 if no purchase is made from supplier i in period t.
X_t Expected beginning inventory level in period t.
Y_t Expected beginning available inventory level in period t, and $Y_t = X_t + \sum_{i=1}^{I} F_{it} \times Q_{it}$.
z_t Standard normal value of ending inventory level in period t.
$L(z_t)$ Standardized number of units short of ending inventory level in period t
β_{itv} A binary variable for calculating the purchase quantity from supplier i in period t.
G_{itw} A binary variable for calculating the time of transportations from supplier i in period t.
U_{itk} A binary variable, set equal to 1 if a certain quantity is purchased, and 0 if no purchase is made, with price break k supplier i in period t.

The above information is used to develop a MIP model and a GA model to solve the lot-sizing problem with multiple suppliers and quantity discounts so that an appropriate inventory level for each period can be determined. The total cost for each period can be calculated by adding up the relevant costs, including ordering cost, holding cost, and purchase cost with quantity discounts. The total cost in a planning horizon includes all the total costs in each period.

3.1 Relevant Costs

The ordering cost for the system is calculated by equation (1), where o_t is the ordering cost per time from supplier i and F_{it} represents whether a quantity is purchased from supplier i in period t.

$$Ordering \quad cost = O = \sum_{t=1}^{T} \sum_{i=1}^{I} o_i \times F_{it} \tag{1}$$

Equation (2) calculates the purchase cost, where $P(Q_{it})$ is the unit purchase cost based on the discount schedule with the order quantity Q_{it}, and F_{it} represents whether a quantity is purchased from supplier i in period t.

$$Purchase \quad cost = P = \sum_{t=1}^{T} \sum_{i=1}^{I} (P(Q_{it}) \times Q_{it} \times F_{it}) \tag{2}$$

Equation (3) calculates the transportation cost of the system, where r_i is the transportation cost per time from supplier i, $\lceil Q_{it}/b_i \rceil$ is the smallest integer greater than or equal to Q_{it}/b_i from supplier i in period t, N_{it} is number of transportations from supplier i in period t and b_i is the maximum transportation batch size from supplier i.

$$Transportation \quad cost = R = \sum_{t=1}^{T} \sum_{i=1}^{I} r_i \times \lceil Q_{it}/b_i \rceil = \sum_{t=1}^{T} \sum_{i=1}^{I} r_i \times N_{it} \tag{3}$$

The shortage cost of the system is calculated by equation (4), where s is the shortage cost per unit per period, $L(z_t)$ is the standardized number of unit shortage function, and σ_t is the pool standard deviation in period t.

$$Shortage \quad cost = S = \sum_{t=1}^{T} s \times L(z_t) \times \sigma_t \tag{4}$$

The holding cost in period t is equal to the holding cost per unit times the ending inventory in period t. Then, the holding cost for a planning horizon is the summation of the holding cost for each period, as in equation (5).

$$Holding \quad cost = H = \sum_{t=1}^{T} h \times (X_{t+1} + \sigma_t \times L(z_t)) \tag{5}$$

3.2 Multi-objective Programming (MOP)

The stochastic lot-sizing problem is formulated into a MOP model for minimizing total cost and maximizing service level. Based on the ε-constraint method, we can set the total cost as an objective and use the service level as a constraint. The proposed model is formulated as follows:

$$\text{Min } TC(x) \tag{6}$$

$$\text{s.t. } x \in E \tag{7}$$

$$Z(x) \geq z_\alpha \tag{8}$$

where z_α is the standard normal value of service level α.

3.3 Mixed Integer Programming (MIP) Model

The multi-objective programming (MOP) problem can be transformed into a MIP model to solve the multi-period inventory problem and to determine an appropriate replenishment policy for each period. The proposed model can be formulated as follows:

Minimize

$$TC=\sum_{t=1}^{T}\Bigg[\sum_{i=1}^{I}o_i\times F_{it}+\sum_{i=1}^{I}P(Q_{it})\times Q_{it}\times F_{it}+\sum_{i=1}^{I}r_i\times N_{it}+s\times L(z_t)\times\sigma_t$$

$$+h\times\left(X_{t+1}+L(z_t)\times\sigma_t\right)\Bigg] \quad (9)$$

$$\text{s.t. } X_{t+1}=Y_t-E(d_t) \quad ,\textit{for all } t \quad (10)$$

$$Y_t=X_t+\sum_{i=1}^{I}Q_{it}\times F_{it} \quad ,\textit{for all } t \quad (11)$$

$$Q_{it}\le M\times F_{it} \quad ,\textit{for all } t \quad (12)$$

$$Q_{it}=\sum_{v=1}^{V}2^{v_{it}-1}\beta_{itv} \quad ,\textit{for all } i, t \quad (13)$$

$$N_{it}=\lceil Q_{it}/b_i\rceil \quad ,\textit{for all } i, t \quad (14)$$

$$N_{it}=\sum_{w=1}^{W}2^{w-1}G_{itw} \quad ,\textit{for all } i, t \quad (15)$$

$$z_t=\left(Y_t-E(d_t)\right)/\sigma_t \quad ,\textit{for all } t \quad (16)$$

$$z_t\ge z_\alpha \quad ,\textit{for all } t \quad (17)$$

$$\hat{\sigma}_t=\theta\times E\left(d_t\right) \quad ,\textit{for all } t \quad (18)$$

$$\sigma_t=\sqrt{\sum_{t'=1}^{t}\hat{\sigma}_{t'}^2} \quad ,\textit{for all } t \quad (19)$$

$$P(Q_{it})=\sum_{k=1}^{K}p_{ik}\times U_{itk} \quad ,\textit{for all } i, t \quad (20)$$

$$q_{ik-1}+M\times(U_{itk}-1)\le Q_{it}<q_{ik}+M\times(1-U_{itk}) \quad ,\textit{for all } i, t, k \quad (21)$$

$$\sum_{k=1}^{K}U_{itk}=1 \quad ,\textit{for all } i, t \quad (22)$$

$$F_{it}\in\{0,1\} \quad ,\textit{for all } i, t \quad (23)$$

$$G_{itw}\in\{0,1\} \quad ,\textit{for all } i, t, w \quad (24)$$

$$\beta_{itv}\in\{0,1\} \quad ,\textit{for all } i, t, v \quad (25)$$

$$U_{itk}\in\{0,1\} \quad ,\textit{for all } i, t, k \quad (26)$$

and all variables are nonnegative.

3.4 Genetic Algorithm (GA) Model

GA is used next to solve the lot-sizing problem with quantity discounts and safety stock so that near-optimal solutions can be produced in a short period of computation time. The procedures of the GA are proposed as follows:

Step 1. Coding scheme
Assume that at most one order can be placed in each period and that a replenishment quantity can serve for an integer number of periods.

Step 2. Initial population of chromosomes
The initial population is generated randomly, and there are two types of chromosomes, which are also determined randomly.

Step 3. Fitness function
The fitness function for each chromosome is Min TC, where TC is the total cost. Min TC is the minimum cost among all the chromosomes across the population.

Step 4. Crossover operation
The standard two-cut-point crossover operator is applied to the selected pair of parent-individuals by recombining their genetic codes and producing two offspring.

Step 5. Mutation operator
A mutation operator is to counteract premature convergence and to maintain enough diversity in the population. It is performed by changing a randomly selected gene in the genetic code (0-1, 1-0). In each generation, all individuals have a set of given genes fixed, called frozen genes.

Step 6. Selection of subsequent population
After the mutation and crossover operations in each generation, a subsequent population is selected for the next generation.

Step 7. Termination
The processes of crossover, selection and replacement are repeated until the objective function of the problem is optimized or the stop criterion is met.

4 CASE STUDY OF A BIKE MANUFACTURER

4.1 Stochastic Lot-sizing Problem

A stochastic lot-sizing problem with quantity discounts and safety stock is solved here. Based on an interview with the management of a bike manufacturer in Taiwan, the following assumptions are made. The ordering cost of supplier A (o_1) and supplier B (o_2) per replenishment is set to be \$220 and \$190, respectively. In addition, we set unit holding cost per period (h), which includes the handling cost, storage cost and capital cost, to be \$0.1. The demand in each period is assumed to be normal distributed with a mean $E(d_t)$ and a coefficient of variation (θ) of 1/3. Table 1 shows the expected demand $E(d_t)$ and its standard deviation $\hat{\sigma}_t$ in each period t.

The ordering cost per time from supplier A and B is \$220 and \$190, respectively. The transportation cost per time is \$21 and \$20.5 from supplier A and B, respectively. The unit shortage cost is \$30, required service level is 95%, and the number of periods is 7. A quantity can be purchased from supplier A and/or B using the discount schedules in Table 2 and Table 3, respectively.

Table 1: Demand of each period in a planning horizon.

t	1	2	3	4	5	6	7
$E(d_t)$	660	700	560	120	650	510	525
Standard deviation ($\hat{\sigma}_t$)	220	233	187	40	217	170	175

Table 2: Discount schedule for supplier A.

Price break (k)	Purchase quantity (q_{1k})	Price per unit (p_{1k})
1	0 – 999	\$4.00
2	1000 – 1999	\$3.92
3	2000 – 2999	\$3.84
4	3000 or more	\$3.76

Table 3: Discount schedule for supplier B.

Price break (k)	Purchase quantity (q_{2k})	Price per unit (p_{2k})
1	0 – 1500	\$4.02
2	1501 – 3000	\$3.89
3	3001 or more	\$3.75

4.2 Experimental Results

The lot-sizing problem is solved by both the MIP model and the GA model. The MIP model is implemented using the software LINGO (2006), and the GA is implemented using the software MATLAB (2007).

The solution of the MIP model is shown in Table 4. Under the MIP model, two purchases are made: 3034 units from supplier B in period 1, 1507 units from supplier B in period 5. The total cost is \$18983.

The GA model is implemented by using the software MATLAB. Two-cut-point crossover for crossover operations is applied, and an inversion mutation operator is used to avoid a solution being trapped in a local optimum and to approach the global optimum. The size of the initial population is set as 35. The crossover rate is set as 0.75, meaning that around 75% pairs of individuals take part in the production of offspring. The mutation rate is set as 0.01, meaning that each gene of a newly created solution is mutated with the probability 0.01. The solutions of the case obtained by the MIP model and by the GA algorithm are the same, and the total cost is \$18983.

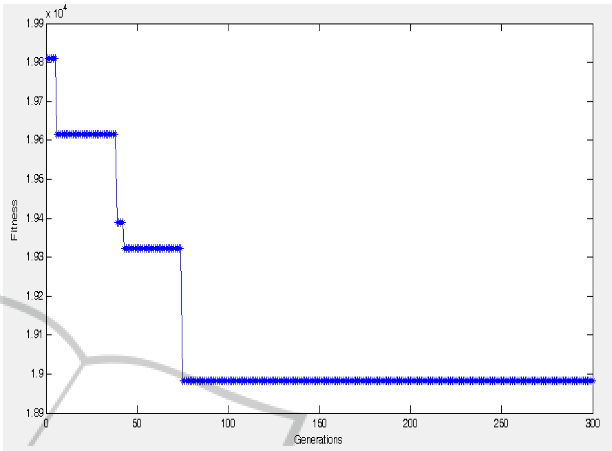

Figure 1: The convergence of GA.

5 CONCLUSIONS

This paper constructs a lot-sizing model with quantity discounts and safety stock to minimize total cost over the planning horizon. A general formulation of the lot-sizing problem is proposed by mixed integer programming (MIP) first to devise appropriate replenishment policies. An efficient genetic algorithm (GA) is introduced next for solving large-scale lot-sizing problem in a very short time. Replenishment level and system cost can be determined after calculating ordering cost, purchase cost, transportation cost, shortage cost and holding cost. The results show that the GA model is effective in searching for solutions, and it can be very useful for managers in real practice.

In the future, a more complete case study of supply chain management can be considered. A model that considers issues, such as variable lead time, probability demand, different priority of orders, backorder and lost sales, can be developed. To incorporate these issues, the assumptions will need to be relaxed by modifying objectives and constraints.

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