

Effective Keyword Search via RDF Annotations

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Abstract: Searching relevant information from Web may be a very tedious task. If people cannot navigate through the Web site, they will quickly leave. Thus, designing effective navigation strategies on Web sites is crucial. In this paper we provide and implement centrality indices to guide the user for an effective navigation of Web pages. We get inspiration from well-know location family problems to compute the center of a graph: a joint use of such indices guarantees the automatic selection of the best starting point. To validate our approach, we have developed a system that implements the techniques described in this paper on top of an engine for keyword-based search over RDF data. Such system exploits an interactive front-end to support the user in the visualization of both annotations and corresponding Web pages. Experiments over widely used benchmarks have shown very good results, in terms of both effectiveness and efficiency.

1 INTRODUCTION

The original perception of the Web by the vast majority of its early users was as a static repository of unstructured data. This was reasonable for browsing small sets of information by humans, but this static model now breaks down as programs attempt to dynamically generate information, and as human browsing is increasingly assisted by intelligent agent programs. With the size and availability of data constantly increasing, a fundamental problem lies in the difficulty users face to find and retrieve the information they are interested in (Li et al., 2001). A relevant problem is that using a search engine probably the retrieved pages are not always what user is looking for. To give an example, let us consider Wikipedia and type “Kenneth Iverson APL”, i.e. we would retrieve information about the developer of APL programming language. As shown Figure 1, the user has to browse the list of all Kenneth Iverson, then manually to solve the disambiguation (i.e. selecting Kenneth E. Iverson) and finally to consume the information about development of APL programming language following the link in the Web page. Such task is time consuming and in case of a long chain of links to follow it could convince the user to quickly leave the Web site. To this aim “Semantic Web” lies in encoding properties of and relationships between objects represented by information stored on the Web (De Virgilio et al., 2010). It envisions that authors of pages

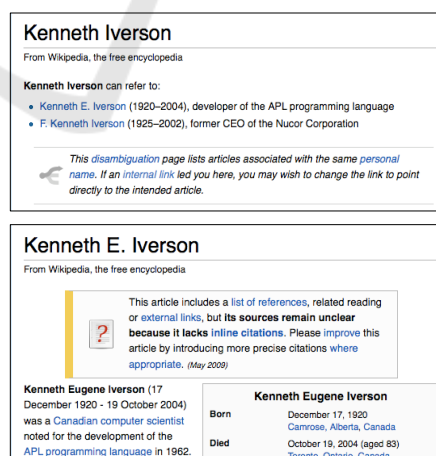


Figure 1: Searching K.E. Iverson, developer of the APL programming language.

include semantic information along with human-readable Web content, perhaps with some machine assistance in the encoding. Referring to the example of Figure 1, we could annotate the corresponding Web pages with the following RDF triples

```
<rdf:Description rdf:about="wiki:Kenneth_Iverson">
  <rdf:type rdf:resource="wiki:Article"/>
  <wiki:redirectsTo rdf:resource="wiki:Kenneth_E._Iverson"/>
  <wiki:redirectsTo rdf:resource="wiki:F._Kenneth_Iverson"/>
  ...
</rdf:Description>
<rdf:Description rdf:about="wiki:Kenneth_E._Iverson">
  <rdf:type rdf:resource="wiki:Article"/>
```

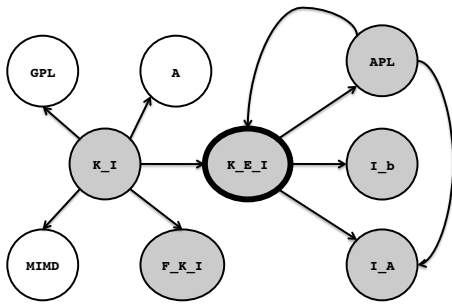


Figure 2: Semantic annotation of K. Iverson pages.

```
<wiki:internalLink
  rdf:resource="wiki:APL_programming_language"/>
<wiki:internalLink rdf:resource="wiki:Iverson_Award"/>
<wiki:internalLink rdf:resource="wiki:Iverson_bracket"/>
...
</rdf:Description>
```

Due to the huge amount of RDF data available on the Web (currently around 7 billion RDF triples and 150 million RDF links), keywords search based systems, e.g. (Cappellari et al., 2011), are increasingly capturing the attention of researchers, implementing IR strategies on top of traditional database management systems with the goal of freeing the users from having to know data organization and query languages. The aim of such approaches is to query semantic annotations instead of making a deep analysis of a large number of Web pages. The result is a set of RDF subgraphs linked to the Web pages of interest. For instance Figure 2 shows the RDF subgraph matching the query “Kenneth Iverson APL”. For the sake of simplicity, we used only initials of URIs and marked matching nodes in gray. However most of the proposals do not analyze in deep how to exploit the resulting RDF annotation to navigate the pages of interest. The user manually has to analyze the RDF graph, selecting the starting node from which begins the navigation of the corresponding Web pages. Of course, in this situation semi-automatic tools would support the analysis but the risk is to guide the user to select a wrong starting point, so far from the most interesting Web pages. For instance looking at the RDF graph of Figure 2, a semi-automatic tool could select K_I as starting point (i.e. it is the source of the graph): in this case we reconduct the user to the same situation of Figure 1. The best choice would be K_{E_I} linking to the Web page of Kenneth E. Iverson.

In this paper we provide and implement centrality indices to guide the user for an effective navigation of Web pages. We get inspiration from well-know location family problems to compute the center of a graph: a joint use of such indices guarantees the automatic selection of the best starting point. To vali-

date our approach, we have developed a system that implements the techniques described in this paper on top of an engine for keyword-based search over RDF data. Such system exploits an interactive front-end to support the user in the visualization of both annotations and corresponding Web pages. Experiments over widely used benchmarks have shown very good results, in terms of both effectiveness and efficiency. The rest of the paper is organized as follows. In Section 2 we discuss the related research. In Section 3 we present our centrality indices. Finally Section 4 illustrates the experimental results, and in Section 5, we draw our conclusions and sketch future works.

2 RELATED WORK

Facility location analysis deals with the problem of finding optimal locations for one or more facilities in a given environment. Location problems are classical optimization problems with many applications in industry and economy. The spatial location of facilities often take place in the context of a given transportation, communication, or transmission system, which may be represented as a network for analytic purposes. A first paradigm for location based on the minimization of transportation costs was introduced by Weber in 1909. However, a significant progress was not made before 1960 when facility location emerged as a research field. There exist several ways to classify location problems. According to Hakami (Herlihy, 1964) who considered two families of location problems we categorize them with respect to their objective function. The first family consists of those problems that use a minimax criterion. As an example, consider the problem of determining the location for an emergency facility such as a hospital. The main objective of such an emergency facility location problem is to find a site that minimizes the maximum response time between the facility and the site of a possible emergency. The second family of location problems considered by Hakami optimizes a minisum criterion which is used in determining the location for a service facility like a shopping mall. The aim here is to minimize the total travel time. A third family of location problems described for example in (Smart and Slater, 1999) deals with the location of commercial facilities which operate in a competitive environment. The goal of a competitive location problem is to estimate the market share captured by each competing facility in order to optimize its location. Our focus here is not to treat all facility location problems. The interested reader is referred to a bibliography devoted to facility location analysis (Domschke

and Drex1, 1985). The aim of this paper is to introduce three important vertex centralities by examining location problems. Then we can introduce a fourth index based not only on “spatial” properties (such as the other centrality indices) but also on the semantics. The definition of different objectives leads to different centrality measures. A common feature, however, is that each objective function depends on the distance between the vertices of a graph. In the following we focus on G as connected directed graph with at least two vertices and we suppose that the distance $d(u, v)$ between two vertices u and v is defined as the length of the shortest path from u to v . These assumptions ensure that the following centrality indices are well defined. Moreover, for reasons of simplicity we consider G to be an unweighted graph, i.e., all edge weights are equal to one. Of course, all indices presented here can equally well be applied to weighted graphs.

3 WEB NAVIGATION

As said in the Introduction, the user is supported by different approximate query processing methods to improve the search of information on the Web. In particular Semantic Web was introduced to annotate the semantic involved into a Web page, making more automatic the interoperability between applications and machines and improving the effectiveness of the results. However a significant issue is to exploit the result (annotation) of the query processing to navigate the corresponding Web pages in an effective way. Then, centrality indices can be computed to quantify an intuitive feeling that in the result some vertices or edges are more central than others. Such indices can support the user to navigate directly the part of the result that best fits the query provided by the user.

3.1 Center Indices

In the following we get inspiration from well-know location family problems to compute the center of G . **Eccentricity.** The aim of the first problem family is to determine a location that minimizes the maximum distance to any other location in the network. Suppose that a hospital is located at a vertex $u \in V$. We denote the maximum distance from u to a random vertex v in the network, representing a possible incident, as the eccentricity $e(u)$ of u , where $e(u) = \max\{d(u, v) : v \in V\}$. The problem of finding an optimal location can be solved by determining the minimum over all $e(u)$ with $u \in V$. In graph theory, the set of vertices with minimal eccentricity is denoted as the center of

G . Hage and Harary (Harary and Hage, 1995) proposed a centrality measure based on the eccentricity

$$c_E(u) = \frac{1}{e(u)} = \frac{1}{\max\{d(u, v) : v \in V\}}$$

This measure is consistent with the general notion of vertex centrality, since $e(u)^{-1}$ grows if the maximal distance of u decreases. Thus, for all vertices $u \in V$ of the center of G : $c(u) \geq c_E(v)$ for all $v \in V$. Based on such method, we define a procedure to compute the center of the graph as described in the Algorithm 1.

Algorithm 1: Center computation by eccentricity.

Input : The graph G

Output: The center c_E

```

1  $n \leftarrow V.length;$ 
2  $L_E \leftarrow InitializeArray(n);$ 
3  $M \leftarrow FloydWarshall(G);$ 
4 for  $i \leftarrow 0$  to  $n$  do
5    $L_E[i] \leftarrow Max(M[i]);$ 
6  $i_{min} \leftarrow MinIndex(L_E);$ 
7  $c_E \leftarrow V[i_{min}];$ 
8 return  $c_E;$ 

```

In the algorithm, by using the function `InitializeArray`, we initialize the eccentricity vector L_E (line[2]). Such vector has length n (the number of nodes in V): for each node with index i we calculate the maximum distance from the nodes of G (lines[4-5]). The distances from each couple of nodes are computed in a matrix M (line[3]) by using the Floyd-Warshall algorithm (Rosen, 2003), that is a graph analysis algorithm for finding shortest paths in a weighted graph. If there does not exist a path between two nodes we set the distance to ∞ . Finally we select the index i_{min} in L_E corresponding to the minimum value (line[6]). The center c_E corresponds to the node with the index i_{min} in V . Referring to the graph of Figure 2, we have the following matrix

$$M = \begin{bmatrix} 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ 1 & 1 & 1 & 0 & 1 & 1 & 2 & 2 & 2 & 2 \\ \infty & \infty & \infty & \infty & 0 & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & \infty & 0 & 1 & 1 & 1 & 1 \\ \infty & \infty & 2 & \infty & \infty & 1 & 0 & 2 & 1 & 1 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 0 \end{bmatrix}$$

where $idx(GPL) = 1$, $idx(MIMD) = 2$, $idx(A) = 3$, $idx(K_I) = 4$, $idx(F_K_I) = 5$, $idx(K_E_I) = 6$, $idx(APL) = 7$, $idx(I_b) = 8$ and $idx(I_A) = 9$. Then the eccentricity vector is $L_E = [\infty \ \infty \ \infty \ 2 \ \infty \ \infty \ \infty \ \infty \ \infty \ \infty]^t$. In

L_E the minimum value is 2, corresponding to the index 4: in this case the center c_E is $K_{\perp I}$.

Closeness. Next we consider the second type of location problems - the minimum location problem, often also called the median problem or service facility location problem. Suppose we want to place a service facility, e.g., a shopping mall, such that the total distance to all customers in the region is minimal. This would make traveling to the mall as convenient as possible for most customers. We denote the sum of the distances from a vertex $u \in V$ to any other vertex in a graph G as the total distance $\sum_{v \in V} d(u, v)$. The problem of finding an appropriate location can be solved by computing the set of vertices with minimum total distance. In social network analysis a centrality index based on this concept is called closeness. The focus lies here, for example, on measuring the closeness of a person to all other people in the network. People with a small total distance are considered as more important as those with a high total distance. Various closeness-based measures have been developed, see for example (Bavelas, 1950; Beauchamp, 1965; Moxley and Moxley, 1974; Sabidussi, 1966) and (Valente and Foreman, 1998). The most commonly employed definition of closeness is the reciprocal of the total distance

$$c_C(u) = \frac{1}{\sum_{v \in V} d(u, v)}$$

In our sense this definition is a vertex centrality, since $c_C(u)$ grows with decreasing total distance of u and it is clearly a structural index. Before we discuss the competitive location problem, we want to mention the radiality measure and integration measure proposed by Valente and Foreman (Valente and Foreman, 1998). These measures can also be viewed as closeness-based indices. They were developed for digraphs but an undirected version is applicable to undirected connected graphs, too. This variant is

$$c_R(u) = \frac{\sum_{v \in V} (\Delta_G + 1 - d(u, v))}{n - 1}$$

where Δ_G and n denote the diameter of the graph and the number of vertices, respectively. The index measures how well a vertex is integrated in a network. The better a vertex is integrated the closer the vertex must be to other vertices. The primary difference between c_C and c_R is that c_R reverses the distances to get a closeness-based measure and then averages these values for each vertex.

Based on such method, we define a procedure to compute the center of the graph as described in the Algorithm 2. As for the eccentricity, we initialize the closeness vector L_C and calculate the matrix M . Then for each node with index i we calculate the sum of

Algorithm 2: Center computation by closeness.

Input : The graph G

Output: The center c_C

```

1  $n \leftarrow V.length;$ 
2  $L_C \leftarrow InitializeArray(n);$ 
3  $M \leftarrow FloydWarshall(G);$ 
4 for  $i \leftarrow 0$  to  $n$  do
5    $L_C[i] \leftarrow \sum_{j=0}^{n-1} M[i][j];$ 
6  $i_{min} \leftarrow MinIndex(L_C);$ 
7  $c_C \leftarrow V[i_{min}];$ 
8 return  $c_C;$ 

```

distances from the other nodes (lines[4-5]). Finally, as for the eccentricity, we calculate the index i_{min} of the minimum value in L_C . Such index corresponds to the center c_C in G . Referring again to our example, given the matrix M by the Floyd-Warshall algorithm, we have the following closeness vector $L_C = [\infty \ \infty \ \infty \ 11 \ \infty \ \infty \ \infty \ \infty \ \infty]^t$. Since the minimum value is 11, i_{min} is 4 (i.e. the center is $K_{\perp I}$).

Centroid Values. The last centrality index presented here is used in competitive settings. Suppose each vertex represents a customer in a graph. The service location problem considered above assumes a single store in a region. In reality, however, this is usually not the case. There is often at least one competitor offering the same products or services. Competitive location problems deal with the planning of commercial facilities which operate in such a competitive environment. For reasons of simplicity, we assume that the competing facilities are equally attractive and that customers prefer the facility closest to them. Consider now the following situation: a salesman selects a location for his store knowing that a competitor can observe the selection process and decide afterwards which location to select for her shop. Which vertex should the salesman choose? Given a connected undirected graph G of n vertices. For a pair of vertices u and v , $\gamma_u(v)$ denotes the number of vertices which are closer to u than to v , that is $\gamma_u(v) = |\{w \in V : d(u, w) < d(v, w)\}|$. If the salesman selects a vertex u and his competitor selects a vertex v , then he will have $\gamma_u(v) + \frac{1}{2}(n - \gamma_u(v) - \gamma_v(u)) = \frac{1}{2}n + \frac{1}{2}(\gamma_u(v) - \gamma_v(u))$ customers. Thus, letting $f(u, v) = \gamma_u(v) - \gamma_v(u)$, the competitor will choose a vertex v which minimizes $f(u, v)$. The salesman knows this strategy and calculates for each vertex u the worst case, that is

$$c_F(u) = \min\{f(u, v) : v \in V - u\}$$

$c_F(u)$ is called the centroid value and measures the advantage of the location u compared to other locations, that is the minimal difference of the number of

customers which the salesman gains or loses if he selects u and a competitor chooses an appropriate vertex v different from u . Based on such method, we define a procedure to compute the center of the graph as described in the Algorithm 3. In the algorithm, we initialize the centroid vector min and the centroid matrix C , i.e. $n \times n$, where each value $[i,j]$ corresponds to $f(i,j)$. We fill C (lines[5-15]) by using the matrix M , calculated by the Floyd-Warshall algorithm. Then for each row i of C we copy the minimum value in $min[i]$ (lines[16-17]). Finally we calculate the index i_{max} corresponding to the maximum value in min (line[18]). The center c_F correspond to the node in V with index i_{max} . Referring again to our example, we have the following matrix

$$C = \begin{bmatrix} 0 & 0 & 0 & -7 & 0 & -4 & -4 & 0 & 0 \\ 0 & 0 & 0 & -7 & 0 & -4 & -4 & 0 & 0 \\ 0 & 0 & 0 & -7 & 0 & -4 & -4 & 0 & 0 \\ 7 & 7 & 7 & 0 & 7 & 0 & 3 & 7 & 7 \\ 0 & 0 & 0 & -7 & 0 & -4 & -4 & 0 & 0 \\ 4 & 4 & 4 & 0 & 4 & 0 & 2 & 4 & 4 \\ 4 & 4 & 4 & -3 & 4 & -2 & 0 & 4 & 4 \\ 0 & 0 & 0 & -7 & 0 & -4 & -4 & 0 & 0 \\ 0 & 0 & 0 & -7 & 0 & -4 & -4 & 0 & 0 \end{bmatrix}$$

from C we compute the following vector $min = [-7 \ -7 \ -7 \ 0 \ -7 \ 0 \ -3 \ -7 \ -7]^t$. In this case the minimum value is 0, corresponding to two indexes: 4 and 6. This means that we have two centroids, i.e. K_I and K_{E_I} .

3.2 Effective Navigation of Web Pages

The methods discussed above compute the center of a graph with respect to the topology and “spatial” information on the nodes. Referring to the example in Figure 2, in any method we have the center K_I (the centroid method reports K_{E_I} also). In this case such center allows to reach all nodes of the graph, but the navigation starting from such center is not effective: K_I corresponds to the Web page with all Kenneth Iverson. The best starting point would be K_{E_I} that is the Kenneth Iverson directly linked to the APL programming language page. Therefore beyond the center based on the spatial information of the graph, we need a “center of interest”, i.e. some vertex that is more closed to the significant pages than others. In other words we need the node that is close to the nodes having high scores. Therefore, we compute the center of interest as described in the Algorithm 4.

In the algorithm, we calculate the closeness of each node, that is the sum of distances from the others but we normalize it with respect to the score of the node (lines[5-13]): the center of interest will have the minimum closeness with the highest score. If the node with index i we are considering has score

Algorithm 3: Center computation by centroid values.

Input : The graph G
Output: The center c_F

```

1  $n \leftarrow V.length;$ 
2  $C \leftarrow InitializeMatrix(n,n);$ 
3  $min \leftarrow InitializeArray(n);$ 
4  $M \leftarrow FloydWarshall(G);$ 
5 for  $i \leftarrow 0$  to  $n$  do
6   for  $j \leftarrow 0$  to  $n$  do
7     if  $i == j$  then
8        $C[i][j] \leftarrow \infty;$ 
9     else
10      for  $h \leftarrow 0$  to  $n$  do
11        if  $h \neq i \wedge h \neq j$  then
12          if  $M[i][h] \leq M[j][h]$  then
13             $C[i][j] \leftarrow C[i][j] + 1;$ 
14          else if  $M[i][h] \leq M[j][h]$ 
15            then
16               $C[i][j] \leftarrow C[i][j] - 1;$ 
16 for  $i \leftarrow 0$  to  $n$  do
17    $min[i] \leftarrow Min(C[i]);$ 
18  $i_{max} \leftarrow MaxIndex(min);$ 
19  $c_F \leftarrow V[i_{max}];$ 
20 return  $c_F;$ 

```

Algorithm 4: Center of interest.

Input : The graph G .
Output: The center of interest c .

```

1  $n \leftarrow V.length;$ 
2  $D \leftarrow InitializeArray(n);$ 
3  $M \leftarrow FloydWarshall(G);$ 
4  $min \leftarrow 0;$ 
5 for  $i \leftarrow 0$  to  $n$  do
6    $D[i] \leftarrow 0;$ 
7   if  $\omega(V[i]) > 0$  then
8     for  $j \leftarrow 0$  to  $n$  do
9       if  $\omega(V[j]) > 0$  then
10         $D[i] \leftarrow D[i] + M[i][j];$ 
11       $D[i] \leftarrow \frac{D[i]}{\omega(V[i])};$ 
12   else
13      $D[i] \leftarrow \infty;$ 
14  $i_{min} \leftarrow MinIndex(D);$ 
15  $c \leftarrow V[i_{min}];$ 
16 return  $c;$ 

```

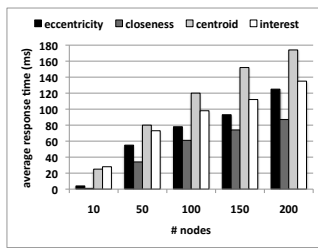


Figure 3: Performance of the approach.

0 then the closeness is ∞ . We store all values into the vector D , initialized by `InitializeArray(line[2])`. Finally we calculate the index i_{min} corresponding to the minimum value in D and the center of interest c will be the node in V with index i_{min} . Referring again to our example we have the following vector $D = [\infty \ \infty \ \infty \ \frac{8}{2} \ \frac{12}{1} \ \frac{6}{2} \ \frac{9}{1} \ \frac{10}{1} \ \frac{9}{1}]^t$. Since the minimum value is 3 (i.e. $\frac{6}{2}$) the center of interest has index 6, that is the node `K_E_I`. The joint use of the center calculated by spatial methods (i.e. eccentricity, closeness and centroid values) and the center of interest allows an effective navigation of the result.

4 EXPERIMENTAL RESULTS

We implemented our framework in a Java tool¹. The tool is according to a client-server architecture. At client-side, we have a Web interface based on GWT that provides (i) support for submitting a query, (ii) support for retrieving the result and (iii) a graphical view to navigate the Web pages via the resulting annotation. At server-side, we have the core of our query engine. We used YAANII (De Virgilio et al., 2009), a system for keyword search over RDF graphs. We have executed several experiments to test the performance of our tool. Our benchmarking system is a dual core 2.66GHz Intel with 2 GB of main memory running on Linux. We have used *Wikipedia3*, a conversion of the English Wikipedia into RDF. This is a monthly updated data set containing around 47 million triples. The user can submit a keyword search query Q to YAANII that returns the top-10 solutions. Each solution is a connected subgraph of *Wikipedia3* matching Q . In Figure 3 we show the performance of our system to compute the centrality indices. In particular we measured the average response time (ms) of ten runs to calculate the center in any method. Then, publishing the system on the Web, we asked to several and different users (i.e. about 100) to test the tool by providing a set of ten keyword search queries and to

¹A video tutorial is available at <http://www.youtube.com/watch?v=CpJhVhx3r80>

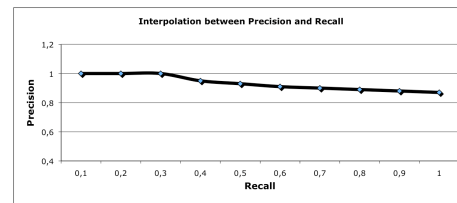


Figure 4: Effectiveness of the approach.

indicate if the centers are really effective. In this way we calculate the interpolation between precision and recall as shown in Figure 4. All these results validate the feasibility and effectiveness of our approach.

5 CONCLUSIONS AND FUTURE WORK

In this paper we discussed and implemented an approach for an effective navigation of Web pages by using semantic annotations. The approach is based on defining and implementing centrality indices, allowing the user to automatically select the starting point from which to reach the Web pages of interest. Experimental results demonstrate how it is significant the use of semantic annotations for surfing the Web effectively. In particular, an effective visualization of the annotation matching the user request improves the quality of the navigation. For future work, we are investigating new methods to individuate the starting point in distributed architectures.

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