State Estimation and Send on Delta Strategy Codesign for Networked Control Systems*

Ignacio Peñarrocha, Daniel Dolz, Julio A. Romero and Roberto Sanchis
Dep. Enginyeria de Sistemes Industrials i Disseny, University Jaume I, Campus de Riu Sec, Castellón, Spain

Keywords: State Estimation, Networked Control, Send on Delta, Event based Estimation and Control.

Abstract: In this work, a new strategy to minimize the use of the network in state estimation over networks is addressed, leading to a co-design procedure of both the observer and the policy for message sending. The sensor nodes implements a send-on-delta approach, sending new data only when there is a considerable deviation from the last sent measurement. The estimator node implements a gain scheduling approach that takes into account the availability of new received data. The performance of the observer is analyzed through $\mathcal{H}_\infty$ norm in both deterministic and stochastic data transfer rate. This norm is used to design both the observer gains and the output variations that induce the sensors to send new outputs to the estimator node, while guaranteeing a given level of performance on the state estimation error. The design approach is based on an optimization procedure with linear and bilinear matrix inequalities constraints that is solved iteratively.

1 INTRODUCTION

The reduction of data traffic through the communication networks while obtaining acceptable closed loop performance, is the main goal in many networked control systems design methodologies. Co-design strategies, where communication can be optimized with respect to the controller's stability and/or performance, represent in the last years a widely accepted approach to deal with this problem, (Wang and Lemmon, 2009; Dai et al., 2010; Gaid et al., 2006; Irwin et al., 2010).

State estimation play a key role in networked control systems, because in most of the practical applications the full state of the plant is not available for control purposes. Many algorithms have been proposed considering remote sensor nodes that send data over the network to an estimation node. Co-design has also been extended, taking into account both the estimation quality and the communication issues, to the solution of this problem: optimize the network usage while guaranteing a given estimation requirement such as a prescribed state estimation error covariance. (Marck and Sijs, 2010) proposes the design of a measurement sampling protocol that is used in combination with an event-based state-estimator. The protocol minimizes communication resources and the state estimation is accurate and remains stable even when no samples are sent.

In this paper a new methodology for estimator co-design is presented, which considers the send-on-delta (SOD) transmission between the sensor nodes and the estimator node. The SOD method consists of transmitting data from the sensor to the estimator node only if the measurement value changes more than a given specified Δ value, (Miskowicz, 2006).

Previous works on SOD based estimator design are (Nguyen and Suh, 2007) and (Suh et al., 2007) where two approaches are presented to improve the Kalman Filter (KF) when SOD transmission method with a pre-established value of Δ is used. In the last of those works, the required value of Δ is calculated by using the stationary Kalman filter equations for the worst case, in order to reach a given estimation error. Other works on SOD based estimation are (Nguyen and Suh, 2008; Nguyen and Suh, 2009; Staszek et al., 2011) where new algorithms are proposed but without following a codesign approach.

In this work, the proposed approach considers the value of Δ as a trade-off parameter between the network usage and the estimation performance. The design is then addressed by means of an optimization problem whose solution includes the estimator gain and the value of Δ for the sensor nodes, in order to fulfil the estimation requirements with the lower data transmission load. Two alternatives are considered.

*This work has been supported by MICINN project number DPI2011-27845-C02-02.

Peñarrocha I., Dolz D., A. Romero J. and Sanchis R..
State Estimation and Send on Delta Strategy Codesign for Networked Control Systems.
DOI: 10.5220/0004038304990504
In *Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2012), pages 499-504
ISBN: 978-989-8565-21-1

In the first one, a deterministic approach is used that guarantees poly-quadratic stability and an $\mathcal{H}_\infty$ attenuation level, assuming that no information about the derivative of the output is known, leading to a value of Δ that is lower than the one obtained in (Suh et al., 2007), but resulting in a much lower computational cost algorithm than the Kalman filter. In the second one, some information about the output derivatives is assumed to be known, and the optimization problem is formulated in terms of the probabilities of output transmission, assuring mean square stability, and leading to a value of Δ that is larger to the one in (Suh et al., 2007), i.e., leading to a lower traffic over the network. Furthermore, the computational cost of the resulting estimator is also much lower than the Kalman filter one.

The paper is organized as follows. In Section 2 the problem is defined, and the different approaches are presented. In Section 3 the gain-scheduled approach is analyzed in depth, and an optimization procedure is presented to obtain the observer gains that assure stability and a given disturbance attenuation. In section 4 an iterative procedure is presented to obtain the largest Δ that guarantees a given level of performance for the state estimation error. In section 5 an example is developed showing the main differences between the addressed approaches and, finally, in section 6 the main conclusions are summarized.

2 PROBLEM STATEMENT

Consider a networked control system, in which the control action is assumed to be updated synchronously with the output measurement. The plant is also assumed to be modeled by a linear time invariant system described by the following equations:

$$x[t+1] = Ax[t] + B_u u[t-1] + Bw[t], \quad (1a)$$
$$y[t] = Cx[t] + v[t], \quad (1b)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^{n_u}$ is the known input vector, $w \in \mathbb{R}^{n_w}$ is the unmeasurable state disturbance vector, $y \in \mathbb{R}$ is the measured output, and $v \in \mathbb{R}$ the measurement noise. The root mean square norms of the disturbance and noise are assumed to be known (i.e., $\|w\|_{RMS}$ and $\|v\|_{RMS}$). At a given period $t = t_k$, a measured plant output is assumed to be sent by the sensor node to the estimation node through the communication network. Let us call that sent data as $y_k = y[t_k]$, where k is defined as an integer index to enumerate the sent data. Then, applying the SOD strategy a new measurement will only be sent if

$$|y[t] - y_k| \geq \Delta \quad (2)$$

In that case, the $(k+1)$-th measurement data is sent, and $y[t]$ becomes y_{k+1} for future reference. Let us denote the number of control periods between transmitted outputs as $N_k = t_{k+1} - t_k$.

The purpose of the state estimator node is to estimate the system state using the received output information. The proposed observer equations are

$$\hat{x}[t^-] = A\hat{x}[t-1] + B_u u[t-2], \quad (3)$$
$$\hat{x}[t] = \hat{x}[t^-] + L[t](m[t] - C\hat{x}[t^-]), \quad (4)$$

where $m[t]$ is the estimated measured output, and $L[t]$ is the observer gain to be used as a function of the characteristics of the estimated measured output. $m[t]$ includes both the information of the output value (y_k) and the information of the uncertainty related to that measurement. In this sense, if the observer node receives a new measurement data, $m[t]$ refers to the sensor measurement signal (y_k) modeled by

$$m[t] = y_k = Cx[t] + v[t].$$

But if there is no new measurement data, $m[t]$ refers to the last sensor measurement signal plus an additive noise as

$$m[t] = y_k + \delta[t] = Cx[t] + v[t] + \delta[t],$$

where $\delta[t]$ is a virtual noise signal fulfilling $\delta[t] \in (-\Delta, \Delta)$, i.e., $\|\delta[t]\|_\infty \leq \Delta$, because if no new data is received, the output of the system fulfills $|y[t] - y_k| < \Delta$. Under the assumption of a uniform distribution of $\delta[t]$, it is easy to obtain $\|\delta[t]\|_{RMS} \leq \frac{\Delta}{\sqrt{3}}$. Let us now define $\alpha[t]$ as the availability factor, that is a binary variable that takes a value of 1 if there is a new measurement from the sensor node and 0 otherwise. With this new variable, the available measurement of the output can be modeled as

$$m[t] = Cx[t] + v[t] + (1 - \alpha[t])\,\delta[t]. \quad (5)$$

One of the goals of this work is to define an observer that makes use of the scarcely received data. Two different general approaches can be considered for that purpose. On one hand the Kalman filter approach can be addressed leading to the following equations

$$\hat{x}[t^-] = A\hat{x}[t-1] + B_u u[t-2], \quad (6a)$$
$$P[t^-] = AP[t]A^T + BWB^T, \quad (6b)$$
$$L[t] = P[t^-]C^T\left(CP[t^-]C^T + \sigma_v + (1-\alpha[t])\sigma_\delta\right)^{-1} \quad (6c)$$
$$\hat{x}[t] = \hat{x}[t^-] + L[t](m[t] - C\hat{x}[t^-]), \quad (6d)$$
$$P[t] = (I - L[t]C)P[t^-] \quad (6e)$$

wherr W, σ_v and σ_δ are the covariances of the state disturbance w, the measurement noise v and the virtual noise δ, respectively. Note that σ_δ is related to Δ, as $\sigma_\delta = \|\delta[t]\|_{RMS} \leq \frac{\Delta}{\sqrt{3}}$. Note also that, as $\alpha[t]$ does not reach an stationary behavior, the gain matrix $L[t]$ will not converge to any stationary value.

On the other hand, as an alternative to the Kalman filter, a gain scheduling approach is proposed leading to the algorithm

$$\hat{x}[t^-] = A\hat{x}[t-1] + B_u u[t-2], \tag{7a}$$

$$\hat{x}[t] = \hat{x}[t^-] + L(\alpha[t])(m[t] - C\hat{x}[t^-]), \tag{7b}$$

where a different gain is used depending on the availability of new measurements, according to

$$L(\alpha[t]) = (1-\alpha[t])L_0 + \alpha[t]L_1,$$

i.e., a gain that takes the value L_0 or L_1.

The goal is to design the observer gains and to minimize the use of the network while some estimation performance is guaranteed. This goal can be achieved by maximizing Δ and, therefore, minimizing the instants of time in which (2) is fulfilled (with the corresponding data transmission).

In order to address this objective, one must first notice that the Kalman filter does not reach any stationary value on the observer gains and, therefore, does not allow a priori analysis of the achievable performance. In (Suh et al., 2007) this drawback of the Kalman filter approach is overcome by means of analyzing off-line the steady state Kalman filter for the worst case scenario (i.e., with $\alpha[t] = 0$ in (6c)), leading to an optimization procedure to obtain the Δ value that is then implemented online with the gains $L[t]$ obtained with algorithm (6).

The scheduled-gain strategy used in this work allows a priori analysis of the behaviour related to Δ without the necessity of considering the worst case scenario. The goal is to minimize the use of the network by maximizing Δ, but guaranteing some estimation performance with gain $L(\alpha[t])$. Two different approaches are proposed depending on the a priori available information of the process output, leading to better results than other previous works as (Suh et al., 2007) when assuming the same information knowledge.

Remark 1. *With the state estimation strategy proposed in* (7)*, the following state estimation error* ($\tilde{x}[t] = x[t] - \hat{x}[t]$) *dynamics is easily derived*

$$\tilde{x}[t] = \mathcal{A}_{\alpha[t]}\tilde{x}[t-1] + \mathcal{B}_{\alpha[t]}\begin{bmatrix} w[t-1]^T & v[t] & \delta[t] \end{bmatrix}^T \tag{8}$$

being

$$\mathcal{A}_{\alpha[t]} = (I - L(\alpha[t])C)A,$$

$$\mathcal{B}_{\alpha[t]} = \begin{bmatrix} (I - L(\alpha[t])C)B - L(\alpha[t]) - (1-\alpha[t])L_0 \end{bmatrix},$$

$$L(\alpha[t]) = (1-\alpha[t])L_0 + \alpha[t]L_1.$$

Note that this is a discrete time linear switched system where the parameter $\alpha[t]$ *takes values 0 or 1. The goal of this paper is to design gains* L_0 *and* L_1*, at the same time that the maximum allowable bound on* Δ *is computed such that a certain bound on the error* $\tilde{x}[t]$ *is guaranteed.*

3 OBSERVER DESIGN

Assuming a given SOD policy (i.e., a given Δ), in this section two approaches are presented for the design of an observer that takes into account all the possible scenarios related to the reception of new data from the sensor node. First, in theorem 1, a deterministic strategy is proposed assuring poly-quadratic stability and a given $\mathcal{H}_\infty$ attenuation level. Then, in theorem 2, a stochastic approach assuring mean square stability, as well as an $\mathcal{H}_\infty$ attenuation level is proposed, under the assumption of some knowledge on the output derivatives, similar to the assumptions used in (Suh et al., 2007; Miskowicz, 2006).

Theorem 1. *Let us assume that observer* (7) *is used to estimate the state of system* (1) *whose measured outputs are sent with the SOD policy. If there exist matrices* P_i*,* Q_i*,* X_i *(*$i = 0,1$*), and positive values* γ_w*,* γ_v *and* γ_δ *such that* $P_i = P_i^T \succ 0$*, and*

$$\begin{bmatrix} Q_i + Q_i^T - P_i & \star & \star & \star & \star \\ ((Q_i - X_iC)A)^T & P_j - I & \star & \star & \star \\ ((Q_i - X_iC)B)^T & 0 & \gamma_w I & \star & \star \\ -X_i^T & 0 & 0 & \gamma_v & \star \\ -(1-i)\cdot X_i^T & 0 & 0 & 0 & \gamma_\delta \end{bmatrix} \succ 0 \tag{9}$$

for all $i, j \in \{0,1\} \times \{0,1\}$*, then if the observer gain is defined as* $L_i = Q_i^{-1}X_i$ *(*$i = 0,1$*), the following conditions are fulfilled: under null disturbances, the system is asymptotically stable, and, under null initial conditions, the state estimation error is bounded by*

$$\|\tilde{x}[t]\|_{RMS}^2 < \gamma_w\|w[t]\|_{RMS}^2 + \gamma_v\|v[t]\|_{RMS}^2 + \gamma_\delta\|\delta[t]\|_{RMS}^2. \tag{10}$$

Proof 1. *If* (9) *holds, then, it is obvious that* $Q_i + Q_i^T - P_i \succ 0$*, and, therefore,* Q_i *is a nonsingular matrix. In addition, if* P_i *is a positive definite matrix, it is always true that* $(P_i - Q_i)^T P_i^{-1}(P_i - Q_i) \succeq 0$*, implying that* $Q_i + Q_i^T - P_i \succeq Q_i^T P_i^{-1} Q_i$*. Using this fact, replacing* X_i *by* Q_iL_i*, in* (9)*, performing congruence transformation by matrix* $Q_i \oplus I \oplus I \oplus 1 \oplus 1$ *and applying Schur complements it leads to*

$$\begin{bmatrix} P_j - I & \star & \star & \star \\ 0 & \gamma_w I & \star & \star \\ 0 & 0 & \gamma_v & \star \\ 0 & 0 & 0 & \gamma_\delta \end{bmatrix} - \underbrace{\begin{bmatrix} ((I - L_iC)A)^T \\ ((I - L_iC)B)^T \\ -(L_i)^T \\ -(1-i)\cdot(L_i)^T \end{bmatrix}}_{\star} P_i(\star) \succ 0. \tag{11}$$

Now, let us define a Lyapunov function depending on the sampling scenario ($\alpha[t] = 0$ *or* $\alpha[t] = 1$) *as*

$$V[t] = V(\tilde{x}[t], \alpha[t]) = \tilde{x}[t]^T((1-\alpha[t])P_0 + \alpha[t]P_1)\tilde{x}[t],$$

that can be rewritten as $V(\tilde{x}[t], \alpha[t]) = \tilde{x}[t]P_i\tilde{x}[t]$. *Now, multiplying expression* (11) *by* $[\tilde{x}[t]^T, w[t]^t, v[t]^T, \delta[t]^T]$ *on the left, and by its transpose on the right, and assuming* $\alpha[t+1] = i$ *and* $\alpha[t] = j$, *it leads*

$$\begin{aligned} &\tilde{x}[t+1]^T P_i \tilde{x}[t+1] - \tilde{x}[t]^T P_j \tilde{x}[t] + \tilde{x}[t]^T\tilde{x}[t] < \\ &< \gamma_w w[t]^T w[t] + \gamma_v v[t]^T v[t] + \gamma_\delta \delta[t]^T \delta[t] \end{aligned} \quad (12)$$

for any pair i, j *in* $\{0,1\} \times \{0,1\}$. *Now, if null disturbances are assumed, it leads to* $V[t+1] < V[t]$, *i.e., the asymptotic stability of the observer is assured. Now, if null initial state estimation error is assumed* ($\tilde{x}[0] = 0, V[0] = 0$) *and expression* (12) *is added from* $t = 0$ *to* T *one obtains*

$$\begin{aligned} &V[T+1] + \sum_{t=0}^{T} \tilde{x}[t]^T\tilde{x}[t] < \quad (13) \\ &< \sum_{t=0}^{T} \left(\gamma_w w[t]^T w[t] + \gamma_v v[t]^T v[t] + \gamma_\delta \delta[t]^T \delta[t]\right) \end{aligned}$$

As $V[T+1] > 0$, *dividing by* T *and taking the limit when* T *tends to infinity, one finally obtains* (10)*.*

In the previous theorem, all the possible combinations of consecutive scenarios related to the reception of new data where assumed. If some information about the output dynamics is assumed, the previous result can be relaxed with the following stochastic approach. As proposed in (Miskowicz, 2006; Suh et al., 2007), let us assume that the expected value of the absolute value of the output difference between control periods, given by $\Delta_y = \mathcal{E}\{|y[t] - y[t-1]|\}$ is known. Let us also assume that the probability density function of that variable is such that the probability of sending a new output in a given control period can be approximated by $p_1 = P\{\alpha[t] = 1\} = \frac{\Delta_y}{\Delta + \Delta_y}$, and, hence, the probability of not having a new measurement $p_0 = P\{\alpha[t] = 1\} = 1 - p_1$.

Theorem 2. *Let us assume that observer* (7) *is used to estimate the state of system* (1) *whose measured outputs are sent with the SOD policy. If there exist matrices* P, X_i ($i = 0,1$), *and positive values* γ_w, γ_v *and* γ_δ *such that* $P = P^T \succ 0$, *and*

$$\begin{bmatrix} p_0 P & \star & \star & \star & \star & \star \\ 0 & p_1 P & \star & \star & \star & \star \\ p_0 \bar{\mathcal{A}}_0^T & p_1 \bar{\mathcal{A}}_1^T & P - I & \star & \star & \star \\ p_0 \bar{\mathcal{B}}_0^T & p_1 \bar{\mathcal{B}}_1^T & 0 & \gamma_w I & \star & \star \\ -p_0 X_0^T & -p_1 X_1^T & 0 & 0 & \gamma_v & \star \\ -p_0 X_0^T & 0 & 0 & 0 & 0 & \gamma_\delta \end{bmatrix} \succ 0 \quad (14)$$

where $\bar{\mathcal{A}}_i = ((P - X_i C)A)$, $\bar{\mathcal{B}}_i = ((P - X_i C)B)$.

Then if the observer gain is defined as $L_i = P^{-1}X_i$ ($i = 0,1$), *the following conditions hold: under null disturbances, the system is mean square stable, and, under null initial conditions, the state estimation error is bounded by*

$$\|\tilde{x}[t]\|_{RMS}^2 < \gamma_w \|w[t]\|_{RMS}^2 + \gamma_v \|v[t]\|_{RMS}^2 + \gamma_\delta \|\delta[t]\|_{RMS}^2. \quad (15)$$

Proof 2. *Following similar steps to those in proof 1, and defining a unique Lyapunov function* $V[t] = x[t]^T P \tilde{x}[t]$ *it is easy to demonstrate that* (14) *implies*

$$\begin{aligned} &\mathcal{E}\{V[t+1]\} - V[t] + \tilde{x}[t]^T\tilde{x}[t] < \\ &< \gamma_w w[t]^T w[t] + \gamma_v v[t]^T v[t] + \gamma_\delta \delta[t]^T \delta[t]. \end{aligned} \quad (16)$$

where $\mathcal{E}\{V[t+1]\}$ *is the next expected value for the Lyapunov function over the two possible modes of the switched system* ($\alpha[t] = 0$ *and* $\alpha[t] = 1$ *in* (8)*). Then, if null disturbances are assumed, it leads* $\mathcal{E}\{V[t+1]\} < V[t]$, *i.e., the mean square stability of the observer is assured. Now, if null initial state estimation error is assumed* ($\tilde{x}[0] = 0$, $V[0] = 0$) *and expression* (16) *is added from* $t = 0$ *to* T *it leads to*

$$\begin{aligned} &\mathcal{E}\{V[T+1]\} + \sum_{t=0}^{T} \tilde{x}[t]^T\tilde{x}[t] < \quad (17) \\ &< \sum_{t=0}^{T} \left(\gamma_w w[t]^T w[t] + \gamma_v v[t]^T v[t] + \gamma_\delta \delta[t]^T \delta[t]\right) \end{aligned}$$

As $\mathcal{E}\{V[T+1]\} > 0$, *dividing by* T *and taking the limit when* T *tends to infinity, one finally obtains* (15)*.*

Remark 2. *If the RMS values of the disturbance, noise and virtual noise are assumed to be known, then the minimization of the sum* $\gamma_w \sigma_w^2 + \gamma_v \sigma_v^2 + \gamma_\delta \sigma_\delta^2$ *over LMI* (9), $i, j \in \{0,1\} \times \{0,1\}$ *leads to the gain-scheduled observer that minimizes the RMS value of the state estimation error. If the probability of output reception is also assumed to be known, then the optimization can be done over LMI* (14), *leading to a lower state estimation error. If the RMS values of the disturbances are not available, they can be used as tuning parameters to achieve a given desired behavior.*

4 OBSERVER CODESIGN

The last remark referred to the problem of designing an observer for a given send-on-delta policy, trying to minimize the estimation error. If the estimation error is only desired to be guaranteed to stay under a prescribed level $\|\tilde{x}[t]\|_{RMS,\max}$, then a different strategy can be devised trying to minimize the network resources used. This will improve the network performance, and increase the battery life of sensors over wireless networks. This can be achieved by searching for the maximum Δ for which $\|\tilde{x}[t]\|_{RMS} < \|\tilde{x}[t]\|_{RMS,\max}$ is assured. Note that this optimization approach can be viewed as the search for the maximum acceptable noise signal, as $\delta[t]$ has been interpreted as a virtual noise on the estimator node. The following optimization algorithm allows to find the Δ and the observer gains that minimize the sensor transmission rate subject to the prescribed state estimation performance constraint, if a uniform random signal $\delta[t]$ taking values within $[-\Delta,\Delta]$ and leading to a root mean square $\|\delta[t]\|_{RMS} = \frac{\Delta}{\sqrt{3}}$, is assumed:

$$\max_{P_{0,1},Q_{0,1},X_{0,1},\gamma_v,\gamma_w,\gamma_\delta,\Delta} \quad \Delta \tag{18}$$

$$s.t. \quad (9),\ i,j \in \{0,1\}\times\{0,1\}$$

$$\gamma_w\sigma_w^2 + \gamma_v\sigma_v^2 + \gamma_\delta\frac{\Delta^2}{3} \leq \|\tilde{x}[t]\|^2_{RMS,\max}$$

Note that this optimization problem is non linear due to the second constraint, in which the decision variable Δ appears nonlinearly on the product $\gamma_\delta\Delta^2$. The result of the optimization problem is not affected by the use of Δ or Δ^2, and this can be easily handled. However, the product between γ_δ and Δ leads to a bilinear inequality that implies a non convex optimization problem. However, as there is only one product between decision variables, a linesearch through variable Δ is easy to be implemented to find the optimal solution of the previous optimization problem.

If the expected absolute output increment in a period is assumed to be known, then the following optimization procedure is proposed

$$\max_{P,X_{0,1},\gamma_v,\gamma_w,\gamma_\delta,\Delta} \quad \Delta \tag{19}$$

$$s.t \quad (14), \quad p_1 = \frac{\Delta_y}{\Delta+\Delta_y}, p_0 = 1-p_1$$

$$\gamma_w\sigma_w^2 + \gamma_v\sigma_v^2 + \gamma_\delta\frac{\Delta^2}{3} \leq \|\tilde{x}[t]\|^2_{RMS,\max}.$$

Note that this is again a nonlinear optimization problem due to the facts presented above plus the appearance of the term $\frac{\Delta_y}{\Delta+\Delta_y}$ on constraint (14), but, again, a linesearch procedure can be used to find the optimal solution.

5 EXAMPLES

Consider a discrete-time process with an integrator defined by matrices

$$A=\begin{bmatrix}0.613 & 0.233\\ 0.274 & 0.835\end{bmatrix}, B=B_u=\begin{bmatrix}0.232\\ 0.398\end{bmatrix}, C^T=\begin{bmatrix}0.1207\\ 0.4426\end{bmatrix}$$

Table 1: Comparative results of the three approaches.

Strategy	Δ	$\|L_0\|$	$\|L_1\|$	$\|\tilde{x}[t]\|_{RMS}$
KF	0.6630	-	-	0.0361
(18)	0.1523	1.6920	2.2560	0.0458
(19)	2.0149	$1.67\cdot10^{-5}$	2.2626	0.0751

Assume a disturbance bounded by the norm $\|w\|_{RMS} = 0.1$, and a measurement noise bounded by $\|v\|_{RMS} = 0.01$. Assume that the estimation error is desired to be guaranteed to be under $\|\tilde{x}[t]\|_{RMS,\max} = 0.2$ and that the mean value of the absolute value of the output increment in one period is $\Delta_y = 1$. Applying the procedures presented in section 4 in order to get the maximum Δ according to the imposed restrictions, the results that are summarized in table 1 are obtained. Comparing the two strategies that are based on a worst-case scenario (KF in (Suh et al., 2007) and the gain scheduling obtained with (18)), the first one achieves a higher Δ. This is because zero mean disturbances are used, for which the KF is optimized, while this fact is not taken into account with strategy (18) (it is also valid for non zero mean disturbances). With the strategy presented in (19), the highest Δ is achieved under the assumption of a known mean on the output discrete derivative. The third and fourth columns show the values of the norm of the resulting gains computed offline with the proposed $\mathcal{H}_\infty$ strategies. The fifth column shows the state estimation error when a simulation with the three approaches is carried out over a controlled plant with outputs fulfilling $\Delta_y = 1$. All the approaches lead to a $\|\tilde{x}\|_{RMS}$ lower than the allowed one, and the reason is the intrinsic conservatism on send-on-delta approaches. This means that the assumed $\pm\Delta$ bound when there is no new measurement available, can be far from the expected value of the output when new measurement have been recently received. It can be noticed that for the Kalman filter the difference from $\|\tilde{x}\|_{RMS}$ and the allowed bound is larger than for the other approaches, due to its conservative consideration of maximum virtual noise. Figure 1 shows the implementation of the three approaches (each one with its corresponding Δ), showing that the approach (19) is the one that minimize the number of output transmissions through the network. In order to compare the behaviour of the KF and the approach (19), when dealing with non zero mean state disturbances, different simulations have been carried out to obtain the achieved state estimation error with a fixed RMS norm of the state disturbance, but different mean values. Figure 2 shows the resulting state estimation error when implementing both the KF approach and (19) with a fixed $\Delta = 2.0149$. It can be observed how the performance of the proposed approach improves the one of the KF when the mean value of

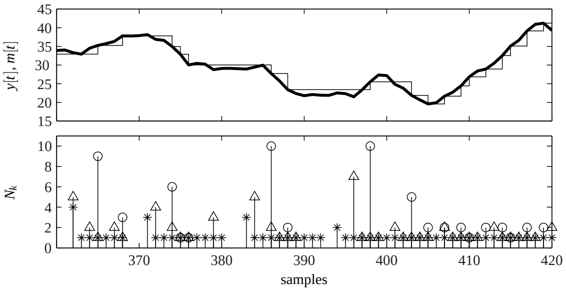

Figure 1: a) Measured ($y[t]$) and received outputs ($m[t]$) for $\Delta = 2.0149$. b) Intersampling periods (N_k) for the three approaches: ('$\triangle$': KF, '$*$': (18), '$\circ$': (19)).

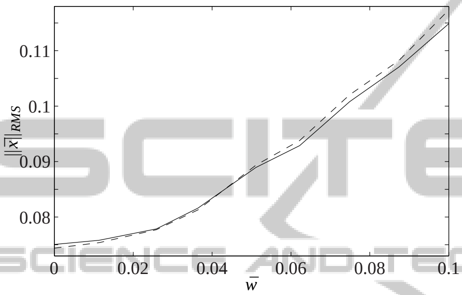

Figure 2: Achieved $\|\tilde{x}[t]\|_{RMS}$ as a function of the mean value of w for $\Delta = 2.0149$ ('- -': KF, '–': (19)).

the disturbance increases.

6 CONCLUSIONS

In this work, an observer codesign procedure for state estimation over networks has been addressed using the send-on-delta methodology (an output measurement is transmitted only when the measured value has changed more than Δ with respect to the last transmitted value). The design procedure consists of obtaining both the observer gains and the maximum value of Δ that guarantees a prescribed state estimation error. The proposed observer is a gain-scheduling one that applies a different gain depending on the availability of new measurements. The resulting closed loop estimator dynamics has been obtained leading to a linear discrete time switching system. Sufficient conditions to assure the stability and a given level of disturbance attenuation have been established under the stated assumptions. Furthermore, a procedure to obtain the maximum value of Δ for a prescribed estimation error has been proposed. Two different alternative approaches have been presented. In the first one, a deterministic approach is used that guarantees poly-quadratic stability and an $\mathcal{H}_\infty$ attenuation level, assuming that no information about the derivative of the output is known, leading to a value of Δ that is lower than the one obtained in other Kalman filter based approaches, but resulting in a much lower computational cost algorithm. In the second one, some information about the output derivatives is assumed to be known, and the optimization problem is formulated in terms of the probabilities of output transmission, assuring mean square stability, and leading to a value of Δ that is larger than the one obtained in other Kalman filter based approaches, i.e., leading to a lower traffic over the network. Furthermore, the computational cost of the resulting estimator is also much lower than the Kalman filter one. A detailed example has illustrated the validity of the approach compared to the Kalman filter based approach.

REFERENCES

Dai, S.-L., Lin, H., and Ge, S. S. (2010). Scheduling-and-control codesign for a collection of networked control systems with uncertain delays. *Control Systems Technology, IEEE Transactions on*, 18(1):66 –78.

Gaid, M., Cela, A., and Hamam, Y. (2006). Optimal integrated control and scheduling of networked control systems with communication constraints: application to a car suspension system. *Control Systems Technology, IEEE Transactions on*, 14(4):776 – 787.

Irwin, G., Chen, J., McKernan, A., and Scanlon, W. (2010). Co-design of predictive controllers for wireless network control. *Control Theory Applications, IET*, 4(2):186 –196.

Marck, J. and Sijs, J. (2010). Relevant sampling applied to event-based state-estimation. In *Sensor Technologies and Applications (SENSORCOMM), 2010 Fourth International Conference on*, pages 618 –624.

Miskowicz, M. (2006). Send-on-delta concept: An event-based data reporting strategy. *Sensors*, 6(1):49–63.

Nguyen, V. H. and Suh, Y. S. (2007). Improving estimation performance in networked control systems applying the send-on-delta transmission method. *Sensors*, 7(10):2128–2138.

Nguyen, V. H. and Suh, Y. S. (2008). Networked estimation with an area-triggered transmission method. *Sensors*, 8(2):897–909.

Nguyen, V. H. and Suh, Y. S. (2009). Networked estimation for event-based sampling systems with packet dropouts. *Sensors*, 9(4):3078–3089.

Staszek, K., Koryciak, S., and Miskowicz, M. (2011). Performance of send-on-delta sampling schemes with prediction. In *Industrial Electronics (ISIE), 2011 IEEE International Symposium on*, pages 2037 –2042.

Suh, Y. S., Nguyen, V. H., and Ro, Y. S. (2007). Modified Kalman filter for networked monitoring systems employing a send-on-delta method. *Automatica*, 43(2):332 – 338.

Wang, X. and Lemmon, M. (2009). Self-triggered feedback control systems with finite-gain stability. *Automatic Control, IEEE Transactions on*, 54(3):452 –467.