New Robust Controller Synthesis Optimization Methodology under Six Sigma Constraint

J. Frechard[1], D. Knittel[1,2] and J. Renaud[1]
[1]LGeCo, INSA, 24 Boulevard de la Victoire, 67084 Strasbourg, France
[2]Institute of Physics and Engineering, University of Strasbourg, 17, Rue du Maréchal Lefèbvre, 67100 Strasbourg, France

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Abstract: Industrial systems usually contain uncertain parameters. These uncertainties come from lack of knowledge of physical phenomena or from evaluation difficulties. The stochastic behaviour of these parameters have to be taken into account during the design process of industrial systems. To tackle with these uncertainties, the Design for Multi-objective Six Sigma method is used. This method permits to fix six sigma constraint and it is applied on the synthesis of web tension controller for an industrial large-scale roll-to-roll system. The key idea is to synthesize the controller in frequency domain by Hinfinity synthesis. Then the six sigma constraint is fixed on output web tension in time domain. The robust controller synthesis is then compared with standard Hinfinity synthesis approach. This paper presents for the first time a controller synthesis including six sigma constraints.

1 INTRODUCTION

Real world optimization problem usually contains uncertainties. These uncertainties come from non modelled dynamics or from a lack of knowledge of the phenomena. Moreover, some parameters can be difficult to estimate and can vary with time. Due to these uncertainties the value of performance criterion can be very different of the expected one. Some industrial applications deal with high parameter variations depending on the processing conditions. Usual optimization methodology only considers the design performance without taking into account parametric uncertainties. It is compulsory, for application with uncertain parameters, to consider the parametric robustness of design. Performance and parametric robustness are antagonist objectives, that means that the improvement of one of them leads to the damage of the other. Therefore, a multi-objective approach has to be used. The key idea is to generate a set of values of uncertain parameters following their distribution law. The fitness function is calculated for each value of the set and the mean value and standard deviation are given as the objectives to minimize in frequency domain. Then the time domain Pareto frontier is build and six sigma constraint is fixed on it. This paper presents for the first time a controller synthesis methodology including six sigma constraint. The proposed approach is applied to the web tension controller synthesis of a large-scale roll-to-roll system.

Roll-to-roll systems are very common in industry, they represent a convenient way to handle web materials such as textile, paper, polymer... The web speed and web tension are the two keys variables to be monitored and controller. The focus of roll-to-roll systems is to displace the material web at the expected speed while keeping the web tension in an acceptable range around the tension reference. The approach developed in this paper is applied on a large-scale roll-to-roll plant that includes a rewinder, an unwinder, two driven rollers, two pendulum dancers and twenty-five idle rollers.

PI or PID controllers are commonly used for web tension control in industrial context, (Gassmann et al., 2011) proposes a convenient way to synthesize PI web tension controller using fixed order and structure H_∞ approach using HIFOO (H-Infinity fixed order optimization, (Burke et al., 2006)). This synthesis method gives good results in term of performances but the obtained controllers are highly sensitive regarding web elasticity variations.

A new robust design optimisation method is proposed in this paper. For our application, the key idea is to consider the web elasticity, Young's modulus, as an uncertain parameter with a normal law.

Firstly, a short state of the art of robust optimiza-

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tion is given. Then the proposed multi-objective robust optimization is presented in details. To finish, the application to the large-scale roll-to-roll system is described.

2 ROBUST OPTIMIZATION IN ENGINEERING

Optimization of mechatronic systems is studied for several years. A survey of the robust optimization methods is given in (Beyer and Sendhoff, 2007).

The field of application of robust optimization is very large. For example, the design of antenna under decision variable uncertainty is studied in (Ben-Tal and Nemirovski, 2002). Mathematical programming methods is used to deal with parametric uncertainty.

Interval programming was successfully implemented for synthesizing modal control in (Khlebalin, 1994). The idea is to give the interval of variation of each uncertain parameters and to directly take them into account in the optimization process.

Another robust optimization method that gives good results in control synthesis using optimal control is the Kharitonov's theorem presented in (Toscano and Lyonnet, 2010). The idea is to verify the stability of the system afterwards the control synthesis using the characteristic polynomial of the closed loop system.

However, the presented methods can not be implemented for optimization of large-scale systems, due to high order of the model or lack of algebraic representation of the fitness function.

The design for six sigma method is studied in (Koch et al., 2004). The major drawback of this method is that the sigma level and the limit value has to be fixed before optimization process runs, and therefore it is not sure that the problem is solvable.

An interesting method that permits to fix the sigma level and the limit value after the optimization process was presented in (Shimoyama and Fujii, 2007). The idea is to use multi objective optimization in order to obtain a set of robust designs.

3 MULTI-OBJECTIVE ROBUST DESIGN OPTIMIZATION METHOD

The multi-objective design for six sigma methods needs many evaluations of the fitness function to be efficient. To measure the performance of mechatronic systems the H_∞ norm can be used. It gives good evaluation of the system behaviour in frequency domain and have an acceptable computational cost.

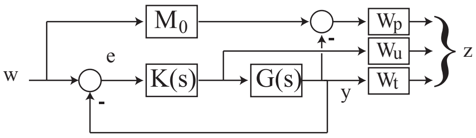

Figure 1: Scheme of the S/KS/T synthesis with reference model M_0.

3.1 Hinfinity Controller Synthesis

The H_∞ approach enables automatic synthesis of controllers (Gassmann et al., 2009). The H_∞ problem consists in finding a stabilizing controller K that minimize the H_∞ norm of the transfer function between a set of exogenous inputs w and the performance outputs z (see Figure 1).

$$\|T_{w\to z}\|_\infty < \gamma \quad (1)$$

The major drawback of H_∞ approach is the high order of the obtained controller. In fact, the order of the controller is equal to the sum of the system order and the weighting functions order. Using a model reduction approach, the controller order can not always be reduced with guarantee of stability and performances. In industrial applications, it is highly relevant to develop a robust design algorithm for PI or PID controllers adjusting. The mathematical problem seems to be difficult because fixed-order controller synthesis can be formulated as a nonsmooth affine problem in the nonconvex cone of stables matrix. The mathematical formulation of such a problem leads to BMI (bilinear matrix inequalities) solving (Benlatreche et al., 2006). Recent progress in nonsmooth algorithm permits to develop relevant synthesis tools like HIFOO released in 2005 (Burke et al., 2006) and more recently *hinfstruct* (Apkarian and Noll, 2006). One drawback of such approaches is that they do not take parametric uncertainties into consideration. In this work, genetic algorithm is used to optimize the H_∞ norm of the system in order to take into account uncertainties. The controller is synthesized with output weighting functions; S/KS/T synthesis scheme is given in Figure 1. Moreover, a reference model M_0 is used. The weighting functions W_p, W_u, W_t and the reference model M_0 appear in the closed loop transfer matrix:

$$T_{wz} := \begin{bmatrix} W_p(M_0 - T) \\ W_u KS \\ W_t T \end{bmatrix} \quad (2)$$

where S is the sensitivity function:

$$S = (I + GK)^{-1} \quad (3)$$

G represents the system, K is the web tension controller. T is the complementary sensitivity function:

$$T = I - S \tag{4}$$

The weighting function W_p has a high gain at low frequency in order to reject low frequency disturbances. The form of W_p is as follows :

$$W_p(s) = \frac{\frac{s}{M} + \omega_B}{s + \omega_B \varepsilon_0} \tag{5}$$

where M is the maximum peak magnitude of $(M_0 - T)$. ω_B is the desired frequency bandwidth, ε_0 is the allowed steady-state error.

The weighting function W_u is used to avoid large control signals and to increase the roll-off of the controller output at high frequencies.

$$W_u(s) = \frac{1+\tau_1 s}{k(1+\tau_2 s)}, \quad \tau_1 > \tau_2 \tag{6}$$

The weighting function W_t increases the roll-off of the system output at high frequencies.

In industry, tension controllers are generally PI controllers:

$$\frac{u}{\varepsilon} = K_p \frac{1+\tau_i s}{s} \tag{7}$$

where u is the control signal and ε is the error signal. K_p and τ_i are the controller design variables to be optimized. The optimization problem can be formulated as follows :

$$\begin{array}{ll} minimize & ||T_{wz}||_\infty \\ subject\ to & \lambda < 0 \end{array} \tag{8}$$

where $||T_{wz}||_\infty$ is the weighted closed-loop H_∞ norm and λ is the maximum real part of the system poles (for the linearised system) also called spectral abscissa.

The uncertain parameter has now to be taken into account in the H_∞ synthesis.

3.2 Robust Controller Synthesis using Hinfinity Approach

Robust optimization methodology used in this work is based on the six-sigma concept introduced by Motorola (Tennant, 2001). The aim is to keep a performance index in an acceptable range for a variation of uncertain parameters of $\pm 6\sigma$, with σ the standard deviation. In fact, a six sigma constraint corresponds to 0.002 defects per millions. In order to solve such problem, the Design For Multi-Objective Six Sigma (DFMOSS) described in (Koch et al., 2004) is used. This methodology was successfully applied in the domain of flying vehicule in (Shimoyama and Fujii, 2007) and in the domain of land vehicule (Koch et al., 2004).

Considering the state-space representation of the uncertain system:

$$\left\{ \begin{array}{lcl} \dot{x}(t) & = & A(\gamma)x(t) + B(\gamma)u(t) \\ y(t) & = & C(\gamma)x(t) + D(\gamma)u(t) \end{array} \right. \tag{9}$$

where γ is a set of uncertain parameters with a normal distribution.

Latin hypercube method is used to generate a set of sample points around the nominal value of uncertain parameters. Then the robust H_∞ optimization problem can be defined as follows.

$$\begin{array}{ll} minimize & \mu_H \\ & \sigma_H \\ under & \lambda < 0 \end{array} \tag{10}$$

where μ_H is the mean of the fitness function (the H_∞ norm in our case) for the set of sample points and σ_H is the standard deviation of the fitness function. This methodology permits to obtain the robust Pareto frontier and then to fix the sigma level and the upper limit constraint after optimization. To deal with this optimization problem, the MOGA-II algorithm is used in the modeFRONTIER commercial software environment.

3.3 Time Domain Pareto Building

The H_∞ optimization process is used to guarantee the frequency domain system behaviour. Moreover the H_∞ norm has a very low computational cost. Nevertheless, time domain optimization allows to take into account the plant non-linearities. It is therefore compulsory to use simulation of the nonlinear model of the system. Therefore the pseudo-Pareto frontier in time domain is also build and the six σ constraint can be applied on it. The non dominated point in frequency domain can not be used alone. In fact, a Pareto optimal design in frequency domain (where the linear model is used), does not guarantee to obtain a Pareto optimal design in time domain (where the nonlinear model is used. This is also true for the linear model). A set of points close to the Pareto frontier in frequency domain has to be chosen and simulated in order to build the pseudo-Pareto frontier in time domain. To select the points useful for time simulation, an area of the search space is defined. The two anchor points (an anchor point is a point where one objective is minimum) are chosen to form a rectangle, this area can be seen in Figure 2.

If this area contains an acceptable number of designs, about a hundred, they will be all simulated. Otherwise only some of them have to be selected. In

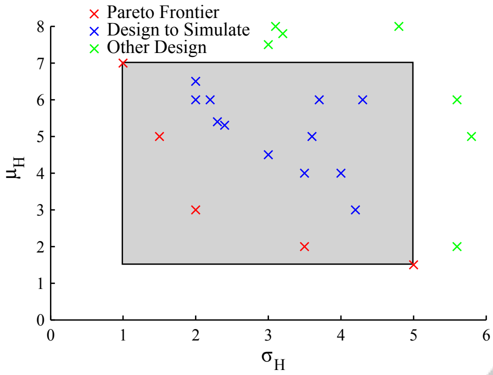

Figure 2: Area around the Pareto obtained from frequency domain optimization.

order to chose the designs for non-linear time simulation we propose to cut the objective space in different areas and to select a point in each of them. The simulation of the non-linear model can be very time consuming and therefore the number of designs has to be limited to one hundred. Moreover, the Pareto proximity has to be taken into account: a point close to the Pareto frontier should have a higher probability to be chosen that a point far from the Pareto frontier.

A method to divide the search space around the Pareto frontier using circles and lines is proposed. Two points in the objective space have to be defined. Firstly, the UTOPIA point which is an unattainable design where all the objectives are minimum. Secondly, the NADIR point is the opposite of the UTOPIA point, it corresponds to the point with the value of the first objective when the second is minimum and vice-versa (see figure 3). These two points are used for the first step of the cut-out method; the circular areas cut-out. The UTOPIA point is taken as circle center, the distance between the utopia point and the closest point is computed and used as minimum value of the circle radius. The distance between UTOPIA and NADIR points is used as maximum circle radius. To guarantee the probability of selection of a point based on Pareto proximity, the difference between minimum and maximum radius is divided in several areas following a non-linear law:

$$R_i = R_{i-1} + \frac{R_{max} - R_{min}}{\frac{1-n^{nb}}{1-n} - 1} n^{(i-1)} \qquad (11)$$

where R_i and R_{i-1} are respectively the radius of the considered circle and the radius of the previous circle. n is a multiplication factor between two consecutive spaces, in this work n is equal to 1.3. nb is the number of circles.

The second phase of the cut-out method uses lines in order to divide the objective space circles. The angle between the utopia point and the two ANCHOR points is divided into several areas using a linear law. Then, in each area a point is chosen using random search. An example of the obtained areas and selected designs can be seen in figure 3.

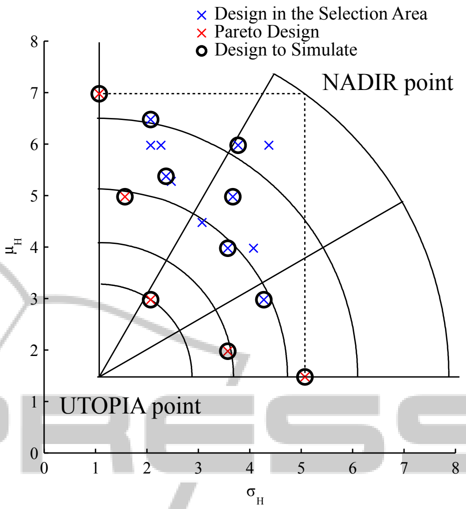

Figure 3: Example of areas and selected points provided by the cut-out strategy (frequency domain).

The cut-out strategy permits to guarantee the dispersion of the simulated design around the frequency domain Pareto (the design close to the Pareto have a higher probability to be selected). The selected designs in the frequency domain are then simulated in time domain, using the non-linear system model.

3.4 Building of the Time Domain Pareto Frontier

In order to evaluate performances of time domain simulations results, many criteria can be used. IAE (Integral of absolute error) and ISE (Integral of squared error) are common criteria. In some applications, the derivative of the error signal can be added to IAE or ISE criterion to minimize oscillation. In several specific applications the error signal does not play a major role but the maximum of the error signal is the major criterion to minimize.

Then a set of thirty values of each uncertain parameter is generated using latin hypercube sampling. The designs selected in the frequency domain Pareto using the cut-out strategy are simulated for each value of the uncertain parameters. The mean value μ_J and standard deviation σ_J of the set of fitness are then calculated. Once the time domain Pareto frontier is build, the six sigma constraint can be fixed on it:

$$\mu_J + 6\sigma_J < USL \qquad (12)$$

where USL is the upper acceptable limit of J.

Figure 4 shows an example of obtained time domain pareto frontier. The red line represents the 6 sigma constraint: the design under it respects the constraint (In other words, the criterion J is greater than the USL value with a probability of only 0.002 per million). Some applications do not need a so high reliability, for them lower sigma level can be used (for example 3 sigma).

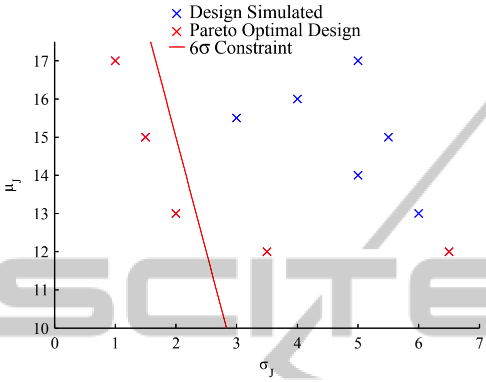

Figure 4: Example of time domain Pareto.

For the choice of the final design, the distance between the six σ constraint line and each design is computed. The more robust design is the design with the higher distance. The presented method can now be applied on a complex large-scale system in order to synthesize robust controllers.

4 APPLICATION TO ROLL-TO-ROLL SYSTEMS

The proposed approach is applied to the synthesis of tension controllers of an industrial roll-to-roll system. This system is composed of four motor, two pendulum dancers, thirty rollers and five load cells. Its model is constructed from the equation describing the speed of each roller, the web tension between two rollers and the pendulum dancers angular position (Koc et al., 2002) (Gassmann et al., 2009). The scheme of the system is given in figure 6.

The tension control of roll-to-roll systems is studied for several years. H_∞ control is used and gives good performance (Gassmann et al., 2011) (Gassmann et al., 2009). However, this method does not take into account the parametric uncertainties.

The approach developped in this work has been compared with the fixed order and structure H_∞ synthesis using the commercial *hinfstruct* algorithm (Apkarian and Noll, 2006) in the Matlab software environment. The results in frequency domain are firstly compared using the linear model, then the time domain simulation are compared for a variation of web elasticity on the non-linear system model.

The frequency domain and time domain criteria depicted below are used. The value of each criterion for the nominal value of web elasticity E_0, the mean and standard deviation of each criterion, are calculated for the set of web elasticity values. A summary of the results is given in table 1.

Table 1: Criteria comparison between hinfstruct and our robust design optimization

	H_∞ (E_0)	J (E_0)	μ_H	σ_H	μ_J	σ_J
(1)	93	11	94	5	12	1.6
(2)	88	8	116	29.2	84	165

The line (1) corresponds to our presented synthesis method whereas the second line marked (2) corresponds to the advanced synthesis using *hinfstruct*. One can see that for our approach the first two criteria are a little greater. Therefore the *hinfstruct* gives slightly better nominal performances (for a fixed nominal web elasticity). However, when web elasticity variations occur, our proposed methodology leads to better results. In fact, mean values and standard deviation of the two criteria are lower.

Then the two set of parameters are compared using non-linear model simulations. The simulation results are presented for three values of web Young's modulus, $E_0/1.5$, E_0 and $E_0.1.5$ in Figure 5 (intermediate web tension).

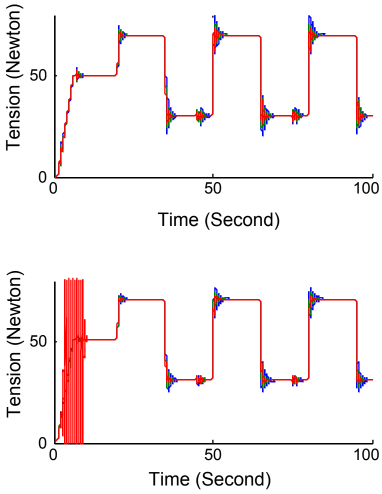

Figure 5: Time domain simulation.

For the *hinfstruct* controller synthesis approach the system becomes unstable (lower representation in figure 5) whereas our approach maintains good performances (upper representation in Figure 5) with

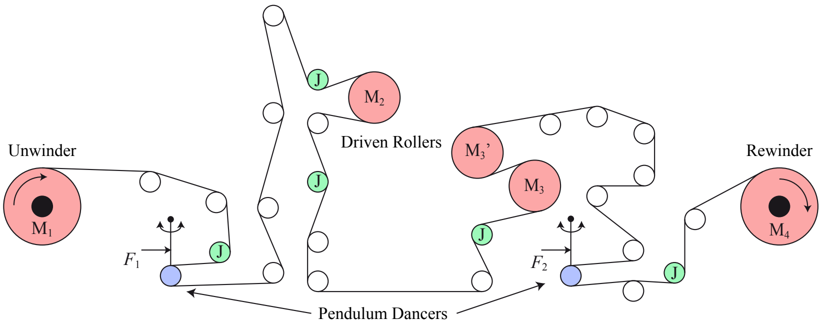

Figure 6: Scheme of the system under study.

web tension errors smaller than 20 N, as expected (six sigma design).

5 CONCLUSIONS

A new robust controller synthesis method is developed using six sigma constraint to tackle with parametric uncertainties. A set of values of the uncertain parameter is computed and statical tools are used in order to build a set of robust design. This set allows to build the time domain Pareto in order to fix six sigma constraint in time domain. The Pareto in closed-loop frequency domain is also calculated. The two Pareto curves enable to select the best design.

The methodology is applied on a complex large scale roll-to-roll model of an experimental plant. The results are compared with advanced commercial robust controller synthesis (*hinfstruct* Robust control toolbox in Matlab software environment). The simulations show that the proposed methodology leads to more robust closed loop system performances regarding the uncertain parameter variations.

Future work consist in integrating other parameters in the optimization process, for example mechanical parameters.

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