On the Temperature Control for a Test Case Short Pipe Network Central Heating System

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Abstract: In the present paper the mathematical representation of a test case central heating system with a short pipng network, three radiators and one boiler heating two apartments is developed in the form of a nonlinear model. A linear dynamic controller achieving independent apartment temperature control and being unaffected from the external temperature is proposed. The controller is developed on the basis of a linear approximant. The closed loop performance is tested through simulations on the original nonlinear model.

1 INTRODUCTION

The problem of modelling and control of central heating systems has attracted considerable attention during the last years (see Cai, 2006; Hansen, 1997; Koumboulis et al., 2007; Koumboulis et al., 2008a and b; Koumboulis et al., 2009a,b and c; Koumboulis and Kouvakas, 2010; Mendi et al., 2002; Morel et al., 2001; Zaheer-Uddin et al., 1994; Zanobini et al., 1998 and the references therein). Significant attention has also been given to the modelling of core components of central heating systems as well as to the application of different control techniques to regulate the performance variables. These techniques range from classical and metaheuristic controllers, fuzzy control schemes, adaptive controllers, optimal controllers to multi delay dynamic controllers satisfying transfer matrix design requirements. In the present paper the mathematical representation of a test case short pipe network central heating system including two apartments is presented in the form of a nonlinear model. The first apartment has one room while the second is considered to have two rooms. The system consists of a short piping network, three radiators, a boiler and three rooms. The main difference between the present model and those in Koumboulis et al., (2007), Koumboulis et al., (2008a and b), Koumboulis, Kouvakas and Paraskevopoulos (2009a-c), and Koumboulis and Kouvakas (2010) is the length of the pipes. The transport delays are small and so they do not significantly to influence the behavior of the plant. Thus, the present mathematical representation does not incorporate time delays. Furthermore, the first and the second room of the second apartment are coupled via direct heat exchange. A linear dynamic controller achieving independent apartment temperature control and being unaffected from the external temperature is proposed. The controller is developed on the basis of a linear approximant. The closed loop performance is examined through simulations on the original nonlinear model.

2 MODEL OF THE SYSTEM

In the present section the mathematical model of the test case central heating system, will be presented. It is similar to that proposed in Koumboulis et al., (2007), Koumboulis et al., (2008a and b), Koumboulis et al., (2009a, b and c), and Koumboulis and Kouvakas (2010). The system consists of the piping network, three radiators, a boiler and three rooms. In each room one of the radiators is installed. The main difference between the model developed here as compared to those presented in Koumboulis et al., (2007), Koumboulis et al., (2008a and b), Koumboulis et al., (2009a-c), and Koumboulis and Kouvakas (2010) is that small length pipes are used. Thus, the transport delays are small and so they can be neglected. Furthermore, the two rooms of the second apartment are considered to be coupled via direct heat exchange. Let $i=1$

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corresponds to the room of the first apartment, while $i=2$ and $i=3$ correspond to the two rooms of the second apartment. Using the results presented in the aforementioned papers, the nonlinear dynamic model of the process can be computed to be in the following general nonlinear form:

$$E(x,u)\dot{x}(t)=F(x,u,\xi) \qquad (1a)$$

$$y(t)=Cx(t),\ \psi(t)=Lx(t) \qquad (1b,c)$$

where

$$x=\left[x_1 \quad x_2 \quad x_3 \;\vdots\; x_4 \quad x_5 \;\vdots\; x_6 \quad x_7 \;\vdots\; x_8 \quad x_9 \;\vdots\; x_{10} \;\vdots\; x_{11} \quad x_{12} \;\vdots\; x_{13} \quad x_{14} \;\vdots\; x_{15} \quad x_{16}\right]=$$
$$=\left[q_1 \quad q_2 \quad q_3 \;\vdots\; T_{1,1} \quad T_{2,1} \;\vdots\; T_{1,2} \quad T_{2,2} \;\vdots\; T_{1,3} \quad T_{2,3} \;\vdots\; T_b \;\vdots\; T_{r,1} \quad T_{f,1} \;\vdots\; T_{r,2} \quad T_{f,2} \;\vdots\; T_{r,3} \quad T_{f,3}\right]^T$$

$$u=\left[u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5\right]^T=$$
$$=\left[Q_{burner} \quad \Delta P \quad k_{f,v_1} \quad k_{f,v_2} \quad k_{f,v_3}\right]^T$$

$$\xi=\left[\xi_1 \quad \xi_2 \quad \xi_3 \quad \xi_4\right]^T=\left[T_{out,1} \quad T_{out,2} \quad T_{out,3} \quad T_e\right]^T$$

and where q_1, q_2 and q_3 are the volumetric flow - rates in the radiators of the first, second and third rooms respectively, $T_{j,i}$ is the j^{th} section temperature of the radiator placed in the i^{th} room, T_b is the boiler effluent temperature, $T_{r,i}$ and $T_{f,i}$ are the ambient air and floor temperatures of the i^{th} room, Q_{burner} is the energy supply to the boiler, ΔP is the pressure applied to the pipe network by the pump, k_{f,v_i} is the pressure drop coefficient of the i^{th} valve, $T_{out,i}$ is the external temperature of the i^{th} room and T_e is the boiler room temperature. Note that Q_{burner}, ΔP and k_{f,v_i} are actuatable inputs while $T_{out,i}$ and T_e are measurable disturbances. Note that $y(t)$ and $\psi(t)$ are the performance and measurable output vectors, respectively, while L and $C\in\mathbb{R}^{3\times 16}$. Their nonzero elements are $c_{1,15}=1$, $c_{2,11}=0.5$, $c_{2,13}=0.5$, $c_{3,11}=1$, $c_{3,13}=-1$, $l_{1,11}=1$, $l_{2,13}=1$ and $l_{3,15}=1$. The nonzero elements of E and F are computed to be:

$$e_{1,1}(x,u)=8L\rho d^{-2}\pi^{-1},\ e_{1,2}(x,u)=16L\rho d^{-2}\pi^{-1}$$

$$e_{2,1}(x,u)=8L\rho d^{-2}\pi^{-1},\ e_{2,3}(x,u)=16L\rho d^{-2}\pi^{-1}$$

$$e_{3,2}(x,u)=8L\rho d^{-2}\pi^{-1},\ e_{3,3}(x,u)=8L\rho d^{-2}\pi^{-1}$$

$$e_{1,3}(x,u)=\frac{4\rho}{\pi}\left(\frac{L_r}{d_r^2}+\frac{6L}{d^2}+\frac{u_5}{\Gamma\,\text{[redacted]}\,d}\right)$$

$$e_{2,2}(x,u)=\frac{4\rho}{\pi}\left(\frac{L_r}{d_r^2}+\frac{4L}{d^2}+\frac{u_4}{\Gamma(x_2,d)d}\right)$$

$$e_{3,1}(x,u)=\frac{4\rho}{\pi}\left(\frac{L_r}{d_r^2}+\frac{2L}{d^2}+\frac{u_3}{\Gamma(x_1,d)d}\right)$$

$$e_{4,4}(x,u)=1,\ e_{5,5}(x,u)=1,\ e_{6,6}(x,u)=1$$

$$e_{7,7}(x,u)=1,\ e_{8,8}(x,u)=1,e_{9,9}(x,u)=1$$

$$e_{10,10}(x,u)=1,\ e_{11,11}(x,u)=1,\ e_{12,12}(x,u)=1$$

$$e_{13,13}(x,u)=1,\ e_{14,14}(x,u)=1,e_{15,15}(x,u)=1$$

$$e_{16,16}(x,u)=1$$

$$e_{10,1}(x,u)=\nu(x)\left[x_2(x_7-x_5)+x_3(x_9-x_5)\right]$$

$$e_{10,2}(x,u)=\nu(x)\left[x_1(x_5-x_7)+x_3(x_9-x_7)\right]$$

$$e_{10,3}(x,u)=\nu(x)\left[x_1(x_5-x_9)+x_2(x_7-x_9)\right]$$

$$e_{10,5}(x,u)=ax_1(a-1)^{-1}(x_1+x_2+x_3)^{-1}$$

$$e_{10,7}(x,u)=ax_2(1-a)^{-1}(x_1+x_2+x_3)^{-1}$$

$$e_{10,9}(x,u)=ax_3(1-a)^{-1}(x_1+x_2+x_3)^{-1}$$

$$f_{1,1}(x,u,\xi)=-2f_p(x_3,L,d)-f_p(x_3,L_r,d_r)-$$
$$2f_p(x_2+x_3,L,d)-2f_p(x_1+x_2+x_3,L,d)-$$
$$f_v(x_3,u_5,d)-K_tx_3^2+u_2$$

$$f_{2,1}(x,u,\xi)=-f_p(x_2,L_r,d_r)-2f_p(x_2+x_3,L,d)$$
$$-2f_p(x_1+x_2+x_3,L,d)-f_v(x_2,u_4,d)-K_tx_2^2+u_2$$

$$f_{3,1}(x,u,\xi)=-f_{\text{[redacted]}}x_1,L_r,d_r)-f_v(x_1,u_3,d)-K_tx_1^2$$
$$-2f_p(x_1+x_2+x_3,L,d)+u_2$$

$$f_{4,1}(x,u,\xi)=\frac{1}{C}60^{-n_1}\left(60^{n_1}NH_qx_1\left[(\varphi-1)x_4-\varphi x_5+x_{10}\right]-(x_4-x_{11})^{n_1}\Phi_0\right)$$

$$f_{5,1}(x,u,\xi)=C^{-1}\left[NH_qx_1(x_4-x_5)-\right.$$
$$\left.-60^{-n_1}(x_5-x_{11})^{n_1}\Phi_0\right]$$

$$f_{6,1}(x,u,\xi) = \frac{1}{C}60^{-n_1}\left(60^{n_1} N H_q x_2 \left[(\varphi - 1) x_6 - \varphi x_7 + x_{10} \right] - (x_6 - x_{13})^{n_1} \Phi_0 \right)$$

$$f_{7,1}(x,u,\xi) = C^{-1}\left[N H_q x_2 (x_6 - x_7) - 60^{-n_1} (x_7 - x_{13})^{n_1} \Phi_0 \right]$$

$$f_{8,1}(x,u,\xi) = \frac{1}{C}60^{-n_1}\left(60^{n_1} N H_q x_3 \left[(\varphi - 1) x_8 - \varphi x_9 + x_{10} \right] - (x_8 - x_{15})^{n_1} \Phi_0 \right)$$

$$f_{9,1}(x,u,\xi) = C^{-1}\left[N H_q x_3 (x_8 - x_9) - 60^{-n_1} (x_9 - x_{15})^{n_1} \Phi_0 \right]$$

$$\begin{aligned} f_{10,1}(x,u,\xi) = \Big(& -a_1 u_1 (x_1 + x_2 + x_3) T_{w,\max} + a_j \big(x_1 \left[a x_5 + (1-a) x_{10} - \xi_4\right] + x_2 \left[a x_7 + (1-a) x_{10} - \xi_4\right] + x_3 (a x_9 + (1-a) x_{10} - \xi_4) \big) T_{w,\max} \\ & + (x_1 + x_2 + x_3) x_{10} \times \left[(1-a) a_2 u_1 + \rho C_w (x_1 + x_2 + x_3) T_{w,\max} \right] \\ & - (x_1 x_5 + x_2 x_7 + x_3 x_9) \times \left[a a_2 u_1 + \rho C_w (x_1 + x_2 + x_3) T_{w,\max} \right] \Big) \\ & / \left[(a - 1) C_b (x_1 + x_2 + x_3) T_{w,\max} \right] \end{aligned}$$

$$\begin{aligned} f_{11,1}(x,u,\xi) = \Big(& 60^{-n_1} \Big[-60^{n_1} N R_{out} (x_{11} - x_{12}) R_{1,2} + R_f \Big(-60^{n_1} N (x_{11} - \xi_1) R_{1,2} + \\ & R_{out} \Big(-60^{n_1} N (x_{11} - x_{13}) + 1000 \left[(x_4 - x_{11})^{n_1} + (x_5 - x_{11})^{n_1} \right] \Phi_0 R_{1,2} \Big) \Big) \Big] \Big) \\ & /(N C_r R_f R_{out} R_{1,2}) \end{aligned}$$

$$\begin{aligned} f_{13,1}(x,u,\xi) = \Big(& 60^{-n_1} \Big[-60^{n_1} N R_{out} (x_{13} - x_{14}) R_{1,2} + R_f \Big(-60^{n_1} N (x_{13} - \xi_2) R_{1,2} + \\ & R_{out} \Big(60^{n_1} N (x_{11} - x_{13}) + 1000 \left[(x_6 - x_{13})^{n_1} + (x_7 - x_{13})^{n_1} \right] \Phi_0 R_{1,2} \Big) \Big) \Big] \Big) \\ & / (N C_r R_f R_{out} R_{1,2}) \end{aligned}$$

$$f_{15,1}(x,u,\xi) = \frac{1}{N C_r R_{out}} 60^{-n_1} \left(-60^{n_1} N (x_{16} - \xi_3) + 1000 R_{out} \left[(x_8 - x_{16})^{n_1} + (x_9 - x_{16})^{n_1} \right] \Phi_0 \right)$$

$$f_{12,1}(x,u,\xi) = \frac{x_{11} - x_{12}}{C_f R_f}, f_{14,1}(x,u,\xi) = \frac{x_{13} - x_{14}}{C_f R_f}$$

$$f_{16,1}(x,u,\xi) = \frac{x_{15} - x_{16}}{C_f R_f}, \ \Gamma(q,d) =$$

$$\left[f_1(q,d)(1 - w(\mathrm{Re}(q,d))) + f_2(q,d) w(\mathrm{Re}(q,d)) \right]$$

$$\mathrm{Re}(q,d) = \frac{4\rho q}{\mu \pi d}, f_1(q,d) = 64/\mathrm{Re}(q,d)$$

$$f_2(q,d) = \zeta_1 / \mathrm{Re}(q,d)^{0.25}$$

$$w(x) = 0.5 \tanh(\beta_1 (x - \beta_2)) + 0.5$$

$$f_p(q,L,d) = \Gamma(q,d) 8 \rho L \pi^{-2} d^{-5} q^2$$

$$f_v(q,u,d) = 8 d^{-4} \pi^{-2} q^2 u \rho,$$

$$\nu(x) = a(a-1)^{-1} (x_1 + x_2 + x_3)^{-2}$$

where $e_{i,j}(x,u)$ and $f_{i,1}(x,u,\xi)$ are the (i,j) and $(i,1)$ elements of $E(x,u)$ and $F(x,u,\xi)$, while d and d_r are the pipe and radiators diameter respectively, L denotes the lengths of the pipes connecting the pump to the first valve, the first valve to the second valve, the second valve to the third valve, L_r denotes the equivalent length of the radiators respectively; ζ_1, β_1 and β_2 are flow condition parameters, K_t is the turbulent pressure drop factor caused to the entrance of the radiators, C_w and ρ are the thermal capacity and density of the water respectively, C and Φ_0 are the heat capacity of the water/radiator material and the nominal heat of the radiator, n_1 and φ are radiator model parameters, N is the number of sections of each radiation, C_b is the thermal capacity of the boiler, a_j is the rate of heat loss from the boiler jacket to the environment, $T_{w,\max}$ is the maximum boiler effluent temperature, a_1 and a_2 are parameters connecting the boiler lumped temperature to the boiler efficiency, a is coefficient connecting the lumped water temperature of the boiler to the inlet and outlet temperature, C_f

and C_r are the floor and room thermal capacities respectively, R_f, R_{out} and $R_{1,2}$ are the thermal resistances between the room and the floor, the room and the environment and the first and second room respectively. From model (1) it is observed that the performance variables are the temperature of the room of the first apartment (1st output), the average of the temperatures of the two rooms of the second apartment (2nd output) and the difference between the temperatures of the two rooms of the second apartment (3rd output).

The linear approximant of the central heating system (1) is computed to be of the form

$$\delta\dot{x}(t) = A\delta x(t) + B\delta u(t) + D\delta\xi(t) \tag{2a}$$

$$\delta y(t) = C\delta x(t), \ \delta\psi(t) = L\delta x(t) \tag{2b,c}$$

After extensive computational experiments for a wide range of inputs and disturbances, it has been observed that the model (2) is an accurate approximant of the original nonlinear model (1). In the next section, the linear approximant will be used to develop a linear controller performing satisfactory to both the linear and the nonlinear model.

3 CONTROLLER DESIGN

Consider the dynamic controller

$$\begin{aligned}\delta U(s) &= K_1(s)\delta\Psi(s) + \\ &+K_2(s)\delta\Xi(s) + G(s)W(s)\end{aligned} \tag{3}$$

where $\delta U(s), \delta\Psi(s), \delta\Xi(s)$ and $W(s)$ denote the Laplace transform of $\delta u(t), \delta\psi(t), \delta\xi(t)$ and $w(t)$ (external command). Clearly, $K_1(s) \in [\mathbb{R}(s)]^{5\times 3}$, $K_2(s) \in [\mathbb{R}(s)]^{5\times 4}$ and $G(s) \in [\mathbb{R}(s)]^{5\times 2}$. The design goal will be that of independent control of the main performance variables (i.e. the temperature of the room of the 1st apartment and the average temperature of the two rooms of the second apartment) with simultaneous disturbance rejection. Furthermore it is required to keep the difference between the first and second room temperatures enough small. If the controller (3) is applied to the approximant (2), the design goal is formulated as a block decoupling with simultaneous disturbance rejection requirement, i.e.

$$P_C(s) = \begin{bmatrix} H_{1,1}(s) & 0 \\ 0 & H_{2,2}(s) \\ 0 & H_{3,2}(s) \end{bmatrix}, P_D(s) = 0_{3\times 4} \tag{4a,b}$$

where $P_C(s)$ and $P_D(s)$ are the transfer matrices mapping the external commands and the disturbances to the performance outputs, respectively, and where $H_{1,1}(s)$, $H_{2,2}(s)$ and $H_{3,2}(s)$ are appropriate transfer functions.

A set of controller matrices satisfying the design goal is:

$$G(s) = \left[sI_{16} - K_1(s)Q(s)\right]P^{\dagger}(s)P_C(s) \tag{5a}$$

$$\begin{aligned}K_2(s) &= -\left[sI_{16} - K_1(s)Q(s)\right]P^{\dagger}(s)\,P_d(s) + \\ &+P(s)\left[I_5 - K_1(s)Q(s)\right]^{-1}K_1(s)Q_d(s)\end{aligned} \tag{5b}$$

$$K_1(s) \text{ is arbitrary and proper} \tag{5c}$$

where

$$Q(s) = L(sI_{16} - A)^{-1}B, \ Q_d(s) = L(sI_{16} - A)^{-1}D$$

$$P(s) = C(sI_{16} - A)^{-1}B, P_d(s) = C(sI_{16} - A)^{-1}D$$

$$\left[P^{\dagger}(s) \quad P^{\perp}(s)\right] = \left(\left[P(s)^T \quad \tilde{P}(s)^T\right]^T\right)^{-1}$$

and $\tilde{P}(s)$ is a 2×5 rational matrix preserving the invertibility of $\left[P(s)^T \quad \tilde{P}(s)^T\right]$. Choose

$$H_{1,1}(s) = \prod_{j=1}^{6}(\gamma_{1,j}s+1)^{-1}, H_{2,2}(s) = \prod_{j=1}^{6}(\gamma_{2,j}s+1)^{-1}$$

$$H_{3,2}(s) = \kappa\prod_{j=1}^{6}(\gamma_{3,j}s+1)^{-1}$$

Let

$$L = 5[\mathrm{m}], \ L_r = 2[\mathrm{m}], \ d = 0.015[\mathrm{m}]$$

$$d_r = 0.0096153[\mathrm{m}], \ \rho = 971.81\left[\mathrm{Kgr/m^3}\right],$$

$$\mu = 0.0003547[\mathrm{Pa\cdot s}], \ \beta_1 = 0.00819, \ \beta_2 = 2300$$

$$N = 2, \ \Phi_0 = 0.395[\mathrm{W}], \ \varphi = 0, \ K_t = 0.00002,$$

$$C = 36[\mathrm{KJ/K}], \ n_1 = 1.25, \ a = 2/9, \ a_1 = 1,$$

$$a_2 = -0.12, \ a_j = 5.06[\mathrm{W/K}],$$

$$C_f = 2175525\left[\mathrm{J}/\mathrm{K}\right], C_r = 43268\left[\mathrm{J}/\mathrm{K}\right],$$
$$R_{out} = 0.01614\left[\mathrm{K}/\mathrm{W}\right],\ R_f = 0.0333\left[\mathrm{K}/\mathrm{W}\right],$$
$$R_{12} = 0.03\left[\mathrm{K}/\mathrm{W}\right],$$
$$T_{w,\max} = 100\left[{}^\circ\mathrm{C}\right]\ C_w = 4170\left[\mathrm{J}/\mathrm{K}\cdot\mathrm{Kgr}\right],$$
$$C_b = 42400\left[\mathrm{J}/\mathrm{K}\right],\ \zeta_1 = 0.316$$
$$\bar{u}_1 = 2000\left[\mathrm{W}\right],\ \bar{u}_2 = 2000\left[\mathrm{Pa}\right],\ \bar{u}_3 = 300,$$
$$\bar{u}_4 = 300,\ \bar{u}_5 = 300,\ \bar{\xi}_1 = \bar{\xi}_2 = \bar{\xi}_3 = \bar{\xi}_4 = 15\left[{}^\circ\mathrm{C}\right]$$
$$\bar{x}_1 = 57.58\left[\mathrm{lt}/\mathrm{hr}\right],\ \bar{x}_2 = 51.18\left[\mathrm{lt}/\mathrm{hr}\right]$$
$$\bar{x}_3 = 49.19\left[\mathrm{lt}/\mathrm{hr}\right],\ \bar{x}_4 = 92.12\left[{}^\circ\mathrm{C}\right]$$
$$\bar{x}_5 = 88.67\left[{}^\circ\mathrm{C}\right],\ \bar{x}_6 = 91.69\left[{}^\circ\mathrm{C}\right],\ \bar{x}_7 = 87.86\left[{}^\circ\mathrm{C}\right]$$
$$\bar{x}_8 = 91.53\left[{}^\circ\mathrm{C}\right],\ \bar{x}_9 = 87.57\left[{}^\circ\mathrm{C}\right],$$
$$\bar{x}_{10} = 95.79\left[{}^\circ\mathrm{C}\right]\ \bar{x}_{11} = 22.41\left[{}^\circ\mathrm{C}\right],\ \bar{x}_{12} = 22.41\left[{}^\circ\mathrm{C}\right]$$
$$\bar{x}_{13} = 22.37\left[{}^\circ\mathrm{C}\right],\ \bar{x}_{14} = 22.37\left[{}^\circ\mathrm{C}\right]$$
$$\bar{x}_{15} = 22.33\left[{}^\circ\mathrm{C}\right],\ \bar{x}_{16} = 22.33\left[{}^\circ\mathrm{C}\right]$$
$$\gamma_{1,1} = 500,\ \gamma_{1,2} = 525,\ \gamma_{1,3} = 550,\ \gamma_{1,4} = 575$$
$$\gamma_{1,5} = 600,\ \gamma_{1,6} = 625,\ \gamma_{2,1} = 500,\ \gamma_{2,2} = 525$$
$$\gamma_{2,3} = 550,\ \gamma_{2,4} = 575,\ \gamma_{2,5} = 600,\ \gamma_{2,6} = 625$$
$$\gamma_{3,1} = 500,\ \gamma_{3,2} = 525,\ \gamma_{3,3} = 550,\ \gamma_{3,4} = 575$$
$$\gamma_{3,5} = 600,\ \gamma_{3,6} = 625,\ \kappa = -0.009$$

Selecting

$$\tilde{P}(s) = \begin{bmatrix} 0.928 & -0.706 & 0 & 0 & 0 \\ 0 & 0 & -1.089 & 1.062 & 0 \end{bmatrix}$$

$$K_1(s) = \begin{bmatrix} -3954.21 & -7278.84 & 3148.84 \\ 8990.94 & -1246.73 & 6284.94 \\ -1435.48 & 3632.79 & 869.98 \\ -8654.17 & 9593.39 & 9528.82 \\ -7409.89 & -451.43 & 8273.09 \end{bmatrix}$$

the controller can be computed. It is pointed out that the resulting closed loop system is asymptotically stable.

To demonstrate the performance of the derived controller, we apply the external commands $w_1(t) = -0.5\left[{}^\circ\mathrm{C}\right]$, $w_2(t) = -1\left[{}^\circ\mathrm{C}\right]$. The disturbances are considered to be all equal to

$$\xi(t) = 15 - 2\left(1 - \frac{1}{3}\mathrm{e}^{-0.002t} + 2\mathrm{e}^{-0.001t} - \frac{8}{3}\mathrm{e}^{-0.0005t}\right)$$

Using the above assumption, the closed loop responses for the performance variables are presented in Figures 1 and 2. From Figure 1 it can readily be observed that the apartment temperatures follow accurately the reference signals with the respective curves being practically identical. The maximum error throughout the simulation was for the first performance variable about $0.012\left[{}^\circ\mathrm{C}\right]$ while for the second performance variable it was about $0.007\left[{}^\circ\mathrm{C}\right]$. Eventhough, in the third performance variable (see Figure 2) the error is significantly larger, it remains extremely small suggesting that the variation cannot be sensed by the occupants of the rooms. Before closing the section it mentioned that state variables and the actuatable input variables remain within acceptable limits.

4 CONCLUSIONS

The mathematical model of a test case central heating system with a short piping network, three radiators, a boiler and three rooms in two apartments has been developed. Based upon the linear approximant of the nonlinear model a dynamic controller, achieving independent control between the two apartment temperatures together with rejection of the influence of the external temperatures and small differences between the temperatures of the rooms of the second apartment, has been derived. The performance of the controller has been examined through simulations on the nonlinear model of the plant.

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APPENDIX

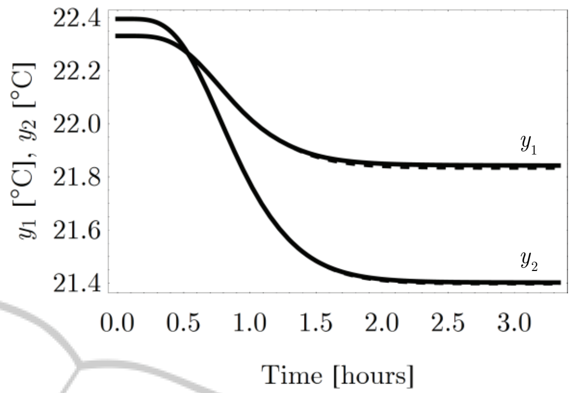

Figure 1: Closed loop responses for y_1 and y_2 (continuous - y_1 and y_2, dashed – reference).

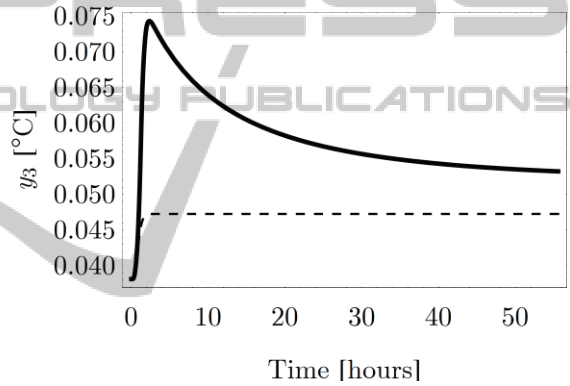

Figure 2: Closed loop response for y_3 (continuous - y_3, dashed – reference).