Behavior Analysis of a Gaussian Beam Optical Trap in the Rayleigh Regime

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Abstract: Recently optical trapping has emerged as a very powerful tool for manipulating micro and nanometer sized particles. In this paper, we present a comprehensive study of the behavior of nanometer sized trapped particles in a Gaussian beam optical trap using Rayleigh model of trapping forces. Along with the working principle of an optical trap, the force equations in the Rayleigh regime have been derived considering focused Gaussian beam. Then numerical simulations are performed for a 30 nm particle with refractive index 1.57 considering water as the surrounding medium. We assume that the wavelength of the light source to be 850 nm easily obtainable from cheap GaAs-based vertical-cavity surface-emitting laser technology. When the light hits a particle, it influences the particle with two forces–the scattering force in the direction of propagation and the gradient force in the direction of gradient of light intensity. We explore the effects of particle size, refractive index of the particle, beam waist radius, position of the particle with respect to the trap center both on scattering and gradient forces. This analysis will be helpful for understanding optical manipulation of nanoparticles and designing suitable trap modules for nanoparticle manipulation.

1 INTRODUCTION

Today the world-wide research deals with nanometer and micrometer sized particles in different field of sciences like physics, chemistry or biology. To manipulate or handle these tiny particles, an effective method had been in a demand. Optical manipulation can serve this purpose without any mechanical damage or contamination, as no physical contact is required here. It is based on the optical trapping phenomenon. To describe the trapping force, we consider three scattering regimes based on the size of the particle under consideration. The three regimes are given in Table 1, where λ is the wavelength and d is the particle diameter.

2 WORKING PRINCIPLE OF AN OPTICAL TRAP

The working principle of an optical trap is based on the light-matter interaction. We know that the light has a momentum p, which is given by

$$p = h/\lambda, \tag{1}$$

where h is the Planck's constant and λ is the wavelength of light. When the light hits a dielectric particle, there is a change of the momentum of the particle due to momentum transfer between the particle and the light. The rate of change of momentum gives rise to a force on the particle. This force can be decomposed into two components– (i) the scattering force in the direction of light propagation and (ii) the gradient force in the direction of the spatial intensity gradient. Fig. 1 shows the working principle of an optical trap. Suppose, a dielectric transparent sphere with several wavelengths of diameter and refractive index higher than that of the surrounding medium is located off-axis in an unfocused Gaussian beam. We consider two parallel rays labeled A and B of the beam hitting the sphere near the center. Ray A has greater intensity than ray B. When these rays hit the sphere, two optical phenomena happen– reflections and refractions at the interfaces of the sphere and surrounding medium. If we take only the refractions into account, these two rays will be refracted as they enter and exit the sphere. The exit rays are in the directions different from their original directions, which means a change of momentum of the incident rays. According to Newton's third law of motion, there must be an equal change of momentum of the sphere in the opposite direction. The rate of change of momentum gives rise to two forces F_A and F_B on the sphere that are in a direction perpen-

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dicular to the direction of the respective ray and these forces act at the center of the sphere. As the intensity of ray A is stronger than that of ray B, F_A is larger than F_B, which pulls the sphere toward the light intensity maximum. Considering all such symmetric pairs of rays incident on the sphere, we find that the net force F_{net} can be resolved into two components: scattering force, F_{scatt} pointing to axial direction of light beam and the transverse gradient force $F_{grad,tr}$ pointing to the radial direction of the beam. For a sphere located on-axis or in a plane wave, $F_A = F_B$ and there is no net transverse force component, the sphere does not move in the transverse direction. But the axial component, which is the scattering force pushes the sphere in the forward direction.

Fig. 2 shows a three-dimensional optical trap, also

Table 1: Different optical scattering regimes based on the size of the particle.

$d \ll \lambda$	Rayleigh regime
$d \approx \lambda$	Mie regime
$d \gg \lambda$	Ray optics regime

known as optical tweezers. With a high numerical-aperture objective, the Gaussian beam is focused resulting an intensity gradient in the axial direction. The net axial gradient force $F_{grad,ax}$ pulls the sphere towards the focus. If reflection of light is considered, there will be scattering force that pushes the sphere in the forward direction. The sphere will be stably trapped, if the net axial gradient force $F_{grad,ax}$ compensates the scattering force.

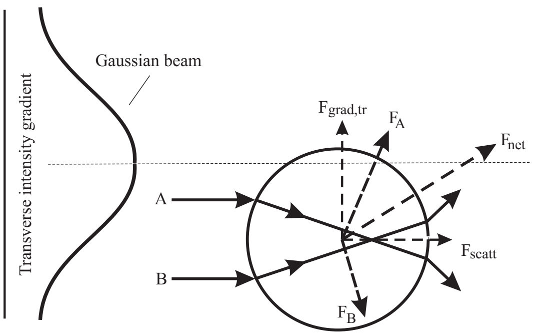

Figure 1: Illustration of scattering and gradient force of an optical trap considering only the refracted beams. The refraction of two rays A and B results in two forces namely F_A and F_B. The resultant force F_{net} can be resolved into two perpendicular components: F_{scatt} in the longitudinal direction and $F_{grad,tr}$ in the transverse direction. F_{scatt} pushes the sphere int the direction of light propagation and the $F_{grad,tr}$ pulls the sphere toward the maximum light intensity; adapted from (Ashkin, 1997).

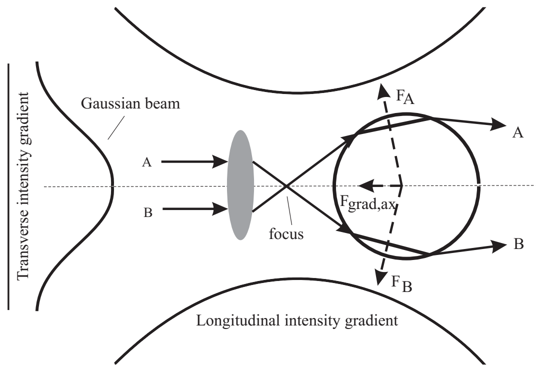

Figure 2: Schematic of a three-dimensional optical trap. The laser light is focused by a high numerical-aperture objective. The axial components of F_A and F_B add to each other and pulls the sphere towards the focus forming a three-dimensional trap.

3 DESCRIPTION OF TRAPPING FORCES IN THE RAYLEIGH REGIME

In the Rayleigh regime, the particle can be treated as an electric point dipole. As a result, we need to consider the polarizability of the dipole. However, in this regime scattering force is generated by the change of momentum of light due to scattering of light by the particle and the gradient force is generated by the Lorentz force acting on the induced dipole (Malagino et al., 2002). The direction of the Lorentz force is in the direction of the intensity gradient of light. Fig. 3 illustrates gradient force in the Rayleigh regime. The particle is shown as a dipole. The force direction is along the gradient direction of electric field and particle moves towards the highest intensity of the light beam. For a particle of radius a, the scattering force

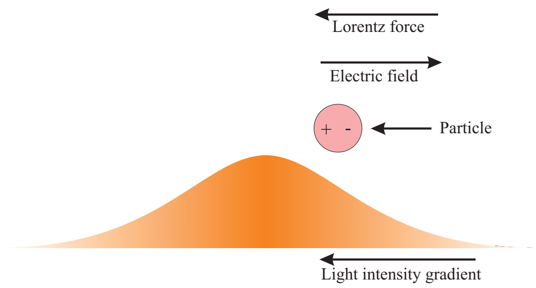

Figure 3: Schematic illustration of the transverse force in the Rayleigh regime, where dielectric particles can be treated as perfect dipoles. The Lorentz force caused by the gradient of light intensity attracts the particle towards the maximum intensity of the beam; adapted from (Schaevitz, 2006).

and gradient force is given by Neuman et al. (Neuman and Block, 2004)

$$F_{\mathrm{scatt}} = \frac{I\sigma n_{\mathrm{m}}}{c}, \tag{2}$$

with

$$\sigma = \frac{128\pi^5 a^6}{3\lambda^4}\left(\frac{m^2-1}{m^2+2}\right)^2, \tag{3}$$

where I is the intensity of the incident light, σ is the scattering cross section of the particle, n_{m} is the index of refraction of the medium, c is the speed of the light in vacuum, m is the effective refractive index defined as the ratio of the index of refraction of the particle to the index of refraction of the medium ($n_{\mathrm{p}}/n_{\mathrm{m}}$), and λ is the wavelength.

The gradient force is expressed as

$$F_{\mathrm{grad}} = \frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\nabla I, \tag{4}$$

where α is the polarizability of the sphere and expressed as

$$\alpha = {n_m}^2 a^3\left(\frac{m^2-1}{m^2+2}\right). \tag{5}$$

As the gradient force is caused by the Lorentz force, it always acts in the direction of gradient of intensity of the light as seen in Fig. 3.

4 MODELING OF TRAPPING FORCE FOR A GAUSSIAN BEAM OPTICAL TRAP

We consider a focused Gaussian beam as the three-dimensional optical trap with following intensity distribution as seen in Fig. 4

$$I(r,z) = I_0\left(\frac{w_0}{w(z)}\right)^2 \exp\left(\frac{-2r^2}{w^2(z)}\right), \tag{6}$$

with

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_{\mathrm{r}}}\right)^2}, \tag{7}$$

where I_0 is the intensity of light at the beam center, $r = \sqrt{x^2+y^2}$ is the radial distance from the center axis of the beam or the so-called beam-axis, z is the axial distance from the beam center, $w(z)$ is the spot size as a function of z and the beam waist $w_0 = w(z)$ at $z = 0$. z_{r} is the Rayleigh length and is given by

$$z_{\mathrm{r}} = \frac{\pi{w_0}^2}{\lambda}. \tag{8}$$

A particle of radius a ($a \ll \lambda$) is located at $z = z_1$, $x = x_1$ and $y = 0$ as in Fig. 4. The particle will experience the following forces: (i) the scattering force along $+z$ direction (F_{scatt}), (ii) the gradient force along $+x$ direction ($F_{\mathrm{grad,tr}}$) and (iii) the gradient force along $-z$ direction ($F_{\mathrm{grad,ax}}$). As $y = 0$, there will be no gradient force component along y direction. F_{scatt} is directly given by Eq. (2). To get the other two gradient forces we refer to Eq. (4), which can be written as

$$F_{\mathrm{grad}} = \frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)I, \tag{9}$$

where i, j and k are the unit vectors along the x, y and z axes, respectively. From Eq. (9), we can write

$$F_{\mathrm{grad}} = iF_{\mathrm{x,grad}} + jF_{\mathrm{y,grad}} + kF_{\mathrm{z,grad}}, \tag{10}$$

where

$$F_{\mathrm{x,grad}} = F_{\mathrm{x,grad,tr}} = \frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\left(\frac{\partial I}{\partial x}\right), \tag{11}$$

$$F_{\mathrm{y,grad}} = F_{\mathrm{y,grad,tr}} = \frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\left(\frac{\partial I}{\partial y}\right), \tag{12}$$

$$F_{\mathrm{z,grad}} = F_{\mathrm{grad,ax}} = \frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\left(\frac{\partial I}{\partial z}\right). \tag{13}$$

In this case $F_{\mathrm{y,grad}} = 0$. However, after some calculations we reach the magnitudes of the gradient forces as

$$F_{\mathrm{grad,tr}} = F_{\mathrm{x,grad}} = -I_0\frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\left(\frac{4{w_0}^2 x}{w^4(z)}\right)\exp\left(\frac{-2x^2}{w^2(z)}\right), \tag{14}$$

and

$$F_{\mathrm{grad,ax}} = -I_0\frac{2\pi\alpha}{c{n_{\mathrm{m}}}^2}\left[\frac{2{w_0}^2 z}{w^2(z)}\exp\left(\frac{-2x^2}{w^2(z)}\right) - \frac{4x^2{w_0}^2 z}{w^3(z)}\exp\left(\frac{-2x^2}{w^2(z)}\right)\right]. \tag{15}$$

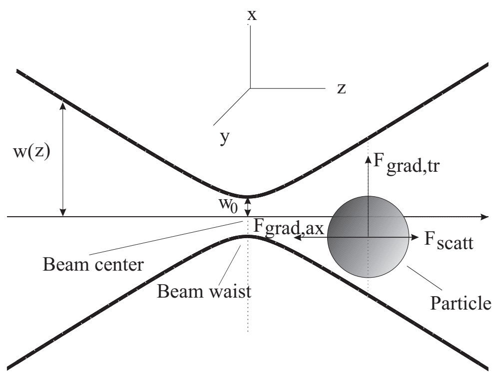

Figure 4: Modeling of trapping force exerted on particle located in a focused Gaussian beam in the Rayleigh regime.

5 SIMULATION RESULTS AND DISCUSSIONS

In this section, we present the simulation results for the particle located in the optical trap in the Rayleigh regime as shown in Fig. 4. The simulations are done for a particle with refractive index of 1.57 with water as the surrounding medium ($n_{\mathrm{m}} = 1.33$). The diameter of the particle is 30 nm. Also, the effects of different beam waist radii of 40 nm, 60 nm, 80 nm and 100 nm were observed. A focused Gaussian beam is assumed as the optical trap. Here the beam-axis is the z-axis. The simulation results, along with discussions are presented below for the forces, which are experienced by the particle.

5.1 Axial Gradient Force

Fig. 5 shows the dependence of axial gradient force as a function of the axial distance of the particle from the beam center. When the particle is located at left side of the beam center, it experiences a force towards the beam center along the positive z-axis indicated by the positive magnitude of force at the left of the origin. On the other hand, if the particle is located at the right side of the bean center, it experiences the force along the negative z-axis. The effect of beam waist radius is also observed. Very close to the beam center, the less the beam waist radius, the more the axial gradient force. Fig. 6 depicts the effect of transverse

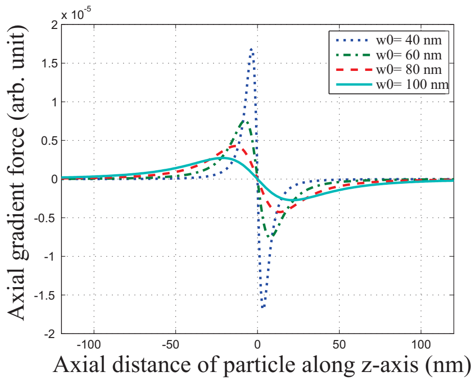

Figure 5: Axial gradient force versus axial distance z of the particle located at $x = 0$, $y = 0$ from the beam center for different beam waist radii.

distance from the beam-axis on axial gradient force. It is observed that the maximum axial gradient force which acts along the negative z-axis occurs when it is located on the beam-axis. In this case, we get highest axial gradient force at a beam waist radius of 60 nm. This is because, at $z = 14$ nm the axial gradient force is highest in magnitude for 60 nm beam waist radius as seen in Fig. 5. Axial gradient force versus refrac-

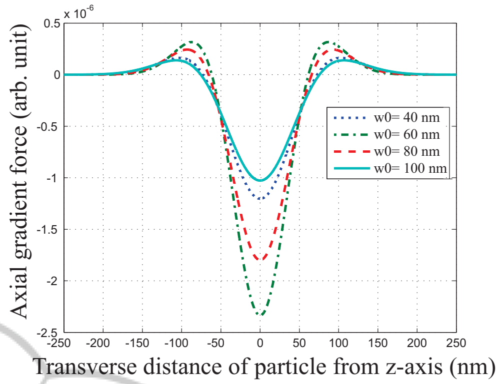

Figure 6: Axial gradient force versus transverse distance x of the particle located at $y = 0$, $z = 14$ nm from the beam-axis for different beam waist radii.

tive index of particle is plotted in Fig. 7. The force direction depends on the effective refractive index m of the particle. The trap pulls the particle towards the beam center, if its refractive index is more than the refractive index of its surrounding medium ($m > 1$) and vice versa if its refractive index is less than that of its surrounding medium ($m < 1$). Highest axial gradient force occurs at 60 nm beam waist radius. From

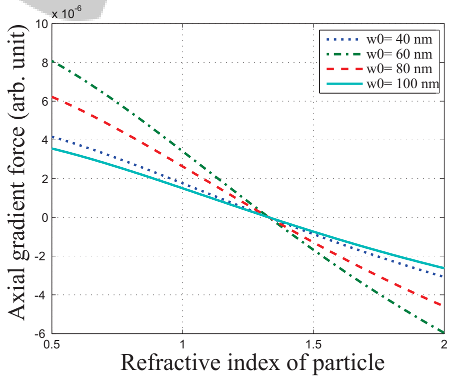

Figure 7: Axial gradient force versus refractive index of particle located at $x = 0$, $y = 0$, $z = 14$ nm for different beam waist radii.

Fig. 8, it is clearly visible that the axial gradient force increases in magnitude with increase in the radius of particle because of the increasing overlap between the optical field and the particle.

5.2 Transverse Gradient Force

Fig. 9 shows the dependence of transverse gradient force on the axial distance of the particle from the beam center. Positive forces indicate that the force

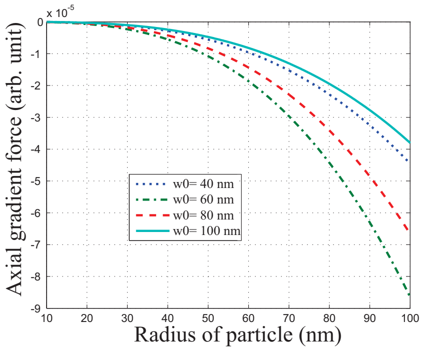

Figure 8: Axial gradient force versus radius of the particle located at $x = 0$, $y = 0$, $z = 14$ nm for different beam waist radii.

direction is towards the positive x-axis in Fig. 4. As a result the particle is pulled towards the beam-axis where the light intensity is the highest. Form Fig.

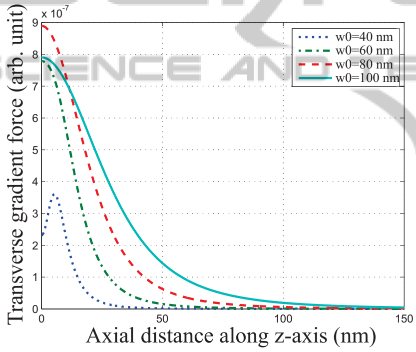

Figure 9: Transverse gradient force versus axial distance z of the particle located at $x = -54$ nm, $y = 0$ from the beam center for different beam waist radii.

10, the dependence of transverse gradient force on the transverse distance of the particle from z-axis can be seen. When the particle is located on the beam-axis, it experiences no forces. But when it moves away from the beam-axis, it experiences a force which pulls it back towards the beam-axis. On both sides, at a certain distance from the beam-axis, this force becomes maximum. The behavior of transverse gradient force with respect to refractive index of particle and particle radius is same as the behavior of axial gradient force, except the force directions indicated by the force magnitudes as seen in Figs. 11 and 12.

5.3 Scattering Force

Fig. 13 shows that, unlike Fig. 5, the scattering force is always positive for any axial position of the particle with respect to the beam center. That means, scattering force always pushes the particle away from the

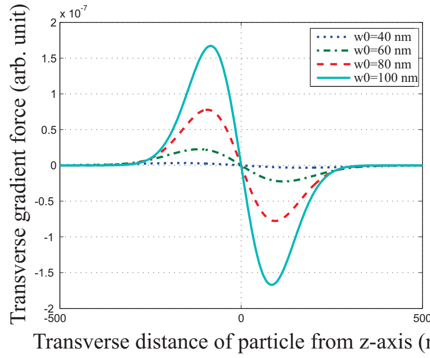

Figure 10: Transverse gradient force versus transverse distance x of the particle from z-axis at $y = 0$, $z = 14$ nm for different beam waist radii.

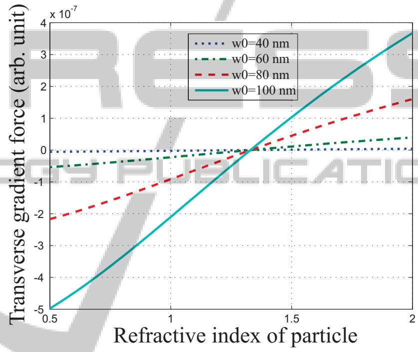

Figure 11: Transverse gradient force versus refractive index of the particle located at $x = -54$ nm, $y = 0$, and $z = 14$ nm for different beam waist radii.

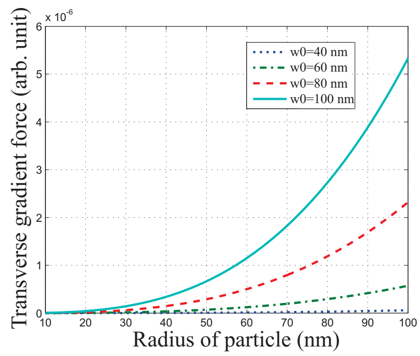

Figure 12: Transverse gradient force versus radius of the particle located at $x = -54$ nm, $y = 0$, and $z = 14$ nm beam waist radius.

beam center towards the forward direction. Fig. 14 tells that when the particle is located on the beam-axis, it experiences the highest scattering force. Also in contrast to Fig. 7, Fig. 15 shows that the direction of scattering force does not depend on the effective refractive index of the particle. It always pushes the particle towards the forward direction irrespective

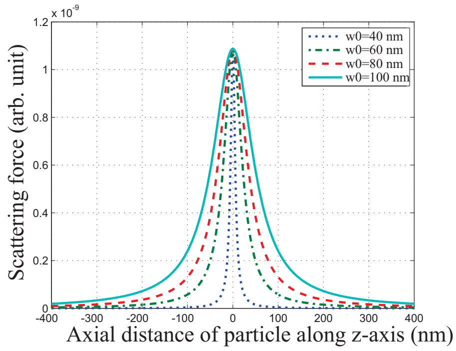

Figure 13: Scattering force versus axial distance z of the particle located at $x = 0$, $y = 0$ from the beam center for different beam waist radii.

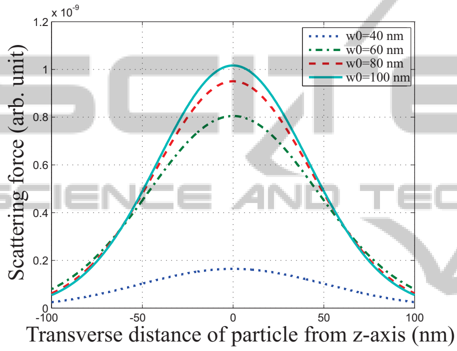

Figure 14: Axial force versus transverse distance x of the particle from z-axis at $y = 0$, $z = 14$ nm for different beam waist radii.

of $m > 1$ or $m < 1$. Similar to the gradient forces,

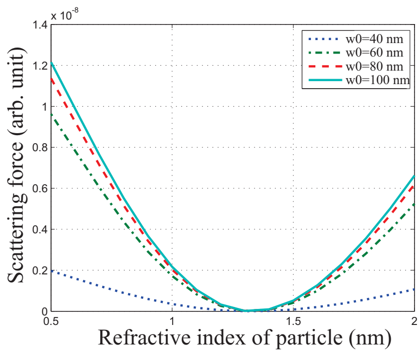

Figure 15: Scattering force versus refractive index of the particle located at $x = 0$, $y = 0$, and $z = 14$ nm for different beam waist radii.

the scattering force also increases with particle size as seen in Fig. 16. However, the scattering force always increases with increasing beam waist radius, which is due to the increased scattering cross-section σ in Eq. (3). Moreover, the magnitude of axial gradient is around 100 -1000 times larger than that of

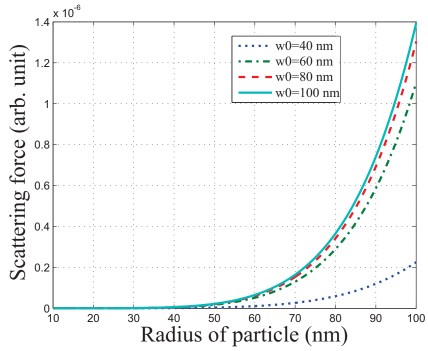

Figure 16: Scattering force versus radius of the particle located at $x = 0$, $y = 0$, and $z = 14$ nm for different beam waist radii.

the scattering force. The more the beam is focused (smaller beam waist radius), the more dominant the axial gradient force becomes with respect to the scattering force, which is necessary to design a stable three-dimensional trap.

6 CONCLUSIONS

In this paper, a thorough study of the trapping behavior of particles in the Rayleigh regime has been carried out. Force equations for a focused Gaussian beam have been derived and then numerical simulations were done to study the effects of the position of the particle in the trap, refractive index of the particle, particle size, beam waist radius on the trapping forces. Gradient forces were observed to be greater in magnitude than the scattering forces.

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