Semantics of Logical Relations in Topological Functioning Model

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Abstract: The Topological functioning model (TFM) captures system functioning specification in the form of topological space consisting of functional features and cause-and-effect relations among them and is represented in a form of directed graph. The formal foundation of TFM makes it as a primary model which should be developed when implementing a software system. The functional features together with topological relationships contain the necessary information to create diagrams of other type, e.g., Activity or Communication diagrams. To specify the behaviour of system execution a new artefact is added to TFM – logical relations. The presence of logical relations denotes forking, branching, decision making, and joining during execution of system. Thus, it is needed to carefully analyse these new relations in TFM to have all the necessary information to transform it to other diagrams. The paper concludes with an example of TFM analysis and logical relationship identification within it.

1 INTRODUCTION

The way software is built still remains surprisingly primitive (by meaning that major software applications are cancelled, overrun their budgets and schedules, and often have hazardously bad quality levels when released) (Jones, 2009). This is due that the very beginning of software development lifecycle is too fuzzy and lacking a good structure since the software developers have limited analysis and modelling of systems (Donins and Osis, 2011). Instead of analysing the system software developers set the main focus on analysis and modelling of software thus leading to a gap between the system and its supporting software (Osis and Asnina, 2008). This issue can be overcome by formalizing the very beginning of the software development lifecycle (Donins and Osis, 2011).

By having too fuzzy beginning of the software development and lacking a good structure of it, for example, the CIM-to-PIM (Computation independent model to Platform independent model) conversion in the context of Model Driven Architecture (MDA) (Miller and Mukerji, 2003) depends much on designers' personal experience and knowledge. Thus the quality of PIM cannot be well controlled (Osis et al., 2007). There are a number of researches (e.g., (Debnath et al., 2008)) which try to enforce the initial phase in software development by strengthening it with various models like use cases (Yue et al., 2009), goal based models (Letier, van Lamsweerde, 2002), behavioral models (Diaz et al., 2005), and structural models (Insfran et al., 2002).

In (Asnina, 2009) a transformation from TFM to "simple" Unified Modeling Language's (UML) (OMG, 2011) Activity diagrams consisting of action nodes and edges is shown. The word "simple" is used while the (Asnina, 2009) states that "it is impossible to create fork and join nodes automatically because the TFM does not hold information of concurrency". This research introduces a new element in TFM – logical relations which hold im-portant information when transforming TFM into other diagram types, for example, Activity or Use Case diagrams. The analysis of logical relations within TFM helps to validate causality between functional features and the logic embedded in TFM. The logical relations contains information of decision making and concurrency thus allowing to formally define decision, merge, fork, and join nodes while transforming TFM into Activity diagram.

This paper is organized into following sections. Section 2 gives mathematical foundations of TFM together with formal definitions of its elements

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(including the logical relations). Section 3 explores semantics of logical relations in TFM and gives method together with example on identification of these relations. Section 4 gives a method of TFM to Activity diagram transformation. In addition it shows an example of formal Activity diagram development. The paper is concluded with conclusions of this research which sketches also future research directions.

2 MATHEMATICAL FOUNDATIONS OF TOPOLOGICAL FUNCTIONING MODEL AND LOGICAL RELATIONS

The TFM holistically represents a complete functionality of the system from the computation independent viewpoint (in the context of MDA). It considers problem domain information separate from the solution domain information. The TFM is an expressive and powerful instrument for a clear presentation and formal analysis of system functioning and the environment the system works within. This means that the TFM of the system validates functional requirements and can be partially changed by those requirements. (Osis and Asnina, 2011) and (Osis and Donins, 2010)

An example of TFM is given below in Figure 1.

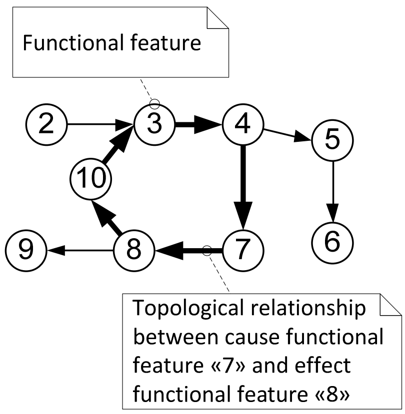


Figure 1: Example of Topological Functioning Model.

TFM has strong mathematical basis and is represented in a form of a topological space. The TFM has four topological characteristics: connectedness, closure, neighbourhood, and continuous mapping; and four functional characteristics: cause-effect relations, cycle structure, and inputs and outputs. TFM enables careful analysis of system's operation and communication with the environment through analysis of functional cycles. (Osis and Asnina, 2011).

While the formal definition of TFM (functional features, topological space, closure operation) is well defined in (Osis and Asnina, 2011) and functional features in (Osis and Donins, 2010) this paper formally defines pre- and post- conditions of functional features, cause-and-effect (i.e., topological) relationships, and the new element within TFM – the logical relations.

2.1 Definition of Topological Functioning Model Elements

Formal Definition of Preconditions and Postconditions. Each precondition or post-condition is a condition Cid described by unique tuple given in equation (1). Condition can be considered as an atomic business rule.

$$C_{id} = \langle Id, Cond, oCond \rangle, \text{ where} \quad (1)$$

- **Id** – identifier of condition,
- **Cond** – condition or an atomic business rule, and
- **oCond** – identifier of opposite condition, i.e., $C_i = \neg C_j$ (*optional*).

Formal Definition of Topological Relationships. Cause-and-effect relationship Tid is a binary relationship relating two functional features Xid and are represented as arcs of a directed graph that are oriented from a cause vertex to an effect vertex. The synonym for cause-and-effect relationship is topological relationship. Each cause-and-effect relationship is a unique tuple represented by equation (2):

$$T_{id} = \langle Id, X_c, X_e, L_{out}, L_{in} \rangle, \text{ where} \quad (2)$$

- **Id** – unique identifier of topological relation,
- $\mathbf{X_c}$ – cause functional feature,
- $\mathbf{X_e}$ – effect functional feature,
- $\mathbf{L_{out}}$ – set of logical relationships between topological relationships on outgoing arcs of cause functional feature X_c (*optional*), and
- $\mathbf{L_{in}}$ – set of logical relationships between topological relationships on incoming arcs of effect functional feature X_e (*optional*).

Formal Definition of Logical Relations. Logical relation Lid shows the logical rela-tionship conjunction (and), disjunction (or), or exclusive or (xor) between two or more topological relationships Tid. The type of logical relation denotes system execution behavior (e.g., decision making, parallel actions). Each logical relation is a unique tuple represented by equation (3):

$$L_{id} = \langle Id, T, Rt \rangle \text{, where} \quad (3)$$

- **Id** – identifier of logical relationship,
- **T** – set of topological relationships belonging to this logical relationship, and
- **Rt** – logical relationship type (*and*, *or*, or *xor*).

Identification of logical relations L_{id} between cause-and-effect (i.e., topological) relationships T_{id} consists of two activities:

1) identification of logical relations L_{out} between topological relationships T_{id} that are outgoing from functional feature X_{id} (see Section 3.1), and
2) identification of logical relations L_{in} between topological relationships T_{id} that are incoming to functional feature X_{id} (see Section 3.2).

Example of logical relations between topological relationships is given in Figure 2.

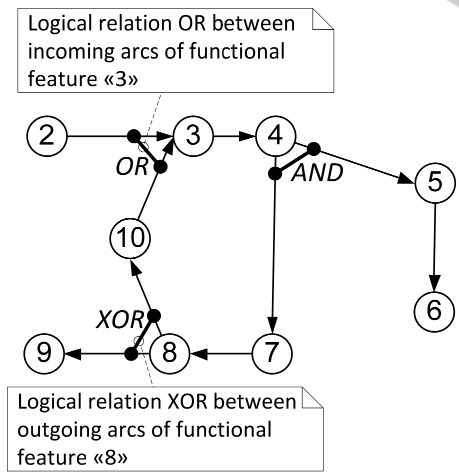


Figure 2: Example of logical relations between topological relationships.

2.2 Application of Topological Functioning Model

Construction of TFM can be iterative. Iterations are needed if the information collected for TFM development is incomplete or inconsistent or there have been introduced changes in system functioning or in software requirements. The development of TFM consists of four steps. Within previous researches (Osis et al., 2008) there are defined three steps (step 1 to step 3) for developing TFM of system functioning. This research adds fourth step – identification of logical relations. The steps of TFM development are as follows:

1) Definition of functional characteristics – functional features,
2) Introduction of topology Θ, which means establishing cause-and-effect relationships between functional features (i.e., development of topological space),
3) Separation of TFM X from the topological space, by applying the closure operation over a set of system's inner functional features, and
4) Identification of logical relations L_{in} and L_{out} within TFM.

The identification of logical relations makes additional check of correctness of developed TFM. There might be situation when conflicting logical relations are identified. If such situation arises then it is needed to review and refine TFM in order to eliminate conflicting logical relations. Refinement of TFM can include addition of functional features and topological relationships, or redefinition of pre- and post- conditions.

3 LOGICAL RELATIONS BETWEEN TOPOLOGICAL RELATIONSHIPS

Between topological relationships exist two kinds of logical relationships – one kind is between arcs that are outgoing from functional features and the other kind is between arcs that are incoming to functional features. The logical relationships between outgoing arcs are denoted with L_{out} and the logical relationships between incoming arcs – L_{in}. Logical relations L_{out} indicates necessity of decision making or branching. In the case of making decision only part of effect functional features X_{id} is executed, but in the case of branching all of the effect functional features X_{id} are executed (i.e., system performs parallel processing); while logical relations L_{in} indicates that there are decision or branching made before the effect functional feature X_{id}. If there was branching before the effect functional feature X_{id}, then before executing this functional feature there should be joining and system can continue its execution only after all arcs are joined. This reflects the mathematical foundations of Petri nets (Desel and Juhás, 2001).

Within TFM can be defined three types of logical relations L_{id}: conjunction (*and*), disjunction (*or*), and exclusive or (*xor*). Within each logical relation L_{id} can participate two or more topological relationships T_{id}. The following two subsections cover the identification of logical relations L_{out} and L_{in}.

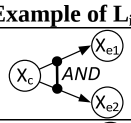
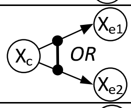
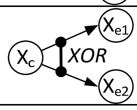
3.1 Relations between Outgoing Arcs

Depending on the relationship type Rt of logical relation L_{id} on outgoing topological relationships T_{id} from cause functional feature X_c, system execution behaviour is defined as follows:

- **AND** – system executes in parallel by executing all functional features X_e of topological relationships T_i participating in this logical relation L_{id},
- **OR** – system can be executed in parallel by executing one, part of or all functional features X_e of topological relationships T_i participating in this logical relation L_{id}, and
- **XOR** – only one functional feature X_e of topological relationships T_{id} participating in this logical relation L_{id} is executed.

The rules for identification of logical relations L_{out} between outgoing arcs of functional features are given in Table 1, where Rt denotes relation type, X_e – effect functional features, and C_{id} – preconditions of X_e.

Table 1: Rules for identification of logical relations Lout between outgoing arcs.

Rt	X_e	C_{id}		Example of L_{id}
AND	X_{e1}	Ø		
	X_{e2}	Ø		
OR	X_{e1}	C_1	$C_1 \neq C_2$ & $C_1 \neq \neg C_2$	
	X_{e2}	C_2		
XOR	X_{e1}	C_1	$C_2 = \neg C_1$	
	X_{e2}	C_2		

The logical relations L_{out} contains necessary information within TFM that denotes decision making and forking in problem domain workflows. Thus the logical relations L_{out} should be analyzed and identified before the TFM transformation into Activity diagram.

3.2 Relations between Incoming Arcs

Depending on the relationship type Rt of logical relation L_{id} on incoming topological relationships T_{id} of effect functional feature X_e, system execution behavior is defined as follows:

- **AND** – system is executing in parallel thus effect functional feature X_e can be executed only when all direct predecessor functional features (i.e., all cause functional features X_c in the distance d=1) of topological relationships T_i participating in logical relation L_{id} are executed,
- **OR** – system can be executing in parallel by executing one, part of or all cause functional features X_c of effect functional feature X_e at the distance d=1 of topological relationships T_i participating in this logical relation, and
- **XOR** – only one cause functional feature X_c of effect functional feature X_e at the distance d=1 of topological relationships T_{id} participating in this logical relation L_{id} is executed.

Relation type Rt of logical relations L_{in} is denoted by corresponding logical relation L_{out} and the inputs and outputs of TFM (this defines the base rule set for identifying L_{in}). Additional rule is used for definition of logical relation which contains both topological relationships connecting input functional feature (can be a chain of input functional feature) with other functional features of TFM and topological relationships connecting functional features within TFM. In such situation a logical relation with type OR is added.

The logical relations L_{in} contains necessary information within TFM that denotes merging (after decision making) and joining in problem domain workflows. Thus the logical relations L_{in} should be analysed and identified before the TFM transformation into Activity diagram.

3.3 Example of Logical Relationships Identification

To better illustrate identification of TFM logical relations a case study is used in which an enterprise data synchronization system is developed (Donins and Osis, 2011). The case study includes development of TFM (without analysis of logical relations), Use Case, Sequence, and Topological class diagram development, while this research analyses logical relations within developed TFM and investigates formal development of Activity diagrams. Within case study have been defined 30 functional features. After definition of functional features the topology Θ (cause-and-effect relationships) are identified between those functional features thus creating topological space representing functioning of the problem domain and relations with external environment.

In order to get all of the system's functionality – the set X – the closuring operation (Osis, Asnina,

2011) is applied over the set of internal system functional features (the set N). The obtained TFM (the set X) after applying closuring operation is as follows: X={2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 24, 25, 26, 27, 28, 29}. The resulting graph is given in Figure 3 which shows functional features (vertices), cause-and-effect relationships (arcs between vertices), and logical relations.

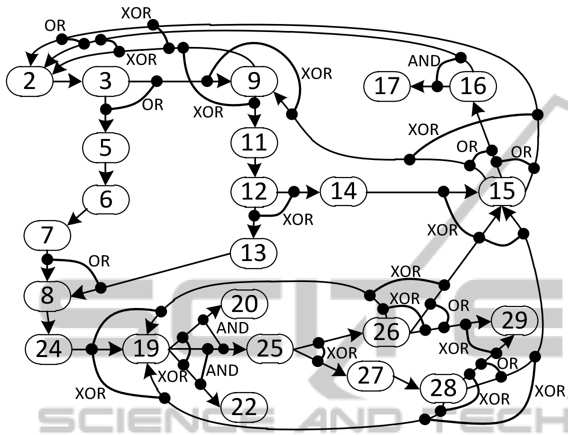


Figure 3: TFM representing enterprise data synchronization system functioning and logical relations between cause-and-effect relationships.

To better understand identification of logical relations a small fragment of TFM given in Figure 3 is used consisting of four functional features: 19, 20, 22, and 25 (see Table 2). The functional feature 20 has a precondition C_1 ("If data from the particular row exists") and functional feature 22 has a precondition C_2 ("If data from the particular row does not exist") while functional feature 25 has no preconditions.

The relation between preconditions C_1 and C_2 is as follows: $C_1 = \neg C_2$; thus indicating that between the arcs that are outgoing from feature 19 to features 20 and 22 (19→20 and 19→22) the logical relation with type exclusive disjunction (XOR) exist. Since functional feature 25 has no preconditions a logical relations with type conjunction (AND) is added between topological relationship 19→20 and 19→25, and 19→22 and 19→25.

Table 2: Part of functional features defined for enterprise data synchronization system.

ID	Object Action	Precondition
19	Checking if data from a particular row already exists in target data base	-
20	Updating existing data in target data base	If data from the particular row exists
22	Insert new data in target data base	If data from the particular row does not exist
25	Logging data row from temporal table	-

4 APPLICATION OF TFM LOGICAL RELATIONS

Application of TFM logical relations within topological functioning modelling allows formally developing Activity diagrams representing workflows in problem domain. The input of this activity is Use Cases, TFM, and mappings between functional features and functional requirements. The scope of each Activity diagram is set by the scope of corresponding Use case (i.e., the Activity diagram contains the description of the same functionality that is included into corresponding Use Case).

The TFM and mappings between functional features and Use cases allows establishing actions and the control flow between actions – functional features are transformed into action nodes and

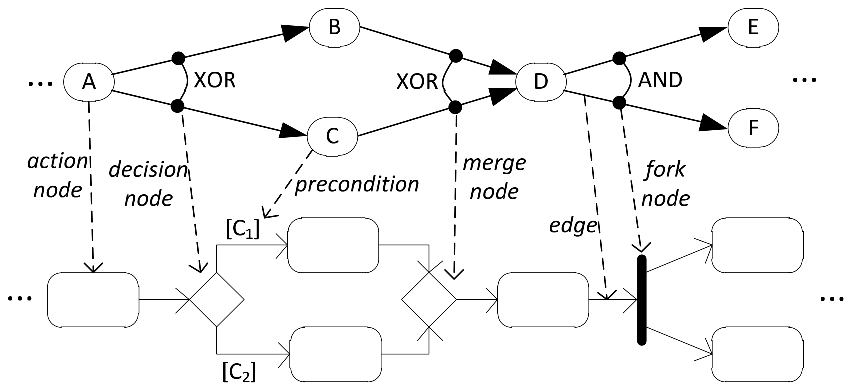


Figure 4: Example of TFM to Activity diagram transformation.

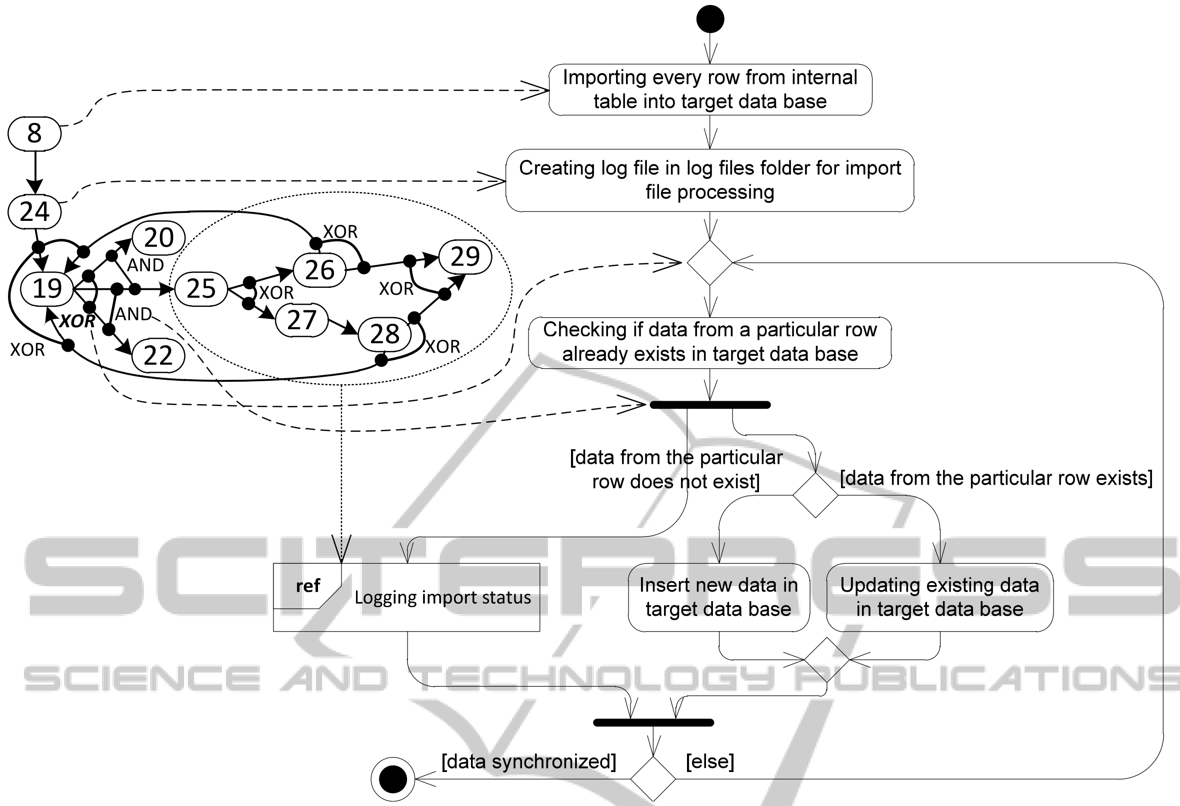


Figure 5: Part of TFM representing functioning of enterprise data synchronization system and activity diagram representing workflow of Use Case "Importing data in target data base".

topological relationships into activity edges.

The logic of control flow (i.e., decision, merge, fork, and join) is defined in accordance with the TFM logical relations. While depending on the type of logical relation L_{out} fork node (for relation type *and*) and decision node (for relation types *or* and *xor*) is added to Activity diagram, the type of logical relations L_{in} denotes join node (for relation type *and*) and merge node (for relation types *or* and *xor*) addition to the Activity diagram. Figure 4 gives an example of TFM to Activity diagram transformation.

In the context of Use Case diagram these logical relations defines *«include»* and *«extend»* relationships between Use Cases. Thus by using logical relations it is possible to build advanced Activity and Use Case diagrams. This is in opposition to the opinion in (Donins and Osis, 2011) that TFM contains information sufficient to create only basic Activity diagrams (i.e., without fork and join nodes).

4.1 Example of Formally Developing Activity Diagram

To better illustrate formal analysis of problem domain workflows a case study is used described in Section 3.3 is used. According to the mappings between functional features and requirements and logical relations in TFM the *«include»* and *«extend»* relationships are automatically established between Use Cases (Donins and Osis, 2011). The scope and count of activity diagrams are denoted by the Use cases and their mappings with functional features. According to the defined Use cases and established mapping, a total set of seven Activity diagrams is created. Activity diagram representing the use case "Importing data in target data base" is given in Figure 5 showing the part of TFM that is transformed into Activity diagram and a trace links from elements of TFM to elements of Activity diagram. As FR1/5 mappings includes also functional requirement FR1/6, the corresponding Activity diagram contains interaction use to Activity diagram "Logging import status". The mappings between functional requirements FR1/5, FR1/6, and functional features are as follows:

- FR1/5 "*Importing data in target data base*" = {8, 24, 19, 20, 22, FR1/6}; and
- FR1/6 "Logging import status" = {25, 26, 27, 28, 29}.

5 CONCLUSIONS

This research introduces a new element into TFM – logical relations. Logical relations in TFM are crucial when transforming TFM into other diagrams. Thus the analysis of logical relations takes an important part of TFM development and problem domain specification. Within each logical relation can participate two or more logical relationships and each logical relation has its type – conjunction (*and*), disjunction (*or*), or exclusive or (*xor*). Logical relations exist between topological relationships and denote the system functioning behavior. Depending on logical relation type system functioning behavior is specified by means of decision making, merging, forking, and joining. While depending on the type of logical relation L_{out} fork node (for relation type *and*) and decision node (for relation types *or* and *xor*) is added to Activity diagram, the type of logical relations L_{in} denotes join node (for relation type *and*) and merge node (for relation types *or* and *xor*) addition to the Activity diagram.

In addition this research shows that by adding additional efforts at the very beginning of software development life cycle it is possible to create a model that contains sufficient and accurate information of problem domain. By “sufficient” meaning that this model can be transformed into other diagrams without major re-analysis of problem domain and by “accurate” meaning that the model precisely reflects the functioning and structure of the system.

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