

# A New Metaheuristic for Float Management in Resource-constrained Project Scheduling

## *A Bi-criteria Approach*

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**Abstract:** In this paper, we present a new unified theoretical model and the conception of the corresponding heuristic algorithm to solve several "what if" like float management problems in resource-constrained project scheduling. The traditional time-oriented resource-constrained project scheduling model for makespan minimization gives an optimal starting time set therefore an activity movement, may be able to destroy the resource-feasibility. The float management, as a starting base, needs a so-called forbidden-set oriented model (a forbidden-set oriented heuristic), which gives an optimal resource conflict repairing relation set. After inserting the additional predecessor-successor relations, in an optimal schedule every movable activity can be moved without destroying the resource feasibility. In the other side, when we have a forbidden-set oriented schedule, then according to the total free float, we have some freedom to redistribute the float among activities to answer several "what if" like questions. For example, in the planning phase we can investigate the consequences of a delay or a longer duration which may be caused by a notorious element of the "critical" activity subset. The unified float management as a new tool was built into the forbidden-set oriented Sounds of Silence (SoS) metaheuristic frame (Csébfalvi et al., 2008a). From theoretical point of view, float management is invariant to the applied heuristic frame; therefore it can be built into any other heuristic which is developed to solve forbidden-set oriented resource-constrained project scheduling problem (RCPSp). The toolbox can be completed by any other new element (float measure), which can be described as a linear programming (LP) or a simple mixed integer linear programming (MILP) problem on the set of the forbidden-set oriented (freely movable without resource-conflicts) solutions as a problem-specific redistribution of the total free float of the project. The essence and viability of our unified approach is illustrated by a set of examples.

## 1 INTRODUCTION

Critical path has long been central to the analysis of non-resource constrained projects. This issue becomes more crucial when resource constraints are introduced. Even in simple resource constrained projects, alternative resource allocations are often possible, resulting in a choice of schedules with identical project durations, but different critical sequences. An activity may be critical in one schedule, but have considerable float (flexibility) in another. In such situations an analysis of floats plays an important and crucial role, making the development of new float measures a central issue in project scheduling.

The desirability of additional float measures has been noted in reviews of project scheduling literature conducted by and Ragsdale (1989). As part of this endeavor, Weist (1967) proposed the "critical sequence" as an extension of the critical path. This concept was employed by Bowers (1995) in the development of a set of heuristics for determining a resource constrained float. Raz and Marshall (1996) explored a definition of resource constrained float involving the generation of two different schedules. Bowers (2000) proposed a float definition for multiple alternative resource constrained schedules.

In a previous study (Levi, 2004) we presented a resource constrained total project float model to for this problem, where the resource constrained total

project float measure (RCTPF) was defined as the sum of the activity floats. In the proposed primary-secondary criteria approach, we maximized the RCTPF value on the set of makespan minimal resource-constrained schedules.

Theoretically, the optimal schedule searching process was formulated as a mixed integer linear programming (MILP) problem, which can be solved for small-scale projects in reasonable time. A conflict repairing harmony search metaheuristic for the proposed primary-secondary criteria approach was presented by Csébfalvi et al. (2008b).

The RCTPF is in essence a flexibility measure which is geared toward enhancing the schedule robustness by maximizing the project total float. The greater the RCTPF is, the better the solution (the robustness) is.

In this paper, we argue that not only the existence of float or its amount is important, but in many cases the distribution of the total amount of the float within the activities is even more significant.

In the recent study, we introduce a model family connected directly or indirectly to the RCTPF, which can be a useful to cope with the several "what if" like questions.

In the proposed bi-criteria approach, a resource-feasible schedule is characterized by the project makespan and the current value of the selected float measure from the measure family. In the presented bi-criteria approach it is characterized by its Pareto front. We have to note, that there is a "natural" conflict between these performance measures (makespan - float) because the longer the "playfield" the higher the chance that we are able to reach a good float measure value and vice-versa.

The theoretical model will be shown in Section 2. In Section 3 we summarise the most important elements of the heuristic algorithm which is based on the resource conflict repairing version of Sounds of Silence (SoS) harmony search heuristic developed by Csébfalvi et al. (2008b) and Csébfalvi and Láng (2011). In Section 4 we present some motivating examples. Finally, Section 5 draws conclusions from this study.

## 2 THEORETICAL MODEL

The core element of the forbidden-set oriented mathematical model which is able to handle float management problems is very simple. It is a straightforward modification of the conflict repairing model developed by Alvarez-Valdés and Tamarit

(1993) omitting unnecessary elements, replacing the starting time variable with an early (late) starting variable for each real activity, and rewriting the original network and the potential conflict repairing relations according to the early (late) starting time variables. The free float is defined as the amount of time that an activity can slip without delaying the start of its successors and while maintaining resource feasibility. The resource constrained total free float measure (RC-TFF) is defined as the sum of the free floats of activities.

The developed resource-constrained float model family consists of the following approaches:

- Total free float (RC-TFF) maximization for a given resource-feasible makespan (RC-MS).
- Uniform free float redistribution (RC-UFF) according to the given RC-MS and RC-TFF.
- Maximization the cardinality of the floatable (shiftable) activities (RC-CFF) for a given RC-MS.
- Makespan minimization subject to the desired activity floats (RC-PFF) for a given "critical" activity subset.

## 3 HEURISTIC APPROACH

Harmony Search (HS) algorithm was recently developed by Lee and Geem (2005) in an analogy with music improvisation process to obtain better harmony. In the HS algorithm, the optimization problem is specified as follows:

$$\text{Max}\{f(\mathbf{X}) \mid \{\mathbf{X} \mid \mathbf{X}^L \leq \mathbf{X} \leq \mathbf{X}^U\}\} \quad (1)$$

In the language of music, vector  $\mathbf{X}$  is a melody, which aesthetic value is represented by  $f(\mathbf{X})$ . In the band, the number of musicians is  $N$ , ( $\mathbf{X}=[X_1, X_2, \dots, X_N]$ ), and musician  $i$ , is responsible for sound  $X_i$ . The improvisation is driven by two parameters. (1) Repertoire consideration rate (RCR): each musician is choosing a sound from his/her repertoire with probability RCR, or a totally random value with probability (1-RCR). (2) Sound adjusting rate (SAR): the sound, selected from his/her repertoire, will be modified with probability SAR. The algorithm starts with a totally random "repertoire upload" phase, followed by improvisations. During the improvisations, when a new melody is better than the worst in the repertoire, it will be replaced by the better one. The two most important parameters of HS algorithm are the repertoire size and the number of improvisations. The HS algorithm is an "explicit" one, because it operates directly on the sounds. In the case of RCPSP, we can only define an

“implicit” algorithm, and without introducing a “conductor” we can not manage the problem efficiently.

First, we show how the original problem can be transformed into the world of music. Here, the resource profiles form a “polyphonic melody”. So, assuming that in every phrase only the “high sounds” are audible, the transformed problem will be the following: find the shortest “Sounds of Silence” melody by improvisation! Naturally, the “high sound” in music is analogous to the overload in scheduling.

In the harmony searching process, the improvisation is fundamentally driven by the “ideas of the musicians”, but each of the possible decisions is made by the conductor (hierarchy is hierarchy). At the start of an improvisation step, the conductor selects a “promising” melody from the repertoire or leaves the musicians to improvise freely. In our magic world, the task of the musicians is very simple: they only have a slider to select (modify) a value from interval  $[-1, 1]$ . A large positive (negative) value means that the musician wants to enter into the melody as early (late) as possible.. Naturally, the magic conductor is able to resolve the possible conflicts between the fighting ideas and define the final entering order for the musicians by solving very simple linear programming problem which gives a scheduling order. If, the new melody “sounds good” (it is shorter than the longest “Sounds of Silence” melody in the repertoire) than it will be memorized.

The forbidden set can be defined as a set of activities that could be scheduled concurrently, and if activities of the set scheduled in parallel, they would violate the resource constraints. To resolve the resource conflicts caused by minimal forbidden sets, it needs to introduce explicit resource conflict repairing relationships. It is important to note that the introduction of an explicit conflict repairing relation might resolve (correct) more than one forbidden set implicitly.

In the traditional time-oriented model, according to the explicit resource conflict handling an activity shift might destroy the schedule. In the presented model the resource feasibility is not affected by the feasible activity moves, because of the implicit resource conflict handling.

In the SoS algorithm, the conductor uses a simple (but fast and effective) “thumb rule” to decrease the time requirement of the forbidden set computation:

- In the forward-backward list scheduling process the conductor (without explicit forbidden set

computation) inserts a precedence relation  $i \rightarrow j$  between an already scheduled activity  $i$  and the currently scheduled activity  $j$  whenever they are connected without lag. The result will be schedule without “visible” conflicts.

- After that, the conductor (in exactly one step) repairs all of the hidden (invisible) conflicts, inserting always the “best” conflict repairing relation for each forbidden set. In this context “best” means a relation  $i \rightarrow j$  between two forbidden set members for which the lag is maximal.

The result of the conflict repairing process will be a resource-feasible solution set, in which every movable activity can be shifted without affecting the resource feasibility. It is well-known that the crucial point of conflict repairing model is the forbidden set computation. In the improved algorithm we combined the “hidden conflict repairing step” with a pre-processing step to decrease the time requirement of the forbidden set computation and to speed up the problem solving process. The essence of the pre-processing step is very simple: In a cyclically repairable process, we select the incompatible activity pairs (triplets) which have exactly one conflict repairing relation and insert the precedence relations. The process is terminated when the relation set will be empty (Csébfalvi and Szendrői, 2012).

## 4 MOTIVATING EXAMPLES

In this section we show the answers for two “what if” like questions for a very simple project with one resource and eight real activities, to demonstrate its usefulness. Figure 1 shows the makespan minimal solution with “distributed” floats. Dark grey background colour means critical activity, the lighter grey means “freely movable” activity, and light grey describes its “playfield”. When we assume that our uncertainty about the real duration of activity 3 is large enough (it is a notorious one) then it would be good to know, how we can schedule activity 3 more safely in the price of a two periods longer project makespan (in the makespan minimal solution, activity 3 is critical, so from the project manager point of view, the schedule is a terrible bad nightmare). In Figure 2 we show the “conformist” solution.

In each case, SoS reached the optimal solution very quickly, using a repertoire consisting of only ten melodies. The number improvisation cycles was also ten (a cycle means repertoire size

improvisations). We run SoS, in each case, with "frozen" golden numbers (tunable parameters).

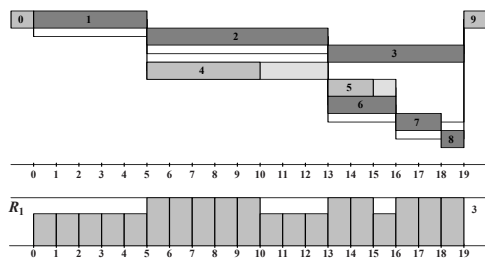


Figure 1: A makespan minimal solution.

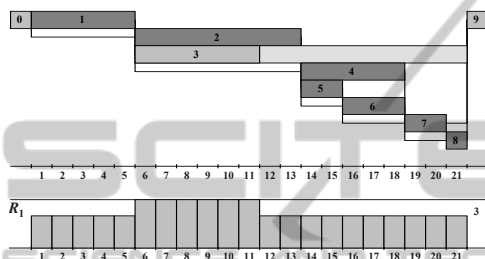


Figure 2: A "conformist" solution for activity 3.

## 5 CONCLUSIONS

In this paper, a new unified theoretical model and the conception of the corresponding heuristic algorithm were presented to solve "what if" like float management problems in RCPSp. The unified float management as a new tool was built into the SoS metaheuristic frame. From theoretical point of view, the float management is invariant to the applied heuristic frame; therefore it can be built into any other heuristic which is developed to solve forbidden-set oriented RCPSp. The toolbox can be complete by any new element (float measure), which can be described as an LP on the set of the forbidden-set oriented solutions as a problem-specific redistribution of the total free float. The essence and viability of our unified approach was illustrated by simple motivating examples. The test of the new float-oriented elements of SoS is under progress using the J30 subset of PSPLIB (Kolisch and Sprecher, 1996) varying the maximal allowed makespan increase. The algorithm was programmed in Compaq Visual FORTRAN 6.5. To solve the benchmark problems to optimality, as a MILP solver, the callable version of Cplex 12.2 was used. The benchmark results for J30 will be presented in a forthcoming paper.

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