Non-linear System Identification by a Fuzzy Takagi-Sugeno System Approach based on Reusable Fuzzified Inputs

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Abstract: An approach to fuzzy identification of discrete time nonlinear dynamical systems based on the Takagi-Sugeno (TS) model with a economical computation formulation is proposed. Number of rules and membership functions positions are fixed for all inputs. This allows to avoid the fuzzification proccess of delayed inputs. Rule base evaluation is avoided for delayed inputs by the Reusable Fuzzified Inputs approach. Consequent parameters are trained or estaimated using least squares approach. This method is intended to be trained in an off-line manner and used in programmable devices. Finally, simulations are performed on two diffrerent problems, the approach shows consistency, tracking of the output that vary with time and a high accuracy of the output estimate, properties requiered in control design applications.

1 INTRODUCTION

One of the many advantages of Fuzzy Inference Systems (FIS) is that they can be used to approximate closely any nonlinear input-output mapping by means of a series of IF-THEN rules (Rong et al., 2006). One of the major tasks in the design of FIS is the structure identification. Structure identification determines the input-output space partition, antecedent and consequent variables of IF-THEN rules, number of such rules, and initial positions of membership functions (Rong et al., 2006). The input-output space partition and initial positions of membership functions has been optimized using genetic algorithms (Surmann and Maniadakis, 2001), clustering techniques(Serra, 2010) and adaptive procedures (Bottura and de Oliveira Serra, 2004). The adjustment of consequent parameters and rules generation are based on adaptive techniques, such as (Nounou and Nounou, 2005), (Rong et al., 2006), (Rezaee and Zarandi, 2010) and (Abdelazim and Malik, 2005). Most of these approaches require a learning rule to adjust the parameters, increasing the calculation effort for the identification task. Some devices, such as, programmable logic controllers and microcontrollers are not enough fast and memory available to run adaptive Takagi-Sugeno systems for the identification of nonlinear systems.

This paper proposes an approach to fuzzy identification of discrete-time nonlinear dynamical systems. Based on the Takagi-Sugeno model, with a suitable formulation for off-line scheme identification. Consequent parameters are estimated by the least square method. Computational load is reduced by fixing the number of rules and the positions of membership functions at delayed version of inputs. The concept of reusable fuzzified inputs is introduced for the reduction of fuzzification calculations. Finally an approach that can be easily implemented on programmable controllers is presented.

2 TAKAGI-SUGENO SYSTEM

The Fuzzy Takagi-Sugeno (TS) model, was first introduced in (Takagi and Sugeno, 1985), this method has been successfully applied to the problem of non-linear identification (Abdelazim and Malik, 2005). Assume a sequence of input-output $\{\mathbf{x}[k], y[k]\},\ k = 1,\ldots,K$ data is collected, the output $y \in \mathbb{R}$ and the vector of inputs $\mathbf{x} \in \mathbb{R}^q$ which contains the premises variables. Each input q is distributed over the interval $[a_q, b_q]$ and partitioned by N_q membership functions F_q^k, for $k = 1,\ldots,N_q$. The TS model is composed of a fuzzy IF-THEN rule base generated by using all possible combinations among the antecedents and the AND operator. The l-th TS rule has the following form:

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$$R^l\text{: IF } x_1 \text{ is } F_1^l \text{ AND } \dots \text{ AND } x_q \text{ is } F_q^l$$
$$\text{THEN } y^l = f^l(\mathbf{x}),\ l = 1,2,\dots,L \qquad (1)$$

where L is the total number of rules. Additionally, each premise variable x_j generates a truth value given by a membership function μ_j^l. Thus, the truth value μ^l for the complete rule l is computed as:

$$\mu^l(\mathbf{x}) = \prod_{j=1}^{q} \mu_j^l(x_j) \qquad (2)$$

the normalized degree of activation for rule l is

$$w^l(\mathbf{x}) = \frac{\mu^l(\mathbf{x})}{\sum_{l=1}^{L} \mu^l(\mathbf{x})}$$

the general expression for the output of the fuzzy TS system with L rules is given by

$$f(\mathbf{x}) = \sum_{l=1}^{L} w^l(\mathbf{x}) y^l \qquad (3)$$

this output can be seen as a linear parameter varying (LPV) system (Bottura and de Oliveira Serra, 2004). Also, the consequent part is an affine dynamic model, which has several advantages (Johansen et al., 2000). This two properties makes TS models in a robust linear system framework ideal for identification tasks. A general schematic representation of the process explained before is summarized in figure 1.

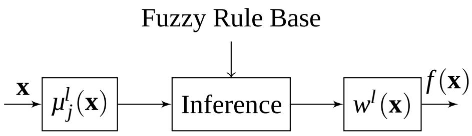

Figure 1: General schematic representation of TS fuzzy inference system.

2.1 Fuzzy Structure Model Approach

Most of fuzzy models for identification of discrete-time nonlinear systems are based on nonlinear autoregressive with exogenous input (NARX) structure model (Bottura and de Oliveira Serra, 2004). The NARX model establishes a relation between past input-output data and the predicted output

$$\hat{y}[k+1] = H\left(y[k], \dots, y[k-n_y+1], u[k], \dots \right.$$
$$\left. \dots, u[k-n_u+1]\right) \qquad (4)$$

where $H()$ is a nonlinear function, k denotes the current discrete time sample, n_y and n_u are integers related to the delay order of output and input samples, respectively. In terms of rules, the function $f^l(\mathbf{x})$ given in equation 1 is

$$f^l(\mathbf{x}) = \sum_{j=1}^{n_y} \alpha_j^l y[k-j] + \sum_{j=1}^{n_u} \beta_j^l u[k-j] + c^l \qquad (5)$$

where α_j^l and β^l are the consequent parameters at fuzzy output for each rule l. In this sense, $\mathbf{x} = [y[k-1], \dots, y[k-n_y], u[k-1], \dots, u[k-n_u]]$, with $q = n_y + n_u$. The nonlinear system presented in equation 4 can be approximated by the linear combination defined in equation 3, as follows

$$\hat{y}[k+1] = \sum_{l=1}^{L} w^l(\mathbf{x}) f^l(\mathbf{x}) \qquad (6)$$

this equation is the standard form to represent TS fuzzy system for the approximation of discrete-time nonlinear systems. Here we propose a different method. First we assume that equation 4 has the following form

$$\hat{y}[k] = H_1(y[k-1]) + \dots + H_{n_y}(y[k-n_y]) +$$
$$G_1(u[k-1]) + \dots + G_{n_u}(u[k-n_u]) \qquad (7)$$

where $H_{k_y}()$ and $G_{k_u}()$ are nonlinear functions, with $k_y = 1, \dots, n_y$ and $k_u = 1, \dots, n_u$. Thus, each nonlinear function can be approximated by its input variable, for example, $H_1(y[k])$ can be approximated by a TS fuzzy system with input $y[k]$. Managing one input to approximate an output reduce the complexity of the rule base. Each rule l per input has the form

$$R^l\text{: IF } x_1 \text{ is } F_1^l \text{ THEN } y^l = f^l(\mathbf{x}),\ l = 1,2,\dots,L$$

in this case, L equals the number of membership functions that partition the input space. Moreover, each rule has only one antecedent, then, the calculation required in equation 2 is avoided, because there are no AND connectors. Furthermore, to reduce more computations, we assume that input variables $u[k-1], \dots, u[k-n_u]$ are equally partitioned with N_u membership functions over the interval $[a_u, b_u]$. The latter is also extended to output variables $y[k-1], \dots, y[k-n_y]$, but they are equally partitioned with N_y membership functions over the interval $[a_y, b_y]$. If $u[k]$ and $u[k-1]$ has the same membership functions configuration, then a delayed version of $\mu^l(u[k])$ will serve to evaluate the rule base system of $u[k-1]$, and the process goes on and on. The last relation can be expressed as

$$\mu^l(u[k-1]) = z^{-1}\mu^l(u[k])$$

where z^{-1} is the unit delay operator. This allows to overcome the calculation of truth values $\mu^l()$ for delayed versions of input and output variables. The latter concept, we name it, reusable fuzzified inputs, because we re-use the fuzzified input values. A systematic view of our approach is shown in 2.

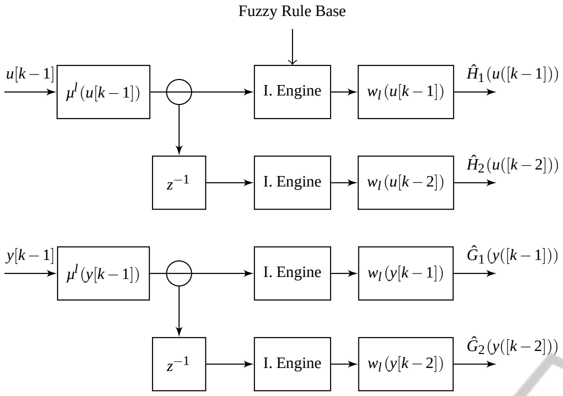

Figure 2: Schematic view of TS fuzzy approach.

2.2 Consequent Parameters Estimation with Least Squares

In this paper, we assume a fixed number of rules L for each input. Thus, combining equations 6, 5 and 7, the problem estimation can be expressed as

$$\hat{y}[k] = \sum_{l=1}^{L}\left(\sum_{i=1}^{n_y}\left(w_i^l y[k-i]\alpha_i^l\right) + \sum_{i=1}^{n_u}\left(w_{n_y+i}^l u[k-i]\beta_i^l\right)\right) = \mathbf{w}_k\Theta \quad (8)$$

where $\mathbf{w}_k$, is the row k of matrix $\mathbf{W}$, and is defined as

$$\mathbf{w}_k = [w_1^1 y[k-1], \ldots, w_1^L y[k-1], w_2^1 y[k-2], \ldots, w_{n_y}^L y[k-n_y], w_{n_y+1}^1 u[k-1], \ldots, w_{n_y+n_u}^L u[k-n_u]]$$

and

$$\Theta = [\alpha_1^1, \ldots, \alpha_1^L, \alpha_2^1, \ldots, \alpha_{n_y}^L, \beta_1^1, \ldots, \beta_{n_u}^L]^T$$

The whole set of output $\mathbf{Y}$ can be calculated

$$\mathbf{Y} = \mathbf{W}\Theta + \mathbf{E}$$

where vector $\mathbf{E}$ is the approximation error. Using least squares, the vector parameter Θ that minimize the norm of vector $\mathbf{E}$ is calculated by the following expression (Espinosa et al., 2004)

$$\Theta = \left(\mathbf{W}^T\mathbf{W}\right)^{-1}\mathbf{Y}^T\mathbf{W}$$

3 RESULTS

In this section, some simulations will be presented in order to illustrate the advantages of the proposed approach in identification for control applications. The criterion of mean square error (MSE) is used to measure the performance of our approach for fuzzy modelling. MSE is defined as

$$MSE = \frac{1}{K}\sum_{k=1}^{K}(y[k] - \hat{y}[k])^2$$

Two different experiments are performed to evaluate this approach. For both experiments, the number of rules L is changed, and a summary performance from results is shown and analysed. Membership functions are configured as triangular form and positioned in a way that the whole discourse universe of each input variable is covered. Procedures were implemented and tested in Matlab using Fuzzy Toolbox.

3.1 Identification Problem I

Four different simulations were performed on the Box and Jenkin's gas furnace data which is a common benchmark (Surmann and Maniadakis, 2001). Membership functions configuration is fixed over the universe of discourse of each input. For example in Figure 3 is shown the disposition of membership functions for input $u[k-1]$ while, the disposition of membership functions for input $y[k-1]$ is shown in Figure 4. The input set is composed by $u[k-1]$,$u[k-2]$,$y[k-1]$ and $u[k-2]$ for the prediction of $y[k]$. MSE values are calculated for the prediction obtained by the trained of consequents using 294 samples (whole dataset samples).

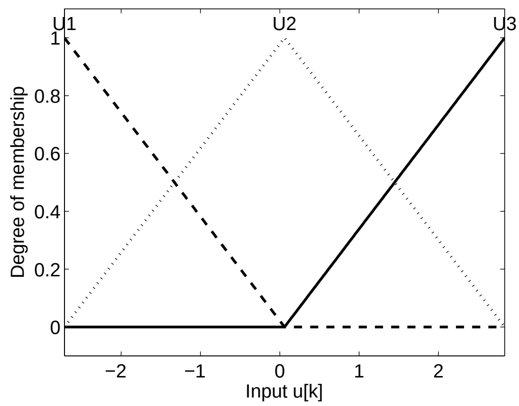

Figure 3: Membership functions for input $u[k]$ with 3 rules.

MSE values for different number of rules in system identificacion performance are shown in Table 1.

There is no a significant improvement in the MSE value for a number of rules higher than 2. The identification approach presents a good performance following the real output of the gas furnace dataset as shown in Figure 5.

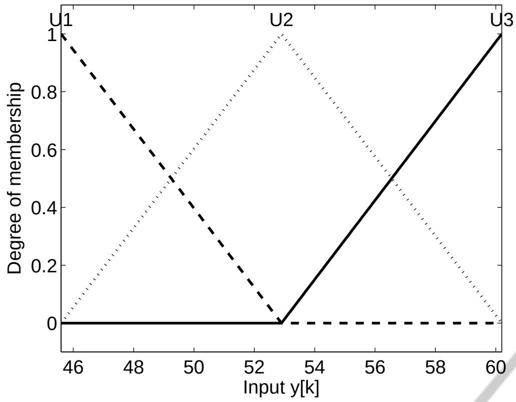

Figure 4: Membership functions for input $y[k]$ with 3 rules.

Table 1: MSE for Gas Furnace experiment performance.

Number of rules	MSE value
2	0.2612
3	0.2505
4	0.2459
5	0.2461

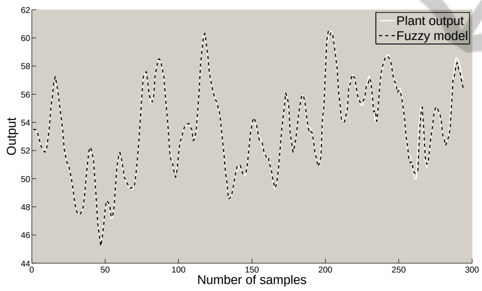

Figure 5: Gas furnace identification performance, with 3 rules per input.

3.2 Identification Problem II

The second nonlinear dynamic system to be identified has been used in (Rong et al., 2006) and (Bottura and de Oliveira Serra, 2004), for testing of their approaches. The nonlinear modelling problem is defined as

$$y[k] = \frac{y[k-1]y[k-2](y[k-1]-0.5)}{1+y^2[k-1]+y^2[k-2]} + u[k-1]$$

The input set is composed by $u[k-1]$,$y[k-1]$ and $y[k-2]$. In the training stage the input $u[k]$ is given by $u[k] = \sin(2k/25)$. The total number of samples generated is 120. The disposition of each membership function is equally distributed between input limits, in a similar manner to the previous experiment. MSE values for different number of rules in system identification performance are shown in Table 2.

Table 2: MSE for Gas Furnace experiment performance.

Number of rules	MSE value
2	0.1027
3	**0.0738**
4	0.0663
5	0.0571

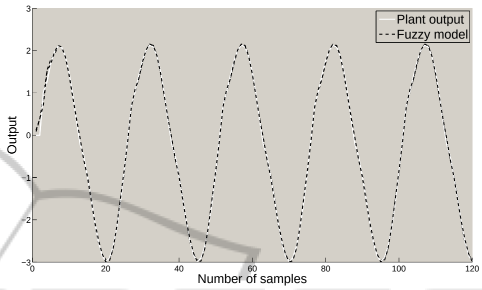

Figure 6: Nonlinear problem II performance obtained with 3 rules per input.

In this case, a big improvement is obtained when passing from two number of rules to three. This indicates that the new added membership function helps to fit better with the data inputs distribution, allowing a better estimation of the real output.

4 CONCLUSIONS

In this paper, a fuzzy inference system is developed based on the reuse of fuzzified inputs. In this method, the fuzzy membership functions are fixed for each input, this allows to even save lots of computation. The performance of the presented approach has been tested on two nonlinear systems identification benchmark problems. Results indicate that this approach is easy to implement, has less computation load and is accurate for a low number of rules per input.
The performance of the approach is sensible to membership functions location. An optimization of the location of membership function would improve the performance and obtain a higher suited model for the input data.

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