SOME EFFECTS OF THE ASSUMPTION OF ALL-POLE FILTER, USED TO DESCRIBE PROCESSES OF TYPE “PULSE SOURSE - FILTER”, ON THE PROPERTIES OF THE GENERATD SIGNAL

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Abstract: In practice, when analysing, processing and generating signals, it is often assumed, that the process is of type “pulse source - filter”. Examples include the speech production process according to the theory of Fant, analysis of shockwaves, ECG, EEG, seismology. For determination of the parameters of the filter many methods exist, most of which require the assumption of linear, all-pole model of the filter. It dates from the time when the computational power of the processing systems was very low. From the 50-s on, many computational effective algorithms have been created. Their complexity is almost an order smaller compared to those, using other models of the filter. In frequency domain the all-pole filter describes very well the processes, for which it has been created. In most of the practical solutions it has became classics, and what follows is his application for other purposes, for which it may be inappropriate. In this paper, some general properties in the application of linear all-pole filter and pulse-source for generating of periodical signal are reviewed. These properties explain some phenomena of the modelled real process and give better interpretation for the constraints, which come out from the implementation of such model.

1 INTRODUCTION

A “pulse source – filter” model could be represented as shown on fig.1:

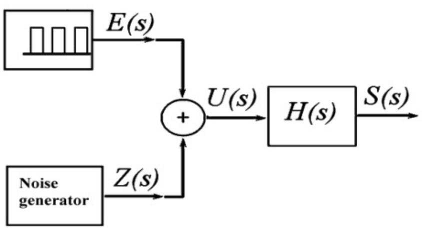

Figure 1: A “pulse source – filter” model.

The observed signal S(s) is defined by the parameters of the filter H(s), excitation pulses E(s) and by the noise Z(s) (Fant G., 1990. In practice statistical methods of autocorrelation and autocovariation are used (Epsy-Willson et.al, 2006, Prasana, S., et.al, 2006). If for sake of clarity we don’t take into account the additive input noise, the generation of the signal in z-domain could be written as:

$$S(z) = E(z)H(z) \tag{1}$$
$$S(z) = Z\{s(nT)\},\ s(nT) = s(t)\big|_{t=nT}$$

Without ignoring the importance of the derived conclusions, we can assume H(z) as a all-pole filter (Titze, 1984):

$$H(z) = \frac{1}{1+\sum_{i=1}^{M} a_i z^{-i}} \tag{2}$$

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The problem of finding the coefficients of the filter $a_i, i = \overline{1,M}$ can be defined as signal analysis. If the input of the filter with transfer function H(z) is a delta impulse, the output will be an envelope of a signal element, modelled with the current filter coefficients. The model of the signal analysis:

$$E(z) = S(z)A(z) \tag{3}$$

uses a filter, inverse with the exciting one, with transfer function:

$$A(z) = \frac{1}{H(z)} = 1 + \sum_{i=1}^{M} a_i z^{-i} \tag{4}$$

Two approaches for finding of the coefficients of the filter are possible – assuming asynchronous excitation, and assuming synchronous one. The first one assumes that the length of the analysed quasistationary intervals is set and known apriory, and the second assumes that the length of the analysed quasistationary intervals is multiple of the of the excitation period.

The method of linear prediction of M-th order (Wiener, 1966) approximates the current value of the signal $s(i)$ from a discrete time series $\{s(i)\}$ with a linear combination of M preceding values with the corresponding weighting coefficients a_i:

$$\hat{s}(n) = \sum_{i=1}^{M} a_i s(n-1) \tag{5}$$

The prediction error is:

$$e(n) = s(n) - \hat{s}(n) \tag{6}$$

For a signal segment, containing N samples, the weighting coefficients can be optimized in such way, that the sum of squares of the errors of prediction for all N samples is minimal. In this case the objective function for the optimization is:

$$\sum_{N} e^2(n) = \sum_{N} \left[s(n) - \sum_{i=1}^{M} a_i s(n-i) \right]^2 \overset{!}{=} min \tag{7}$$

Setting the partial derivative of the sum of squares equal to zero we have the equation:

$$\sum_{i=1}^{M} a_i \sum_{N} s(n-i)s(n-k) = \sum_{N} s(n)s(n-k), k = \overline{1,M} \tag{8}$$

Two common methods, differing in the limits of summation are known: autocorrelation and autocovariation.

The range of summation of the autocorrelation is $-\infty < n < \infty$:

$$\sum_{i=1}^{M} a_i \Phi(|i-k|) = \Phi(k), \qquad k = \overline{1,M} \tag{9}$$

with the coefficients of autocorrelation :

$$\Phi(|i-k|) = \sum_{n=-\infty}^{\infty} s(n-i)s(n-k) \tag{10}$$

The interval to be analysed is actually $0 < n < N$, the samples outside it could be eliminated with an appropriate window function $w(n)$, and the autocorrelation coefficients $R(i)$, can be evaluated as follows:

$$R(i) = \sum_{n=0}^{N-1-i} \zeta(n)\zeta(n+i), \quad \zeta(n) = w(n)s(n), \quad w(n) = \begin{cases} \neq 0, & 0 \le n < N \\ 0 \end{cases} \quad i = \overline{0.M} \tag{11}$$

The equality from the condition for unconditional optimization becomes:

$$\sum_{i=1}^{M} a_i R(|i-k|) = R(k), \qquad k = \overline{1,M} \tag{12}$$

and as a matrix notation:

$$\mathbf{R}(a_1, a_2, \ldots, a_M)^T = (R(1), R(2), \ldots, R(M))^T \tag{13}$$

The matrix **R** of the coefficients $R(|i-k|)$ is a *Toeplitz* matrix – it is symmetrical and the elements in the diagonals are identical (Grenader. U. et. al. , 1958) :

$$\mathbf{R} = \begin{pmatrix} R(0) & R(1) & \cdots & R(M-1) \\ R(1) & R(0) & \cdots & R(M-2) \\ R(2) & R(1) & \cdots & R(M-3) \\ \cdots & \cdots & \cdots & \cdots \\ R(M-1) & R(M-2) & \cdots & R(0) \end{pmatrix} \quad (14)$$

There are a lot of methods for solving the system of equations. The most effective is the recursive method of *Durbin*, where the number of operations grows only with the square of the weighting coefficients (Makhoul, J., 1975).
Because their value is always less than one, the poles of the filter will always be within the unit circle on the z-plane, which guaranties its stability.
When using the covariation, the prediction error is minimized within the interval $0 < n < N$. The matrix of the coefficients in general isn't a *Toepliz* one and the methods for obtaining the filter coefficients aren't so effective (the Cholesky method for example (Werner, H, 1975)) and the stability of the filter isn't guaranteed.

Both the autocorrelation and covariation use the same two steps for evaluating the filter coefficients. – first they find the coefficients matrix, and then solve the system of linear equations (Madisetti V., Williams D, 1999). There are other possible methods, (for example using lattice structures), which combine the two steps. In can be proven (Makhoul, J., 1975) that the most effective method is the one of Durbin, which is the most preferred autocorrelation method.

2 IMPACT OF THE DURATION OF THE EXCITATION PHASE TO THE PERIOD OF THE SPECTRAL PEAK

We assume model of the filter is of order two:

$$H(s) = \frac{k_1\omega_1^2}{s^2 + \omega_1^2} \quad (15)$$

This means that the signal will contain only one spectral peak ω_1. If the filter is excited by a sequence of rectangular pulses, described by:

$$e(t) = \begin{cases} 1, (m-1)T_g \le t < t_{\text{excitation\_phase}} + (m-1)T_g \\ 0, t_{\text{excitation\_phase}} + (m-1)T_g \le t < mT_g \end{cases} \quad m = \overline{1, N_{imp} - 1} \quad (16)$$

Where T_g is the excitation period, and $t_{\text{excitation\_phase}}$ is the duration of the excitation phase. The output signal for the first excitation period (m=1) is:

$$s_{\text{excitation\_phase}}^{1F}(t) = AO_{\text{excitation\_phase}}^{1F} - A_{\text{excitation\_phase}}^{1F} \sin(\omega_1 t + \varphi_{\text{excitation\_phase}}^{1F}) \quad (17)$$

for $t < t_{\text{excitation\_phase}}$, i.e. in excitation phase, and:

$$s_{free\_vibration}^{1F}(t) = A_{free\_vibration}^{1F} \sin(\omega_1 t + \varphi_{free\_vibration}^{1F}) \quad (18)$$

for $t \ge t_{free\_vibration}$, i.e. in free vibration phase, where:
$AO_{\text{excitation\_phase}}^{1F} = k_1$ is the constant component of the signal in the excitation phase
$A_{\text{excitation\_phase}}^{1F} = k_1$ and
$A_{free\_vibration}^{1F} = 2k_1 \sin\left(\frac{\omega_1 t_{\text{excitation\_phase}}}{2}\right)$ are the amplitudes of the signal in excitation phase and in the phase of free vibration
$\varphi_{\text{excitation\_phase}}^{1F} = \frac{\pi}{2}$ and
$\varphi_{free\_vibration}^{1F} = 2\pi - \frac{\omega_1 t_{\text{excitation\_phase}}}{2}$ are the angular phases of the signal in excitation phase and in the phase of free vibration
$\omega_1 = 2\pi f_1$ are the circular frequency, which corresponds to the spectral peak f_1.
We can observe the following:

- The amplitude of the signal in th excitation phase depends only on the gain constant of the filter
- The amplitude of the signal in the phase of free vibration depends again on the gain constant, but also in a complicated way on the ratio of duration of the preceding phase of excitation to the period of the spectral peak.
- The later holds true also for the angular phases.

This means that changes in the duration of the excitation phase can increase or decrease the amplitudes of the spectral peaks , without changing the parameters of the filter. To illustrate this impact, we define a dimensionless coefficient, proportional to the ratio of duration of the excitation phase to the period of the spectral peak :

$$r_{\omega_1 t_{\text{excitation\_}phase}} = \omega_1 t_{\text{excitation\_}phase} \quad (19)$$

For the relation of the amplitudes of the signal in the excitation phase and in the phase of free vibration we define the coefficient:

$$r_{A^{1F}_{\text{excitation\_phase}}A^{1F}_{\text{free\_vibration}}} = \frac{A^{1F}_{\text{excitation\_phase}}}{A^{1F}_{\text{free\_vibration}}} \quad (20)$$

Here $t_{\text{free\_vibration}} = T_g - t_{\text{excitation\_phase}}$ means the duration of the free vibration phase of one excitation period. The relation between these coefficients is:

$$r_{A^{1F}_{\text{excitation\_phase}}A^{1F}_{\text{free\_vibration}}} = \left|2\sin\left(\frac{\omega_1 t_{\text{excitation\_phase}}}{2}\right)\right| \quad (21)$$

Obviously this relation is periodical, and the first two periods are shown in fig.2

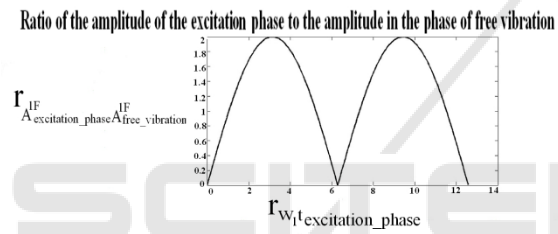

Figure 2: The relation of the amplitude of the generated signal in the excitation phase to the amplitude of the phase of free vibration as function of the coefficient $r_{\omega_1 t_{\text{excitation\_phase}}}$.

As one can see, varying the duration of the excitation phase, without changing the filter coefficients, the amplitude of the spectral peak of the generated signal in the free vibration phase can take any value from zero (fig.3) to two times the amplitude in the excitation phase (fig.4).

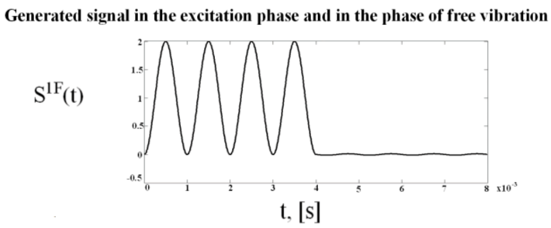

Figure 3: The generated signal in the excitation phase and in the phase of free vibration with $r_{\omega_1 t_{\text{excitation\_phase}}} = 2\pi n, n = 0, \pm 1, \pm 2$ .... and $r_{A^{1F}_{\text{excitation\_phase}}A^{1F}_{\text{free\_vibration}}} = 0$

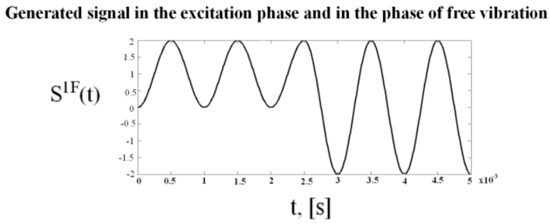

Figure 4: The generated signal in the excitation phase and in the phase of free vibration with $r_{\omega_1 t_{\text{excitation\_phase}}} = \pi n, n = \pm 1, \pm 3$ .... and $r_{A^{1F}_{\text{excitation\_phase}}A^{1F}_{\text{free\_vibration}}} = 2$

This effect becomes more apparent within a signal segment, containing more than one excitation period. In this case not only the coefficient $r_{\omega_1 t_{\text{excitation\_phase}}}$, but also the ratio of the duration of the excitation phase to the excitation period, which is actually the is mark-to-space ratio, will be of importance for the ratio of the amplitudes:

$$k_{full\_imp} = \frac{t_{excitation\_phase}}{T_g} \quad (22)$$

The derived analytical relations lead us to important conclusions. The ratio of the amplitudes of the signal in the excitation phase to the phase of free vibration for the second excitation period is given by the relation:

$$r_{A^{1F}_{\text{excitation\_phase}}A^{1F}_{\text{free\_vibration}}} = \left[4k_1 \sin\left(\frac{r_{\omega_1 t_{\text{excitation\_phase}}}}{2}\right) \cos\left(\frac{r_{\omega_1 t_{\text{excitation\_phase}}}}{2k_{full_{imp}}}\right)\right] \left[\left(k_1 \sin\left(r_{\omega_1 t_{\text{excitation\_phase}}}\right) + k_1 \sin\left(2\pi - \frac{r_{\omega_1 t_{\text{excitation\_phase}}}}{k_{full_{imp}}}\right)\right)^2 + \left(-2k_1 \sin^2\left(\frac{r_{\omega_1 t_{\text{excitation\_phase}}}}{2}\right) + k_1 \sin\left(\frac{r_{\omega_1 t_{\text{excitation\_phase}}}}{k_{full_{imp}}} - \frac{\pi}{2}\right)\right)^2\right]^{-\frac{1}{2}} \quad (23)$$

The graphical representation of this relation is shown in fig 5.

Obviously for the next periods the calculation of this ratio is getting more and more complicated and some numerical methods are needed. Nevertheless, the following important observation can be made:

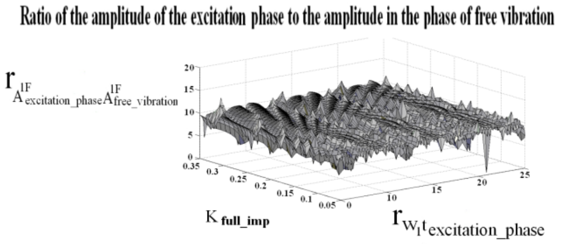

Figure 5: The relation of the amplitudes of the generated signal in the excitation phase to the phase of free vibration for the second excitation period.

Varying the ratio of the duration of the excitation phase to the duration of the period of excitation, and without changing the parameters of the filter, one can generate segments, in which the amplitude of the spectral peak for every following period of excitation increases, decreases, doesn't change considerably, or follows some analytical relation.

3 IMPACT OF THE DURATION OF THE EXCITATION PHASE WITH MORE THAN ONE SPECTRAL PEAK

The way, that the parameters of the excitation change the ratio between the amplitudes of the different spectral peaks in the generated signal, is similar to the one, presented in the previous chapter. We assume, that the we have all-pole filter of fourth order, and the poles all lie of the unit circle:

$$H(s) = \frac{1}{(s^2+\omega_1^2)(s^2+\omega_2^2)} \quad (24)$$

If the excitation of the filter is one rectangular pulse, the output signal will contain two spectral peaks ω_1 and ω_2. If we assume, that $\omega_2 = k_2\omega_1$ and$k_2 > 1$, the components of the signal in the excitation phase are:

$$s^{1F}_{\text{excitation\_phase}}(t) = AO^{1F}_{\text{excitation\_phase}} - A^{1F}_{\text{excitation\_phase}} \sin(\omega_1 t + \varphi^{1F}_{\text{excitation\_phase}}) \quad (25)$$

$$s^{2F}_{\text{excitation\_phase}}(t) = AO^{2F}_{\text{excitation\_phase}} - A^{2F}_{\text{excitation\_phase}} \sin(\omega_2 t + \varphi^{2F}_{\text{excitation\_phase}}) \quad (26)$$

Where:

$AO^{1F}_{\text{excitation\_phase}} = \frac{1}{(k_2^2-1)\omega_1^4}$ and $AO^{2F}_{\text{excitation\_phase}} = \frac{1}{-k_2^2(k_2^2-1)\omega_1^4}$ are the constant components of the first and second spectral peaks in the excitation phase

$A^{1F}_{\text{excitation\_phase}} = \frac{1}{(k_2^2-1)\omega_1^4}$ and $A^{2F}_{\text{excitation\_phase}} = \frac{1}{k_2^2(k_2^2-1)\omega_1^4}$ are the amplitudes of the first and second spectral peaks in the excitation phase

$\varphi^{1F}_{\text{excitation\_phase}} = \frac{\pi}{2}$ and $\varphi^{2F}_{\text{excitation\_phase}} = \frac{3\pi}{2}$ are the angular phases of the first and second spectral peaks in the excitation phase

$\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ are the circular frequencies, which correspond to the spectral peaks f_1.and f_2

The signal components in the phase of free vibration are:

$$s^{1F}_{\text{free\_vibration}}(t) = A^{1F}_{\text{free\_vibration}} \sin(\omega_1 t + \varphi^{1F}_{\text{free\_vibration}}) \quad (27)$$

$$s^{2F}_{\text{free\_vibration}}(t) = A^{2F}_{\text{free\_vibration}} \sin(\omega_2 t + \varphi^{2F}_{\text{free\_vibration}}) \quad (28)$$

Where:

$A^{1F}_{\text{free\_vibration}} = \frac{1}{(k_2^2-1)\omega_1^4}\left|\sin\left(\frac{\omega_1 t_{\text{excitation\_phase}}}{2}\right)\right|$ and $A^{2F}_{\text{free\_vibration}} = \frac{1}{k_2^2(k_2^2-1)\omega_1^4}\left|\sin\left(\frac{\omega_2 t_{\text{excitation\_phase}}}{2}\right)\right|$ are the amplitudes of the first and second spectral peaks in the free vibration phase

$\varphi^{1F}_{\text{free\_vibration}} = -\frac{\omega_1 t_{\text{excitation\_phase}}}{2}$ is the angular phase of the first spectral peak in the free vibration phase

if $4l\pi \le \omega_1 t_{\text{excitation\_phase}} < (4l+2)\pi$ and

$\varphi^{1F}_{\text{free\_vibration}} = \frac{\pi}{2} - \frac{\omega_1 t_{\text{excitation\_phase}}}{2}$

if $(4l+2)\pi \le \omega_1 t_{\text{excitation}_{\text{phase}}} < (4l+4)\pi$

$\varphi^{2F}_{\text{free\_vibration}} = -\frac{\omega_2 t_{\text{excitation\_phase}}}{2}$ is the angular phase of the second spectral peak in the free vibration phase

if $4l\pi \le \omega_2 t_{\text{excitation\_phase}} < (4l+2)\pi$ and

$\varphi^{2F}_{\text{free\_vibration}} = \frac{\pi}{2} - \frac{\omega_2 t_{\text{excitation\_phase}}}{2}$

if $(4l+4)\pi \le \omega_2 t_{\text{excitation}_{\text{phase}}} < (4l+4)\pi$

$$l = \pm 1, \pm 2 \ldots$$

$\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ are the circular frequencies, which correspond to the spectral peaks f_1.and f_2

As with the case with one spectral peak, we may expect that the dimensionless coefficients, proportional to the ratio of the duration of the excitation phase to the period of the spectral peak will have big influence on the parameters of the signal components in the phase of free vibration:

$$r_{\omega_1 t_{\text{excitation\_phase}}} = \omega_1 t_{\text{excitation\_phase}} \text{ and } r_{\omega_2 t_{\text{excitation\_phase}}} = \omega_2 t_{\text{excitation\_phase}} = k_2 \omega_1 t_{\text{excitation\_phase}} \quad (29)$$

This can be represented with the dimensionless coefficient of the ratio of the amplitudes of the two spectral peaks in the signal

$$r_{A^{2F}A^{1F}} = \frac{A^{2F}}{A^{1F}} \quad (30)$$

In the excitation phase this coefficient will depend only on the filter parameters:

$$r_{A^{2F}_{\text{excitation\_phase}}A^{1F}_{\text{excitation\_phase}}} = \frac{1}{k_2^2} \quad (31)$$

In the free vibration phase, this ratio will depend of the filter parameters, but also in a complicated manner on the duration of the excitation phase:

$$r_{A^{2F}_{\text{free\_vibration}}A^{1F}_{\text{free\_vibration}}} = \frac{1}{k_2^2}\left|\frac{\sin\left(\frac{k_2 r_{\omega_1 t_{\text{excitation\_phase}}}}{2}\right)}{\sin\left(\frac{r_{\omega_1 t_{\text{excitation\_phase}}}}{2}\right)}\right| \quad (32)$$

This means, that the change of the duration of the excitation phase can substantially change the predetermined from filter parameters constellation of spectral peaks in the signal. This influence can be easily seen from the next numerical example, with typical for a real speech signal values of the filter parameters (Damyanov D., Galabov V., 2012):

- First spectral peak $f_1 = 420\,Hz$;
- Second spectral peak $f_2 = 966\,Hz$; which means $k_2 = \frac{f_2}{f_1} = 2.3$;
- Nominal duration of the excitation phase $t_{excitation\_phase} = 2.8\,ms$;
- Fluctuation of the nominal duration of the excitation phase $\Delta t_{exitation\_phase} = \pm 0.4\,ms$;

In the excitation phase the ratio of the amplitudes depends only on the filter parameters:

$$r_{A^{2F}_{\text{excitation\_phase}}A^{1F}_{\text{excitation\_phase}}} = \frac{A^{2F}_{\text{excitation\_phase}}}{A^{1F}_{\text{excitation\_phase}}} = k_2^{-2} \approx 0.189 \quad (33)$$

In the phase of free vibration with nominal duration of the $t_{excitation\_phase} = 2.8\,ms$; this coefficient will be $r_{A^{2F}_{\text{excitation\_phase}}A^{1F}_{\text{excitation\_phase}}} \approx 0.288$. If the duration of the excitation phase is shortened with 4 ms, the coefficient will increase more than 20 times to $r_{A^{2F}_{\text{free\_vibration}}A^{1F}_{\text{free\_vibration}}} \approx 6.33$, and if the duration of the excitation phase is lengthened with 4ms, the coefficient will decrease more than 20 times to $r_{A^{2F}_{\text{free\_vibration}}A^{1F}_{\text{free\_vibration}}} \approx 0.061$. On fig.6 for the three cases the excitation rectangular pulse with duration, equal to the duration of the excitation phase, the generated signal and its spectra are shown.

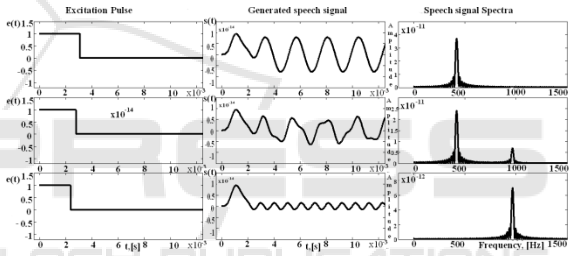

Figure 6: The three cases the excitation rectangular pulse with duration, equal to the duration of the excitation phase, the generated signal and its spectra.

4 CONCLUSIONS

When dealing with periodical and quasiperiodical processes, the “source-filter” model allows simplification of analysis and parameterization and makes the technical implementation easier. This facilitations can be achieved when filter and excitation source are treated independent. In this case for the parameterization of the filter very efficient techniques and methods can be used. This approach gives excellent results in most cases of use of the model – in systems for analysis, synthesis, coding and transmission of speech signals and others. In some cases this description is not relevant enough and additional complicated methods and information sources must be used. This paper shows that the model can be made more effective without further complications, using the cumulative effect of simultaneous treatment of the processes, which happen on the source and the filter.

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