Process Monitoring in Production Systems with Large Diversity of Products

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Abstract: The main objectives of Statistical Process Control (*SPC*) are monitoring and analyzing the capability of processes. Traditionally, the analysis of the process capability is performed at the end of Phase 1 (preliminary) and periodically during Phase 2 (monitoring) of the *SPC*, using the indices *Cp* and *Cpk*. *SPC* and capability analysis of production systems with a large diversity of products present difficulties in implementation. In order to meet the needs required by the current production systems, this paper presents methods for both the statistical control and capability analysis of the processes. These methodologies include two situations, when there are sufficient data to estimate the process parameters (mean, variance) and when it does not exist. In the first case, it is suggested the implementation of control charts *Z* and *W* and capability indices Z_L and Z_U. In the second case, when there is a limited amount of data, the authors suggest the implementation of control charts *Q* and capability indices Q_L and Q_U. The methodologies are illustrated with two case studies, concluding that they allow streamline the statistical control of the various processes and reduce the downside, in Phase 2 of the SPC, where the capability analysis is made only periodically.

1 INTRODUCTION

Statistical methods play a key role in the quality evaluation, allowing, among other things, to verify whether the product meets the technical specification defined. Traditionally, the most common way of making this approach is the use of capability indices C_p and C_{pk}. The simultaneous analysis of these two indices allows fully evaluation of the process' performance, *i.e.*, making the comparison between the technical specifications and tolerances of the natural process (in case the distribution of the quality characteristic is Normally distributed). The introduction of the C_p index is attributed to Juran (1974) and the introduction of the C_{pk} index is attributed to Kane (1986).

The capability analysis of processes is one of the most effective ways to address the issue of customer satisfaction. There are some particularly important tasks prior to the study of C_p and C_{pk}. The control charts are valuable tools that enable the distinction between special causes and common causes of variation, verification of process stability and estimation of its parameters. Once these parameters are estimated, we proceed to the study of process capability.

Control charts were introduced by Shewhart (1931), at Bell Telephone Laboratories and give a valuable contribution to continuous quality improvement. The control charts designed and developed by Shewhart are typically applied to processes that provide a large amount of data. Proper implementation of Shewhart charts is based on the following principles:

- Samples should be homogeneous, *i.e.*, all units are produced under the same conditions.
- The sampling frequency is defined according to the process characteristics; so, it is expected to maximize the opportunity of change between samples.
- The data collected should be independent, so that the observation i of the sample j is defined by $x_{ik} = \mu + \varepsilon_{ik}$ $(i = 1, \ldots, n;\ k = 1, \ldots, m)$, where $\varepsilon \sim N(0, \sigma^2)$ is a random variable designated by white noise.
- The data collected should follow a Normal distribution $\left(X \sim N(\mu, \sigma^2)\right)$.

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- The control limits of the various charts are located to ± 3 standard deviations from the average (Center Line) of the statistical distribution of the sample in examination, corresponding to a significance level of 0.27%.

The current situation of production systems is often very different from that which prevailed when Shewhart theorized statistical quality control. Today it is necessary to consider, in the same system, the simultaneous production of many items in smaller amounts, which leads to the need of developing methodologies adapted to new contexts. This issue has been the subject of study by several researchers, including, for example, Bothe (1988), Wheeler (1991), Pyzdek (1993), Quesenberry (1991, 1997), Montgomery (2012), and Pereira and Requeijo (2012).

Thus, the statistical control of processes dealing with various products/characteristics must be implemented through other control charts that provide an alternative to the Shewhart control charts. This approach is commonly known as the statistical control of small productions ("short runs"). The *Z* and *W* control charts and the Q control charts are the statistical techniques used in this context. The *Z* and *W* charts are dimensionless and are applied when it is possible to estimate the parameters of the different processes. When there are insufficient data to estimate the parameters of the processes, Quesenberry (1991, 1997) proposes the use of control charts Q. The implementation of these two types of control charts (*Z*/*W* and *Q*) is made to take into account - within the same document - all products/characteristics, and provide a quick way to easily control the stability (in-control) of all processes.

In order to continuously evaluate the performance of processes, these techniques present the capability indices, Z_L, Z_U, Q_L, and Q_U introduced by Pereira and Requeijo (2012), which enable the analysis of the same capacity in real time.

This article discusses the two options referred, namely, the existence or not of sufficient data to estimate the parameters of the various processes, considering that in both cases the study variables are continuous, independent and normally distributed.

2 METHODOLOGY

For implementation of statistical process control when large numbers of products/characteristics are or not available, the authors of this paper propose the methodology described in Figure 1.

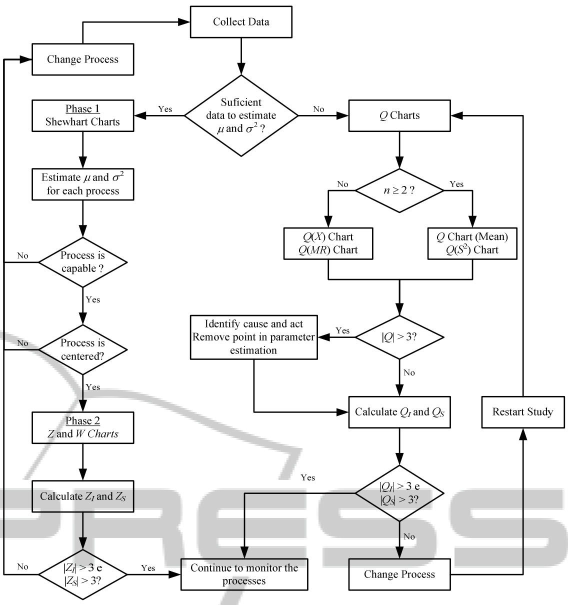

Figure 1: Methodology for Statistical Process Control with large diversity of products.

3 SPC FOR A SIGNIFICANT AMOUNT OF DATA

When there is sufficient data to estimate the processes parameters, one should implement in Phase 1 of the *SPC* the Shewhart control charts, applied to each process and quality characteristic. Usually, for continuous variables, we use the $\overline{X}$ and R, $\overline{X}$ and S or X and MR control charts. Then, in Phase 2 of the *SPC*, the Z and W control charts are implemented, covering all products /characteristics in chronological order of collection of observations.

The analysis of the control charts help to prove that the processes are in-control, *i.e.*, when only exist common causes of variation. The interpretation of Shewhart control charts is based on the existence of any non-random patterns (ISO 8258:1991).

3.1 Phase 1

In Phase 1, the analyst proceeds to the construction of the most appropriate Shewhart control chart for each product/characteristic. The upper control limit (*UCL*), the lower control limit (*LCL*) for monitoring these charts and the center line (*CL*) are determined in Phase 1, using the formulas shown in Table 1.

Table 1: Limits of the Shewhart Control Charts (Phase 1 of *SPC*).

Chart	*LCL*	*CL*	*UCL*
$\overline{X}$ (Average)	$\overline{\overline{X}} - A_2\overline{R}$ or $\overline{\overline{X}} - A_3\overline{S}$	$\overline{\overline{X}}$	$\overline{\overline{X}} + A_2\overline{R}$ or $\overline{\overline{X}} + A_3\overline{S}$
S (Standard Deviation)	$B_3\overline{S}$	$\overline{S}$	$B_4\overline{S}$
R (Range)	$D_3\overline{R}$	$\overline{R}$	$D_4\overline{R}$
MR (Moving Range)	$D_3\overline{MR}$	$\overline{MR}$	$D_4\overline{MR}$

If it is found that the process is stable (statistically in-control) one proceeds to the estimation of the processes parameters using equations (1) and (2), where d_2 and c_4 are coefficients that depend on the sample size *n*.

$$\hat{\mu} = \overline{\overline{X}} \quad \text{or} \quad \hat{\mu} = \overline{X} \tag{1}$$

$$\hat{\sigma} = \frac{\overline{S}}{c_4} \quad \text{or} \quad \hat{\sigma} = \frac{\overline{R}}{d_2} \quad \text{or} \quad \hat{\sigma} = \frac{\overline{MR}}{d_2} \tag{2}$$

The study of the capability of each process is carried out through the classical capability indices:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{3}$$

$$C_{pk} = min\left(C_{pkI}, C_{pkS}\right) \tag{4}$$

$$C_{pkU} = \frac{USL - \mu}{3\sigma} \quad \text{and} \quad C_{pkL} = \frac{\mu - LSL}{3\sigma} \tag{5}$$

The use of equations (3) to (5) is only possible if the quality characteristic is Normally distributed. We suggest the use of the Kolmogorov-Smirnov test to verify the Normality of data distribution.

If a process is not in-control the analyst should investigate the causes that led to this situation and make appropriate corrections. Furthtermore, corrections should also be made in the process when it is stable (in-control), but is not capable.

3.2 Phase 2

After stability is observed and the process capability is analyzed in Phase 1, the statistical process control continues through monitoring. This procedure is commonly referred to as Phase 2 of the *SPC*. It follows, in this Phase 2, the application of *Z* and *W* control charts. These charts are built based on *Z* and *W* statistics calculated from the sample statistics $\overline{X}$ (or *X*) and *S* (or *R* or *MR*), respectively. Table 2 presents the transformed *Z* and *W* statistics for the different control charts, referring to the product/characteristic *j* at time *i*.

The study of the performance of processes is essential in Phase 2 of the *SPC*. So, it is necessary to define the periodic analysis of the processes in this Phase. Thus, it is suggested that this be accomplished in real time, based on two normalized indices (Pereira and Requeijo, 2012).

The limits for *Z* and *W* control charts are:

$$\begin{aligned} UCL_X &= UCL_{\overline{X}} = +3 \\ LCL_X &= LCL_{\overline{X}} = -3 \end{aligned} \tag{6}$$

$$\begin{aligned} UCL_S &= B_4 \\ LCL_S &= B_3 \end{aligned} \tag{7}$$

$$\begin{aligned} UCL_R &= UCL_R = D_4 \\ LCL_R &= LCL_{MR} = D_3 \end{aligned} \tag{8}$$

The new normalized capability indices will be recorded in each time *r* in the *Z* control chart. They are defined for each *j* product/characteristic at instant *r* by equations (9) and (10). A process of the product/characteristic *j* is capable when it satisfies simultaneously the two conditions $\left(Z_U\right)_j > 3$ and $\left(Z_L\right)_j < -3$.

$$\left(\left(Z_r\right)_L\right)_j = \left(\frac{LSL - \mu_r}{k\,\sigma_r}\right)_j \tag{9}$$

$$\left(\left(Z_r\right)_U\right)_j = \left(\frac{USL - \mu_r}{k\,\sigma_r}\right)_j \tag{10}$$

The *k* value is usually 1.33 or 1.25 for bilateral specifications or unilateral specifications, respectively. The values μ_r and σ_r for the product/characteristic *j*, are estimated by equations (11) and (12) using data from the previous Phase 1 and also new data gathered during Phase 2.

Table 2: Transformed *Z* and *W* statistics.

$Z_{\overline{X}}$ Chart	$\left(Z_i\right)_j = \left(\left(\overline{X}_i - \mu\right)/\sigma_{\overline{X}}\right)_j$
Z_X Chart	$\left(Z_i\right)_j = \left(\left(X_i - \mu\right)/\sigma\right)_j$
W_S Chart	$\left(W_i\right)_j = \left(S_i/\overline{S}\right)_j$
W_R Chart	$\left(W_i\right)_j = \left(R_i/\overline{R}\right)_j$
W_{MR} Chart	$\left(W_i\right)_j = \left(MR_i/\overline{MR}\right)_j$

$$\hat{\mu}_r = \overline{\overline{X}}_r \quad \text{or} \quad \hat{\mu}_r = \overline{X} \tag{11}$$

$$\hat{\sigma}_r = \frac{\overline{S}_r}{c_4} \quad \text{or} \quad \frac{\overline{R}_r}{d_2} \quad \text{or} \quad \frac{\overline{MR}_r}{d_2} \tag{12}$$

where:

$$\overline{\overline{X}}_r = \frac{1}{r}\left((r-1)\overline{\overline{X}}_{r-1} + \overline{X}_r\right) \; , \; r = 2, 3, \ldots \tag{13}$$

$$\overline{X}_r = \frac{1}{r}\left((r-1)\overline{X}_{r-1} + X_r\right) \; , \; r = 2, 3, \ldots \tag{14}$$

$$\overline{S}_r = \frac{1}{r}\left((r-1)\overline{S}_{r-1} + S_r\right) \; , \; r = 2, 3, \ldots \tag{15}$$

$$\overline{R}_r = \frac{1}{r}\left((r-1)\overline{R}_{r-1} + R_r\right) \; , \; r = 2, 3, \ldots \tag{16}$$

$$\overline{MR}_r = \frac{1}{r}\left((r-1)\overline{MR}_{r-1} + MR_r\right) \; , \; r = 3, 4, \ldots \tag{17}$$

4 SPC FOR A LIMITED NUMBER OF DATA

When the amount of data for each product/characteristic is not sufficient to adequately estimate the process parameters, it is suggested the use of the *Q* statistics, which result from the transformation of statistical sampling. Table 3 presents the *Q* statistics for the various charts.

The equations in Table 3 consider that X_r is the observation at time *r*, $\overline{X}_{r-1}$ is the average of $(r-1)$ observations, S_{r-1} is the standard deviation of $(r-1)$ observations, MR_r is the moving range calculated at time *r*, $\Phi^{-1}(\bullet)$ is the inverse of the Normal Distribution Function, $G_\nu(\bullet)$ is the T-student distribution Function with ν degrees of freedom, $F_{\nu_1,\nu_2}(\bullet)$ is the Fisher distribution Function with ν_1 e ν_2 degrees of freedom, n_i is the size of the sample *i*, ν_i is the degrees of freedom of the sample *i* $(\nu_i = n_i - 1)$, $\overline{X}_i$ is the average of the sample *i*, $\overline{\overline{X}}_i$ is the sequencial mean of *i* samples, S_i^2 is the variance of the sample *i*, $S_{p,i}^2$ is the pooled variance of *i* samples.

The lower and the upper control limits for the *Q* control charts Q are, respectively, –3 and +3.

The processes capabilities are analyzed at every moment through new capability indices Q_L and Q_U developed by Pereira and Requeijo (2012). The estimates of these indices at time *r* are given by equations (18) and (19).

Table 3: *Q* statistics.

$Q(X)$ Chart
$Q_r(X_r) = \Phi^{-1}\left(G_{r-2}\left(\sqrt{\frac{r-1}{r}}\left(\frac{X_r - \overline{X}_{r-1}}{S_{r-1}}\right)\right)\right)$ $r = 3, 4, \ldots$
$Q(MR)$ Chart
$Q_r(MR_r) = \Phi^{-1}\left(F_{1,\nu}\left(\frac{\nu\,(\mathrm{MR})_r^2}{(MR)_2^2 + (MR)_4^2 + \cdots + (MR)_{r-2}^2}\right)\right)$ $r = 4, 6, \ldots$
$Q(\overline{X})$ Chart
$Q_i(\overline{X}_i) = \Phi^{-1}\left(G_{n_1+\ldots+n_i-i}(\omega_i)\right) = \Phi^{-1}\left(G_{\nu_1+\ldots+\nu_i}(\omega_i)\right)$ $i = 2,\ 3,\ \ldots$ $\omega_i = \sqrt{\frac{n_i(n_1+\ldots+n_{i-1})}{n_1+\ldots+n_i}}\left(\frac{\overline{X}_i - \overline{\overline{X}}_{i-1}}{S_{p,i-1}}\right)$ $S_{p,i}^2 = \frac{(n_1-1)S_1^2 + \ldots + (n_i-1)S_i^2}{n_1+\ldots+n_i-i} = \frac{\nu_1 S_1^2 + \ldots + \nu_i S_i^2}{\nu_1+\ldots+\nu_i}$
$Q(S^2)$ Chart
$Q_i(S_i^2) = \Phi^{-1}\left(F_{n_i-1,n_1+\ldots+n_{i-1}-i+1}(\theta_i)\right)$ $i = 2,\ 3,\ \cdots$ $\theta_i = \frac{(n_1+\ldots+n_{i-1}-i+1)S_i^2}{(n_1-1)S_1^2+\ldots+(n_{i-1}-1)S_{i-1}^2} = \frac{S_i^2}{S_{p,i-1}^2}$

$$\left(\hat{Q}_L\right)_r = \frac{LSL - \hat{\mu}_r}{k\,\hat{\sigma}_r} \tag{18}$$

$$\left(\hat{Q}_S\right)_r = \frac{USL - \hat{\mu}_r}{k\,\hat{\sigma}_r} \tag{19}$$

$$\hat{\mu}_r = \overline{\overline{X}}_r \quad \text{or} \quad \hat{\mu}_r = \overline{X} \tag{11}$$

$$\hat{\sigma}_r = S_r / c_4 \quad \text{or} \quad S_{p,r} / c_4 \tag{20}$$

The statistics $\overline{\overline{X}}_r$ and $\overline{X}_r$ are given by equations (13) and (14). The statistic $S_{p,r}$ is

$$S_{p,r} = \sqrt{\frac{(n_1-1)S_1^2 + \ldots + (n_i-1)S_r^2}{n_1+\ldots+n_r-r}} \tag{21}$$

$$S_r = \sqrt{\left(\frac{r-2}{r-1}\right) S_{r-1}^2 + \frac{1}{r}\left(X_r - \overline{X}_{r-1}\right)^2} \;,\; r = 3,4,\ldots \quad (22)$$

A process is capable if simultaneously it verifies both conditions $Q_L < -3$ and $Q_U > 3$.

The implementation of the Q control charts assumes that the quality characteristic X is independent and Normally distributed.

5 CASE STUDIES

5.1 Example 1

In this section, the authors present an example of application to a production system of components (connection joints) of the wiring of electrical and electronic systems of an automotive industry. Five products (connection joints) were selected, and the traction resistance is the quality characteristic to be studied.

Initially (Phase 1 of the *SPC*) 50 samples (each sample was constituted by 4 connection joints) were obtained for the five products, and the whole procedure was as follows:

- Check whether the data from each component (connection joint) were independent, *i.e.*, $x_{ij} = \mu + \varepsilon_{ij}$ where $\varepsilon \sim N(0, \sigma^2)$, by using the Estimated Autocorrelation Function (*EACF*) and the Estimated Partial Autocorrelation Function (*EPACF*).
- Construction of $\overline{X}$ and S control charts for each product.
- Analysis of $\overline{X}$ and S control charts.
- Estimation of the process parameters, u $\overline{\overline{X}}$ as the estimator of the mean and $\overline{S}/c_4$ as the estimator of the process standard deviation.
- Check the shape of the distributions of data for the 5 products.
- Analysis of the process capability of the five processes, using the classic capacity indices defined by equations (3), (4) and (5).

The implementation of Phase 1 of the *SPC* showed that, for all the five processes:

1) The data for the five quality characteristics are independent (no significant autocorrelation).
2) The processes were in-control, *i.e.*, there were only common causes of variation.
3) The data of these five distributions were approximately Normal; this study was made based on the Kolmogorov-Smirnov test.
4) The five processes had the capability to produce according to specifications.

Table 4 shows the results obtained in the study of the processes of the 5 components (connection joints).

After the referred study in Phase 1, the authors moved on to Phase 2 of the SPC. As there are several products (5 types of connection joints), the control charts Z and W_S are best fit for this situation, since the distributions are approximately Normal and process parameters have been estimated in Phase 1. The authors built up the control charts Z and W_S, including all components (joints) in chronological sequence. The statistics Z and W of the sample i for the product j were determined from the equations of Table 2. As the technical specification concerning the traction resistance test of all five components is unilateral (there is only the *LSL*), the authors determined only the capability index Z_L for each product j, at time r, calculated using equation (9). The control charts were built using the Excel software. Figure 2 shows these control charts constructed from 40 samples, taken in chronological order, referring to the five components. The analysis of these (monitoring) control charts reveals that the five processes are stable, *i.e.*, the patterns are random for each component, and their capability to meet technical specifications remains at a satisfactory level.

5.2 Example 2

The second example relates to the production of two food products, which are referenced by T1 and T2. The weight of each pack is the quality characteristic under study. The technical specification relating to the weight of the product packages T1 is $250 \pm 10\,g$ and the technical specification relating to the weight of the product packages T2 is $500 \pm 15\,g$.

The number of packages of each product produced is very low. In this case, data availability was limited to only twelve samples of T1 and ten samples of T2. Each sample was constituted by 8 packages.

As the amount of data was restricted, this second study applied the $Q(\overline{X})$ and $Q(S^2)$ control charts; once again, Excel software was used to determine the values of the statistics $Q(\overline{X})$, $Q(S^2)$, Q_L and Q_U. The resulting control charts are shown in Figure 3.

Table 4: Study of processes of five components.

	Control Chart		Estimated Parameters			
Connection Joints	$\overline{\overline{X}}$	$\overline{S}$	$\hat{\mu}$	$\hat{\sigma}$	LSL	$\hat{C}_{pk}$
U2	82.95	3.018	82.95	3.275	70	1.318
U3	113.22	2.513	113.22	2.728	100	1.615
U5	90.30	2.391	90.30	2.595	80	1.323
U7	118.91	3.232	118.91	3.508	105	1,322
U9	449.51	9.958	449.51	10.809	400	1.527

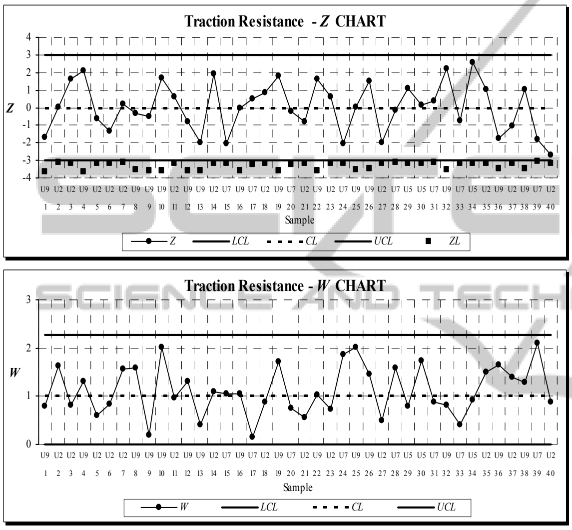

Figure 2: Z and W_S Control Charts for the traction test of 5 components (connection joints).

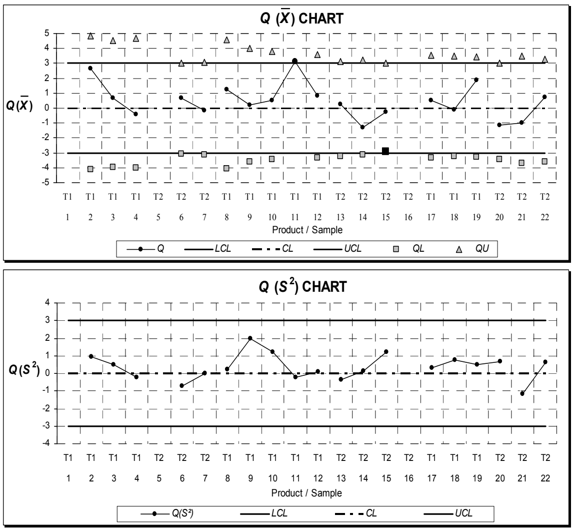

Figure 3: $Q(\overline{X})$ e $Q(S^2)$ Control Charts for package weight of two products T1 and T2.

The analysis of the $Q(\overline{X})$ control chart and $Q(S^2)$ control chart reveals the existence of a special cause at time t = 11 on the average of the T1 product package. The production process is fixed up the process and, consequently, the authors ignored the values of the statistics at time t = 11 in the calculation of Q statistics in the subsequent moments. At time t = 15, it was found that the process for product T2 showed no capability; therefore, an intervention was made to improve the process behavior. The study on the T2 product process was restarted at time t = 16.

6 CONCLUSIONS

The Z and W control charts have advantages over traditional Shewhart charts, namely:

1) they allow statistical control of all products/quality characteristics in the same control chart, as well as the construction of the X and MR control charts for each product.
2) they allow to study different characteristics together (*i.e.*, simultaneously).
3) they dramatically reduce the analysis time.

On the other hand, the possibility of making statistical process control with a limited amount of data is the most important advantage of the Q control charts, solving a difficult issue in the *SPC* domain. In addition to this great benefit, implementation of Q control charts also has the same advantages mentioned above for the Z and W control charts.

The use of capability indices Z_L, Z_U, Q_L and Q_U within the Z and Q control charts allows the study of processes capabilities in real time, thereby decreasing the probability of producing non conform units, *i.e.*, it reduces the chance of producing defective units.

In contrast with the above, a notorious disadvantage of the Z and Q control charts is the difficulty in analyzing the existence of non-random patterns, increasing the complexity of this analysis with the number of products/quality characteristics to be checked.

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