An Algorithm for Checking the Dynamic Controllability of a Conditional Simple Temporal Network with Uncertainty

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Abstract: A Simple Temporal Network with Uncertainty (STNU) is a framework for representing and reasoning about temporal problems involving actions whose durations are bounded but uncontrollable. A dynamically controllable STNU is one for which there exists a strategy for executing its time-points that guarantees that all of the temporal constraints in the network will be satisfied no matter how the uncontrollable durations turn out. A Conditional Simple Temporal Network with Uncertainty (CSTNU) augments an STNU to include observation nodes, where the execution of each observation node provides, in real time, the truth value of an associated proposition. Recent work has generalized the notion of dynamic controllability to cover CSTNUs. This paper presents an algorithm—called a DC-checking algorithm—for determining whether arbitrary CSTNUs are dynamically controllable. The algorithm, which is proven to be sound, is the first such algorithm to be presented in the literature. The algorithm extends edge-generation/constraint-propagation rules from an existing STNU algorithm to accommodate propositional labels, while adding new rules required to deal with the observation nodes. The paper also discusses implementation issues associated with the management of propositional labels.

1 INTRODUCTION

Workflow systems have been used to model business, manufacturing and medical-treatment processes. To meet the needs of such domains, Combi et al. (2010) presented a new workflow model that accommodates tasks with uncertain/uncontrollable durations; temporal constraints among tasks; and branching paths, where the branch taken is not known in advance. Subsequently, Hunsberger et al. (2012) introduced a *Conditional Simple Temporal Network with Uncertainty (CSTNU)* to represent the key features of that workflow model. The important property of *dynamic controllability* for CSTNUs was also defined. A CSTNU is dynamically controllable if there exists a strategy for executing the tasks in the associated workflow in a way that ensures that all temporal constraints will be satisfied no matter how the uncontrollable durations or branching events turn out.

This paper presents a *DC-checking algorithm* for CSTNUs (i.e., an algorithm for checking whether arbitrary CSTNUs are dynamically controllable). It is the first such algorithm in the literature. The algorithm, which is proven to be sound, extends the DC-checking algorithm for a simpler class of networks, called STNUs, developed by Morris and Muscettola (2005). It propagates *labeled values* on graph edges in a way that draws from prior work by Conrad and Williams (2011).

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2 MOTIVATING EXAMPLE

In the following, we will consider, as a motivating example, a process taken from the healthcare domain. More precisely, consider the excerpt from a workflow schema depicted in Fig. 1, which follows the model proposed by Combi and Posenato (2009).

The *workflow schema* is a directed graph where nodes correspond to *activities* and edges represent *control flows* that define dependencies on the order of execution. There are two types of activity: *tasks* and *connectors*. *Tasks* represent elementary work units that will be executed by external agents. Each task is represented graphically by a rounded box and has a mandatory *duration* attribute that specifies the allowed temporal spans for its execution. Typically, the duration of a task is not controlled by the system responsi-

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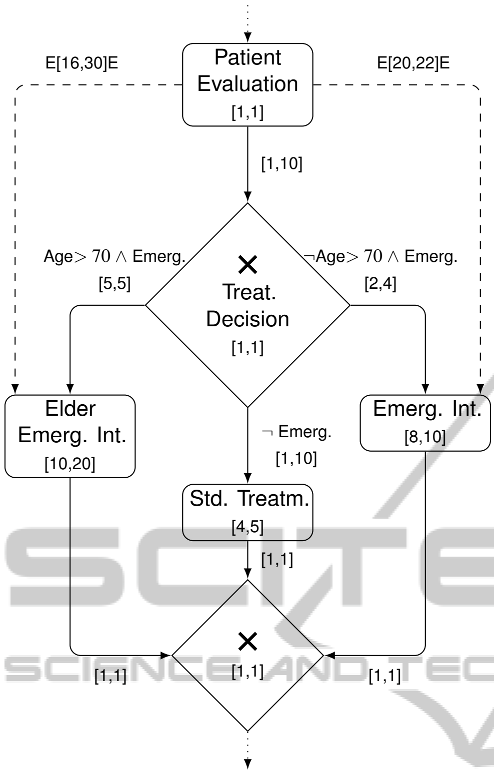


Figure 1: An excerpt of a healthcare workflow schema.

ble for managing the overall execution of the workflow (i.e., the Workflow Management System, WfMS). Unlike a task, a *connector* represents an internal activity whose execution *is* controlled by the WfMS. In particular, the WfMS uses connectors to coordinate the execution of the tasks. Connectors are represented graphically by diamonds. Like tasks, each connector has a mandatory *duration* attribute that specifies allowable temporal spans for its execution. However, unlike tasks, the WfMS can choose the value of each connector duration dynamically, in real time, to facilitate the coordination of the tasks in the workflow.

There are two kinds of connectors: *split* and *join*. *Split* connectors are nodes with one incoming edge and two or more outgoing edges. After the execution of the predecessor, possibly several successors have to be considered for execution. The set of nodes that can start their execution is determined by the kind of split connector. A *split* connector can be: *Total*, *Alternative* or *Conditional*. *Join* connectors are nodes with two or more incoming edges and only one outgoing edge. A *join* connector can be either *And* or *Or*.

Control flow is governed by oriented edges. Each oriented edge connects two activities, where the execution of the first activity (the *predecessor*) must be finished before starting the execution of the second one. Every edge has a *delay* attribute that specifies the allowed times that can be spent by the WfMS for possibly delaying the execution of the second activity.

Besides the temporal constraints associated with the duration and delay attributes of tasks, connectors and edges, a workflow schema can also include *relative constraints*. A relative constraint constrains the temporal interval between (the starting or ending time-points of) two non-consecutive workflow activities. Graphically, a relative constraint is represented by a directed edge from one activity to another, labeled by an expression of the form, $\mathsf{t}_1[\mathsf{MinD},\mathsf{MaxD}]\mathsf{t}_2$, where $t_1 \in \{S,E\}$ specifies whether the constraint applies to the starting or ending time-point of the first activity; $t_2 \in \{S,E\}$ specifies whether the constraint applies to the starting or ending time-point of the second activity; and $[\mathsf{MinD},\mathsf{MaxD}]$ specifies the allowed range for the temporal interval between the specified time-points.

The graph instance in Fig. 1 is a small excerpt from a process in a clinical domain. After the initial task, *Patient Evaluation,* whereby a physician determines whether the patient is in need of immediate medical attention (emergency state), there is an *alternative* connector, labeled *Treatment Decision*, from which three different treatment paths are possible, depending upon the age and emergency status of the patient. The three different treatments involve the following tasks: (1) *Elder Emergency Intervention*, (2) *Standard Treatment*, (3) *Emergency Intervention.* The times at which the *Elder Emergency Intervention* and *Emergency Intervention* tasks must be completed, relative to the initial *Patient Evaluation* task, are restricted by the relative temporal constraints emanating from the *Patient Evaluation* node. These constraints are labeled $\mathsf{E}[16,30]\mathsf{E}$ and $\mathsf{E}[20,22]\mathsf{E}$, respectively, in the figure.

Given a particular workflow schema, it is important to determine in advance whether the WfMS is able to successfully execute the tasks in the schema, while observing all relevant temporal constraints, no matter how the durations of the tasks turn out. (Task durations are typically not controllable by the WfMS.) It is interesting to observe that the overall workflow schema in Fig. 1 may not be successfully executed by the WfMS for some possible task durations, even though each possible workflow subschema (or workflow path) is controllable when age and emergency status are known before execution begins.

A CSTNU is a more general formalism that allows the representation of all kinds of temporal constraints for workflow execution. In the following, after some background on related kinds of temporal networks, we will discuss CSTNUs and a new algorithm for determining the dynamic controllability of CSTNUs.

3 BACKGROUND

Dechter et al. (1991) introduced Simple Temporal Networks (STNs). An STN is a set of time-point variables (or time-points) together with a set of simple temporal constraints, where each constraint has the form $Y - X \leq \delta$, where X and Y are time-points and δ is a real number. The *all-pairs, shortest-paths* matrix for the associated graph is called the *distance matrix* for the STN. For any STN, the following statements are equivalent:

- The STN has a solution (i.e., a set of values for the time-points that satisfy all of the constraints);
- The associated graph has no negative loops; and
- The distance matrix has zeros on its main diagonal.

Morris et al. (2001) presented Simple Temporal Networks with Uncertainty (STNUs) that augment STNs to include *contingent links* that represent uncontrollable-but-bounded temporal intervals. They gave a formal semantics for the important property of *dynamic controllability,* which holds if there exists a strategy for executing the time-points in the network that guarantees that all of the constraints will be satisfied no matter how the contingent durations turn out.[2] Crucially, the durations of contingent links are observed in real-time, as they complete; execution decisions can only depend on past observations.

Morris et al. (2001) also presented a *pseudo-polynomial-time* algorithm—called a *DC-checking algorithm*—for determining whether any given STNU is *dynamically controllable (DC).* Later, Morris and Muscettola (2005) presented the first polynomial DC-checking algorithm, which operates in $O(N^5)$ time. Because this algorithm plays an important role in this paper, it will henceforth be called the MM5 algorithm. Morris (2006) subsequently presented an $O(N^4)$-time DC-checking algorithm for STNUs, but it will not be discussed further in this paper.

Tsamardinos et al. (2003) introduced the Conditional Temporal Problem (CTP) which augments STNs to include *observation nodes*. When an observation node is executed, the truth value of its associated proposition becomes known. They presented a formal semantics for the important property of *dynamic consistency* which holds if there exists a strategy for executing the time-points in the network that guarantees that all of the constraints will be satisfied no matter how the observations turn out. Crucially, the truth values of propositions associated with observation nodes only become known in real time, as the observation nodes are executed. Tsamardinos et al. (2003) showed how to convert the semantic constraints inherent in the definition of dynamic consistency into a Disjunctive Temporal Problem (DTP). They then used an off-the-shelf DTP solver to determine the dynamic consistency of the original network in exponential time.

Hunsberger et al. (2012) combined the features of STNUs and CTPs to produce a Conditional Simple Temporal Network with Uncertainty (CSTNU). They proved that their definition of a CSTNU generalizes both STNUs and CTPs. In addition, they introduced a definition of dynamic controllability for CSTNUs that they proved generalizes the corresponding notions for STNUs and CTPs. They noted that because the existing DC-checking algorithms for STNUs and CTPs work so differently, they could not be easily combined to yield a DC-checking algorithm for CSTNUs. They also suggest that a new kind of algorithm has to be defined that incorporates new edge generation rules that take into account the propositional truth values generated by the observation nodes. In preparation for this kind of algorithm, they presented a *Label-Modification* rule for edges in a CSTNU that loosely resembles the *Label-Removal* rule for STNUs used by Morris and Muscettola.

This paper presents a DC-checking algorithm for CSTNUs that follows the proposal mentioned above. It extends the edge-generation/constraint-propagation MM5 algorithm for STNUs to accommodate observation nodes whose execution makes known the truth values of their associated propositions in real time. The algorithm, called the CSTNU DC-checking algorithm, generates edges that are labeled by propositions associated with observation nodes. Because there can be multiple such labeled edges between any pair of time-points, the algorithm carefully manages the potentially-exponential explosion of labels using techniques inspired by the work of Conrad and Williams (2011).

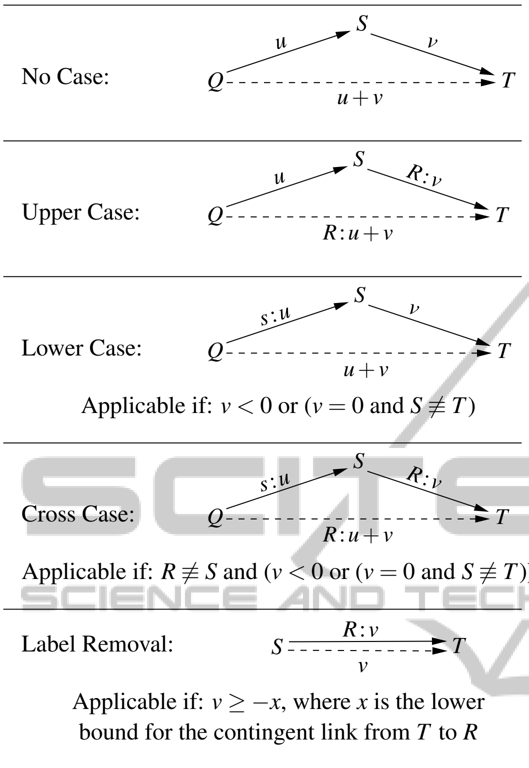
3.1 DC-Checking for STNUs

Following Morris et al. (2001), an STNU is a set of time-points and temporal constraints, like those in an STN, together with a set of contingent links. Each contingent link has the form, (A,x,y,C), where A and C are time-point variables (or time-points) and $0 < x < y < \infty$. A is called the *activation time-point*; C is the *contingent time-point*. Once A is executed, C is guaranteed to execute such that $C - A \in [x,y]$. However, the particular time at which C executes is uncontrollable. Instead, it is only observed as it happens.

Let $\mathcal{S} = (\mathcal{T}, \mathcal{C}, \mathcal{L})$ be an STNU, where $\mathcal{T}$ is a set of time-points, $\mathcal{C}$ is a set of constraints, and $\mathcal{L}$ is a set of contingent links. The graph associated with $\mathcal{S}$ has the form, $(\mathcal{T}, \mathcal{E}, \mathcal{E}_\ell, \mathcal{E}_u)$, where each time-point in $\mathcal{T}$

[2] Hunsberger (2009) subsequently corrected a minor flaw in the semantics of dynamic controllability.

Table 1: Edge-generation rules for the MM5 algorithm. For each rule, the edge generated by the rule is dashed.



serves as a node in the graph; $\mathcal{E}$ is a set of *ordinary* edges; $\mathcal{E}_\ell$ is a set of *lower-case* edges; and $\mathcal{E}_u$ is a set of *upper-case* edges (Morris and Muscettola, 2005).

- Each ordinary edge has the form, $X \xrightarrow{v} Y$, representing the constraint, $Y - X \leq v$.
- Each lower-case edge has the form, $A \xrightarrow{c\,:\,x} C$, representing the *possibility* that the contingent duration, $C - A$, might take on its minimum value, x.
- Each upper-case edge, $C \xrightarrow{C:-y} A$, represents the *possibility* that the contingent duration, $C - A$, might take on its maximum value, y.

The MM5 algorithm works by recursively generating new edges in the STNU graph using the rules shown in Table 1. For each rule, pre-existing edges are denoted by solid arrows and newly generated edges are denoted by dashed arrows. Note that each of the first four rules takes two pre-existing edges as input and generates a single edge as its output. In contrast, the Label-Removal rule takes only one edge as input. Finally, applicability conditions of the form, $R \not\equiv S$, should be construed as stipulating that R and S must be distinct time-point variables, not as constraints on the *values* of those variables.

Procedure : MM5-DC-Check(G).

Input: G: STNU graph instance to analyze.
Output: the controllability of G.
for 1 *to Cutoff_Bound* **do**
  if *(AllMax matrix inconsistent)* **then**
    return *false*;
  generate new edges using rules from Table 1;
  if *(no edges generated)* **then** **return** *true*;
end
return *false*

Note that the edge-generation rules only generate new ordinary or upper-case edges. Unlike the upper-case edges in the original graph, the upper-case edges generated by these rules represent conditional constraints, called *waits* (Morris et al., 2001). In particular, an upper-case edge, $Y \xrightarrow{C:-w} A$, represents a constraint that as long as the contingent time-point, C, remains unexecuted, then the time-point, Y, must wait at least w units after the execution of A, the activation time-point for C.

Procedure : MM5-DC-Check gives pseudocode for the MM5 DC-checking algorithm. The algorithm performs at most $N^2 + NK + K = O(N^2)$ iterations, which is the number of distinct kinds of edges in a graph having N time-points and K contingent links. In each iteration, the algorithm first computes the *AllMax* matrix—which is the distance matrix for the STN formed by all of the original and generated, ordinary and upper-case edges (without their alphabetic labels)—and checks that there is no negative cycles in it and then applies the rules from Table 1 to all relevant combinations of edges of the STNU from the previous iteration. If no new edges are generated in any given iteration and there is no negative cycle at all, the algorithm reports that the network is dynamically controllable. If the algorithm continues generating new, stronger edges after the cutoff bound $N^2 + NK + K$, then the network cannot be DC. Since each iteration can be done in $O(N^3)$ time, the overall complexity of the MM5 algorithm is $O(N^5)$.

3.2 CSTNUs

A Conditional Simple Temporal Network with Uncertainty (CSTNU) is a network that combines the observation nodes and branching from a CTP with the contingent links of an STNU (Hunsberger et al., 2012). There is a one-to-one correspondence between observation nodes and propositional letters: the execution of an observation node generates a truth value for the corresponding proposition. However, nodes and edges in a CSTNU graph may be labeled by conjunctions of propositional literals. The time-point corresponding to a node with label, ℓ, need only be executed in scenarios where ℓ is true. Similarly, the constraint corresponding to an edge with label, ℓ, is only applicable in scenarios where ℓ is true. The *label universe,* defined below, is the set of all possible labels.

Definition 1 (Label, Label Universe)**.** Given a set P of propositional letters, a *label* is any (possibly empty) conjunction of (positive or negative) literals from P. For convenience, the empty label is denoted by $\boxdot$. The *label universe* of P, denoted by P^*, is the set of all labels whose literals are drawn from P.

In the following, when not specified, lower-case Latin letters will denote propositions of P, while Greek lower-case letters will denote labels of P^*.

Definition 2 (Consistent labels, label subsumption)**.**

- Labels, ℓ_1 and ℓ_2, are called *consistent*, denoted by $Con(\ell_1, \ell_2)$, if and only if $\ell_1 \wedge \ell_2$ is satisfiable.
- A label ℓ_1 *subsumes* a label ℓ_2, denoted by $Sub(\ell_1, \ell_2)$, if and only if $\models (\ell_1 \Rightarrow \ell_2)$.

The following definition of a CSTNU is extracted from Hunsberger et al. (2012). The most important ingredients of a CSTNU are: $\mathcal{T}$, a set of time-points; $\mathcal{C}$, a set of *labeled* constraints; $\mathcal{OT}$, a set of observation time-points; and $\mathcal{L}$ a set of contingent links.

Definition 3 (CSTNU)**.** A Conditional STN with Uncertainty (CSTNU) is a tuple, $\langle \mathcal{T}, \mathcal{C}, L, \mathcal{OT}, \mathcal{O}, P, \mathcal{L} \rangle$, where:

- $\mathcal{T}$ is a finite set of real-valued time-points;
- P is a finite set of propositional letters;
- $L : \mathcal{T} \to P^*$ is a function that assigns a label to each time-point in $\mathcal{T}$;
- $\mathcal{OT} \subseteq \mathcal{T}$ is a set of observation time-points;
- $\mathcal{O} : P \to \mathcal{OT}$ is a bijection that associates a unique observation time-point to each propositional letter;
- $\mathcal{L}$ is a set of contingent links;
- $\mathcal{C}$ is a set of *labeled* simple temporal constraints, each having the form, $(Y - X \leq \delta,\ \ell)$, where $X, Y \in \mathcal{T}$, δ is a real number, and $\ell \in P^*$;
- for any $(Y - X \leq \delta,\ \ell) \in \mathcal{C}$, the label ℓ is satisfiable and subsumes both $L(X)$ and $L(Y)$;
- for any $p \in P$ and $T \in \mathcal{T}$, if$p$ or $\neg p$ appears in T's label, then
  - $Sub(L(T), L(\mathcal{O}(p))$, and
  - $(\mathcal{O}(p) - T \leq -\varepsilon,\ L(T)) \in \mathcal{C}$, for some $\varepsilon > 0$;
- for each $(Y - X \leq \delta,\ \ell) \in \mathcal{C}$ and each $p \in P$, if p or $\neg p$ appears in ℓ, then $Sub(\ell, L(\mathcal{O}(p)))$; and
- $(\mathcal{T}, \lfloor \mathcal{C} \rfloor, \mathcal{L})$ is an STNU, where $\lfloor \mathcal{C} \rfloor$ is the following set of unlabeled constraints: $\{(Y - X \leq \delta) \mid (Y - X \leq \delta, \ell) \in \mathcal{C}$ for some $\ell\}$.

The graph for a CSTNU is similar to that for an STNU except that some of the nodes may be observation nodes; and there may be propositional labels on nodes and edges. If p is a proposition, then the observation node whose execution generates a truth value for p shall be denoted by P? The propositional label of a node is usually represented near the node name, enclosed in square brackets. For example, a node labeled by $[cd]$ is only applicable to scenarios where propositions c and d are both true. Since edges in a CSTNU graph can have both propositional labels (associated with observation nodes) and alphabetic labels (associated with lower-case and upper-case edges in an STNU), these different kinds of labels are clearly distinguished in the *labeled values* for an edge, as follows.

Definition 4 (Labeled values)**.** A *labeled value* is a triple, $\langle PLabel,\ ALabel,\ Num \rangle$, where:

- $PLabel \in P^*$ is a propositional label,
- $ALabel$, an alphabetic label, is one of the following:
  - an upper-case letter, C, as on an upper-case edge in an STNU;
  - a lower-case letter, c, as on a lower-case edge in an STNU; or
  - $\diamond$, representing no alphabetic label, as for an ordinary STN edge.
- Num is a real number.

For example, $\langle p\neg q,\ c,\ 3 \rangle$ is a labeled lower-case edge; $\langle pq\neg r,\ C,\ -8 \rangle$ is a labeled upper-case edge; and $\langle \neg p,\ \diamond,\ 2 \rangle$ is a labeled ordinary edge.

Fig. 2 shows a sample CSTNU that represents a possible mapping of the main part of the workflow schema of Fig. 1. Initially, each ordinary edge in the network has only one labeled value, while each edge associated with a contingent link has two labeled values: one representing an ordinary STN constraint and the other representing an upper-case or lower-case STNU constraint. However, the new edge-generation rules given below will typically result in situations where a single edge may have numerous labeled values associated with it. The graph in the figure includes two observation nodes and three contingent links. Observation node A? generates a truth value for the proposition, a, which represents that the patient in question is over age 70. Observation node B? generates a truth value for the proposition, b, which represents that the patient is in need of immediate medical attention. The contingent link, $(C, 10, 20, D)$, represents an *Elder Emergency Intervention* task that takes between 10 and 20 minutes; the contingent link, $(H, 4, 5, I)$, represents a *Standard Treatment* task that takes between 4 and 5 minutes; and the contingent link, $(E, 8, 10, F)$,

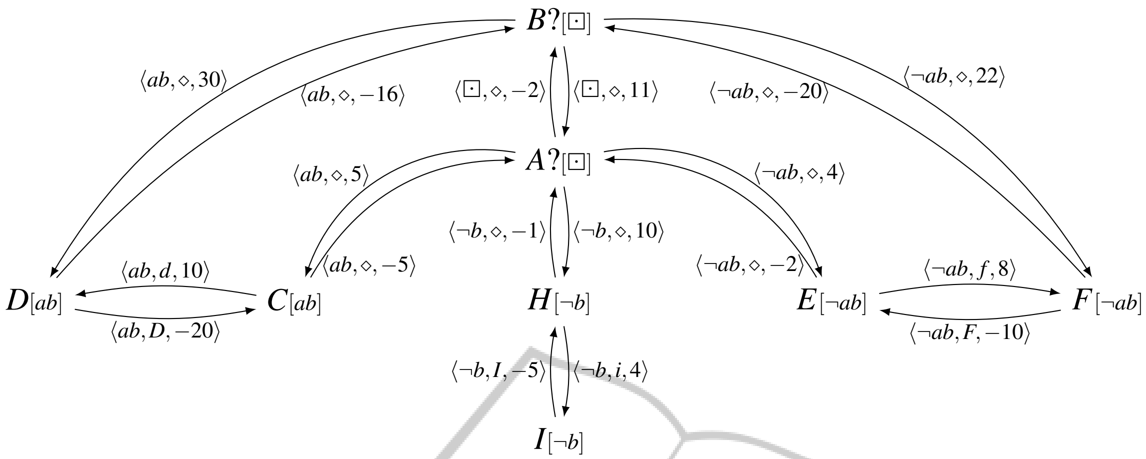


Figure 2: A possible CSTNU graph mapping the main part of the workflow schema of Fig. 1.

represents an *Emergency Intervention* task that takes between 8 and 10 minutes. To simplify the graph, only the lower-case and upper-case edges for each contingent link are explicitly represented.[3] All other edges in the sample CSTNU represent ordinary temporal constraints. For example, the edges between $B?$ and $A?$ represent that the observation of proposition a must occur between 2 and 11 minutes after the observation of proposition b.

As defined in Hunsberger et al. (2012), a *scenario* s is a label that specifies a truth value for every propositional letter. The STNU formed by the nodes and edges (i.e., time-points and constraints) whose labels are true in a given scenario is called a *projection* of the CSTNU onto that scenario. A *situation* ω for an STNU specifies fixed durations for all of the contingent links. A *drama* (s,ω) is a scenario/situation pair that specifies fixed truth values for all of the propositional letters and fixed durations for all of the contingent links.

An *execution strategy* is a mapping from dramas to *schedules.* A schedule assigns an execution time to all of the time-points. Thus, if σ is an execution strategy and (s,ω) is drama, then $\sigma(s,\omega)$ is a schedule. For any time-point X, $[\sigma(s,\omega)]_X$ denotes the execution time assigned to X by the strategy σ in the drama (s,ω). A *dynamic* execution strategy is one in which the execution times assigned to non-contingent time-points only depends on *past* observations. A CSTNU is dynamically controllable if it has a dynamic execution strategy that guarantees the satisfaction of all temporal constraints no matter which drama unfolds in real time.

Note that a constraint whose propositional label is ℓ need only be satisfied in scenarios where ℓ is true. Similarly, a constraint whose alphabetic label is C need only be satisfied while C remains unexecuted.

Each of the STNUs obtained by projecting the sample CSTNU of Fig. 2 onto the scenarios, $ab, \neg ab$ and $\neg b$, is dynamically controllable—as an STNU. However, as will be shown below, the sample CSTNU is not dynamically controllable—as a CSTNU. This conforms to the observation by Combi and Posenato (2010) that the independent controllability of each *path* through a workflow is a necessary, but insufficient condition for the controllability of the entire workflow. For the workflow in Fig. 1, it turns out that there is no execution time for the observation node, $A?$, that will enable the rest of the network to be safely executed no matter how subsequent observations turn out.

4 DC-Checking FOR CSTNUs

This section presents a DC-checking algorithm for CSTNUs. The basic approach is to extend the MM5 algorithm for STNUs to accommodate propositional labels. The presence of observation nodes also requires some new *label-modification* rules. In addition, since the propagation of labeled values involves conjoining labels, which can lead to an exponential number of labeled values, the paper also addresses the management of sets of labeled values.

A *partial scenario* is a scenario that assigns truth values to some subset of propositional letters. Partial scenarios represent the outcomes of past observations. A label ℓ (on a node or edge) is said to be *enabled* in a (possibly partial) scenario s if none of the propositional literals in ℓ is false in s. For example, the label $a\neg c$ is enabled in the partial scenario $b\neg c$, but not in the partial scenario bcd. Note that the truth value of a is not determined in either of these partial scenarios.

[3] As proven elsewhere (Hunsberger, 2013), the ordinary edges associated with contingent links are not needed for the purposes of DC checking.

During execution of a CSTNU instance, the WfMS keeps track of all past observations, which together determine a partial scenario. For any as-yet-unexecuted non-contingent time-point, the WfMS must consider *all* enabled labeled constraints involving that time-point/node and verify that those constraints are satisfiable. For any pair of time-points, X and Y, it is possible that more than one labeled constraint from X to Y is enabled because they are compatible with the current partial scenario and, therefore, all of them have to be satisfiable. Thus, it is necessary to generate all possible constraints/edges for all possible (partial) scenarios in order to evaluate if a CSTNU is DC controllable. Hereinafter, we indifferently refer to the set of labeled constraints/edges for a given pair of time-points as a set of different labeled constraints/edges or as different labeled values of the same constraint/edge.

4.1 Edge Generation for CSTNUs

The edge generation rules for CSTNUs fall into two main groups. The first group extends the edge-generation rules of the MM5 algorithm to accommodate labeled edges; the second group consists of label-modification rules that address interactions involving observation nodes.

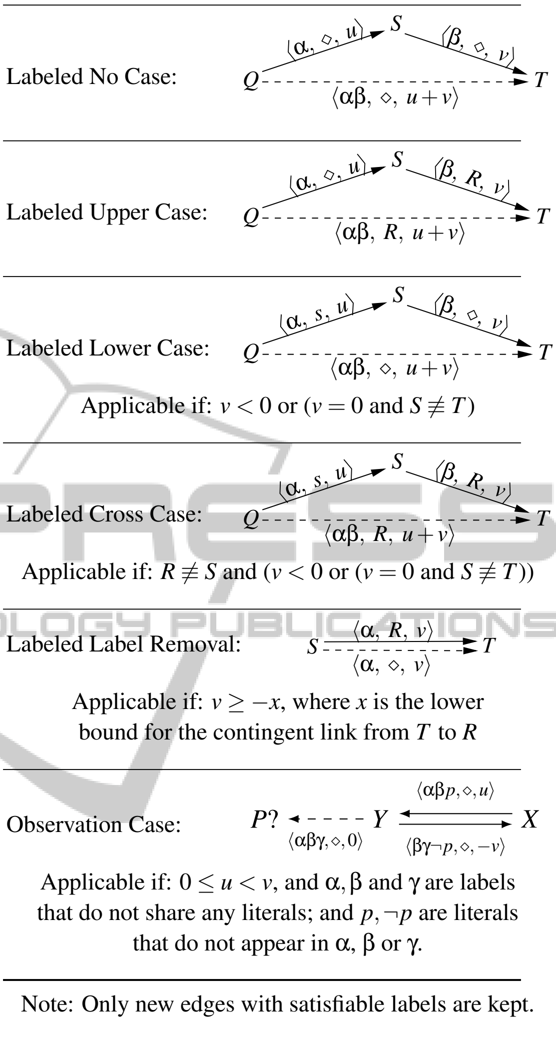
4.1.1 Labeled Constraint Generation

We begin by modifying the edge-generation rules for STNUs (cf. Table 1) to accommodate labeled edges. The new rules are shown in Table 2. Note that each of the first four rules generates an edge whose PLabel is the conjunction of the PLabels of its parent edges. If the resulting PLabel is unsatisfiable (e.g., $p\neg p$), then the new edge is not generated (or kept). The fifth rule effectively removes the upper-case (alphabetic) label, resulting in a labeled ordinary edge.

The sixth rule, the *Observation Case* rule, does not extend any of the MM5 rules; however, it is included here for convenience. This new rule addresses circumstances where an existing labeled edge from X to Y is inconsistent with an existing labeled edge from Y to X. To avoid having to satisfy both of these constraints—which would be impossible—this rule adds a new edge that ensures that the value of the proposition p, which appears in both labels, will be known before having to decide which constraint to satisfy. The soundness of this rule is ensured by the following lemma.

Lemma 4.1 (Observation Case). Let σ be a dynamic execution strategy that satisfies the labeled constraints in Fig. 3-(a). Then σ must also satisfy the labeled constraint, $(P? - Y \leq 0, \alpha\beta\gamma)$, shown in Fig. 3-(b).[4]

Table 2: New edge-generation rules for CSTNUs.



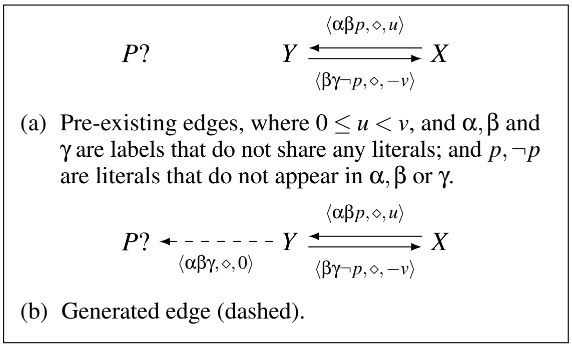


Figure 3: The Observation Case Rule (cf. Lemma 4.1).

Proof. Let σ be as in the statement of the lemma. Suppose there is a drama, (s, ω), such that:

[4] Recall that an execution strategy need only satisfy labeled constraints in scenarios where their labels are true.

- the label $\alpha\beta\gamma$ is true in scenario s; but
- the schedule $\sigma(s,\omega)$ does *not* satisfy the constraint, $(P? - Y \le 0, \alpha\beta\gamma)$.

Then, in that schedule, $P? - Y > 0$ and, hence, $Y < P?$ Next, since $X - Y \le -v < 0$, it follows that $X < Y$. Thus, $X < Y < P?$ (i.e., both X and Y precede $P?$).

Next, let $\tilde{s}$ be the same scenario as s except that the truth value of p is flipped. Let t be the first time at which the schedules, $\sigma(s,\omega)$ and $\sigma(\tilde{s},\omega)$, differ. Thus, there must be some time-point T that is executed in one of the schedules at time t, and in the other at some time later than t. In that case, the corresponding histories (of past observations) at time t must be different. However, since all other propositions and contingent durations are identical in the dramas, (s,ω) and $(\tilde{s},\omega)$, the only possible difference must involve the value of the proposition p, whence $P?$ must be executed in both schedules before the time of first difference, t.

Now, in the schedule $\sigma(s,\omega)$, we have seen that both X and Y are executed before $P?$, and hence before t. Thus, $[\sigma(s,\omega)]_X = [\sigma(\tilde{s},\omega)]_X$ and $[\sigma(s,\omega)]_Y = [\sigma(\tilde{s},\omega)]_Y$. But this is not possible because in one scenario $Y - X \le u$, while in the other $Y - X \ge v$. In particular, both constraints cannot be satisfied, using the same values of X and Y, since $u < v$. □

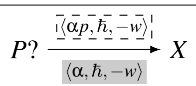
4.1.2 Label Modification

This section introduces a variety of *label-modification* rules that share some resemblance to the Label-Removal rule in Table 2. Thus, we begin with a short description of the Label-Removal rule.

Suppose a CSTNU contains a contingent link, $(A, 5, 12, C)$. In other words, the contingent duration, $C - A$, is uncontrollable, but guaranteed to be within the interval, $[5, 12]$. Suppose further that the network also contains the upper-case edge, $Y \xrightarrow{C:-2} A$, which represents the following *wait* constraint: "As long as the contingent time-point C remains unexecuted, then Y must wait at least 2 units after the execution of its activation time-point, A." Given that the minimum duration of this contingent link is 5, it follows that the contingent time-point C must remain unexecuted until after the wait time of 2 has expired. As a result, the decision to execute Y must, in *every* situation, wait at least 2 units after A. For this reason, the Label-Removal rule generates the ordinary edge, $Y \xrightarrow{-2} A$, which represents the *unconditional* constraint, $A - Y \le -2$ (i.e., $Y \ge A + 2$). This example illustrates that in certain circumstances, a constraint conditioned on an uncontrollable event—in this case, the execution of the contingent time-point C—might have the force of an unconditional constraint because the uncertainty associated with the uncontrollable event will definitely not be resolved at the time a particular execution decision—in this case, the decision to execute Y—must be made.

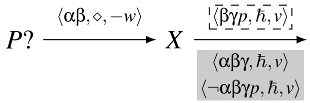
Table 3: Label-modification rules for CSTNUs. Labeled values in shaded boxes replace those in dashed boxes.

R0 Case:



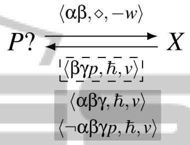
Applicable if: $0 \le w$, p is a literal not in α, and $\hbar$ is either $\diamond$ or an upper-case letter.

R1 Case:



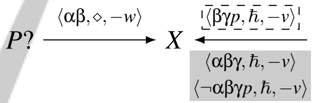
Applicable if: $0 \le w$; $v \le w$; α, β and γ are labels that do not share any literals; p is a literal that does not appear in α, β or γ; and $\hbar$ is $\diamond$ or an upper-case letter.

R2 Case:



Applicable if: $0 \le w \le v$; α, β and γ are labels that do not share any literals; p is a literal that does not appear in α, β or γ; and $\hbar$ is either $\diamond$ or an upper-case letter.

R3 Case:



Applicable if: $0 \le w$; $v \le w$; α, β and γ are labels that do not share any literals; p is a literal that does not appear in α, β or γ; and $\hbar$ is $\diamond$ or an upper-case letter.

The label-modification rules in Table 3 have the same general flavor, except that they deal with the uncertainty associated with observation nodes, rather than contingent links. For example, consider the edge, $P? \xrightarrow{\langle \alpha p,\ \diamond,\ -w\rangle} X$, where neither p nor $\neg p$ appears in α, and $w \ge 0$. This edge represents the conditional constraint that "in scenarios where αp is true, $X - P? \le -w$ (i.e., $X + w \le P?$) must hold." Given that $w \ge 0$, it follows that in scenarios where αp is true, X must be executed before the observation node $P?$ But that, in turn, implies that the truth value of p cannot be known at the time X is executed. And, of course, the truth value of p cannot be known when the decision to execute $P?$ is made either. As a result, decisions about when to execute X and $P?$ cannot depend on the truth value of p. Thus, the PLabel on the edge from $P?$ to X should be modified to remove the occurrence of p, yielding the new edge, $P? \xrightarrow{\langle \alpha,\ \diamond,\ -w\rangle} X$, which represents the constraint that in scenarios where α holds, $X - P? \le -w$ (i.e., $X + w \le P?$) must hold.

This is the idea behind the label-modification rule, R0, shown in Table 3. For each rule in the table, pre-existing labels are represented as usual, labels to be modified (or replaced by new ones) are shown within a dashed box, and newly generated labels are shown within a shaded box. The following lemma shows that Rule R0 is sound.

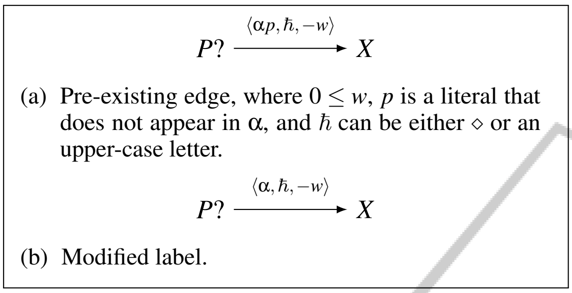


Figure 4: The Label-Modification rule, R0 (cf. Lemma 4.2).

Lemma 4.2 (Label-Modification Rule, R0)**.** Suppose that $w \geq 0$ and α is a label that does not contain the literal p. If σ is a dynamic execution strategy that satisfies the labeled constraint, $(X - P? \leq -w, \alpha p)$, as shown in Fig. 4-(a), then σ must also satisfy the labeled constraint, $(X - P? \leq -w, \alpha)$, as shown in Fig. 4-(b). The rule also applies to upper-case edges (i.e., edges with an upper-case alphabetic label, $\hbar$).

Proof. Let (s, ω) be a drama such that:

- the label $\alpha \neg p$ is true in scenario s; but
- the schedule $\sigma(s, \omega)$ does *not* satisfy the constraint, $(X - P? \leq -w)$.

In that case, $X + w > P?$ Next, let s' be the same scenario as s except that p is true in s'. Then αp is true in s', which implies that $(X - P? \leq -w)$ holds in $\sigma(s', \omega)$. Thus, $X + w \leq P?$ holds in $\sigma(s', \omega)$.

Next, let t be the first time at which the schedules, $\sigma(s, \omega)$ and $\sigma(s', \omega)$, differ. Then there must be some time-point T that is executed in one of the schedules at time t, and in the other at some time after t. But in that case, the corresponding histories at time t must be different. Since the dramas, (s, ω) and (s', ω), are identical except for the truth value of p, it follows that the observation node, $P?$, must be executed before time t. Now, in the drama (s', ω), the constraint, $X + w \leq P?$, is satisfied; thus, both X and $P?$ must be executed before time t in that drama. Since the schedules, $\sigma(s, \omega)$ and $\sigma(s', \omega)$, are identical prior to time t, it follows that the same constraint is satisfied by $\sigma(s', \omega)$, contradicting the choice of (s', ω). ☐

Rule R1 in Table 3 first appeared in Hunsberger et al. (2012). The corresponding lemma, given below, shows that it is sound. Its proof is not repeated here.

Lemma 4.3 (Label-Modification Rule, R1)**.** Let σ be a dynamic execution strategy that satisfies the labeled constraints in Fig. 5-(a). Then σ must also satisfy the labeled constraint $(Y - X \leq v, \alpha\beta\gamma)$. The original constraint, $(Y - X \leq v, \beta\gamma p)$, is replaced by the pair of labeled constraints, $(Y - X \leq v, \alpha\beta\gamma)$ and $(Y - X \leq v, \neg\alpha\beta\gamma p)$, as depicted in Fig. 5-(b).

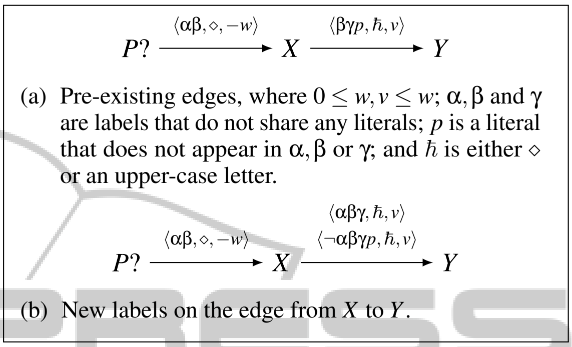


Figure 5: The Label-Modification rule, R1 (cf. Lemma 4.3).

Regarding the proof, here we only remark that when $v > w$ the rule is not needed because, in that case, the execution of Y could be postponed until after the execution of $P?$, in which case the truth value of p would become known.

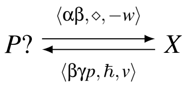
Lemma 4.3 does not analyze the case where $P?$ and Y are the same node. In that case, rule R1 is not needed. In particular, if $v < w$, there would be a negative cycle, implying that the network was not DC. However, we need to consider the case where $P?$ and Y are the same node and $v \geq w$. In that case, the label $\beta\gamma p$ must always be considered before the execution of $P?$ (i.e., before the truth value of p is known) and, therefore, it is necessary to propagate it without p. We call this rule R2.

Lemma 4.4 (Label-Removal Rule, R2)**.** Let σ be a dynamic execution strategy that satisfies the labeled constraints in Fig. 6-(a). Then σ must also satisfy the labeled constraint, $(Y - X \leq v, \alpha\beta\gamma)$. The original constraint, $(Y - X \leq v, \beta\gamma p)$, is replaced by the pair of labeled constraints, $(Y - X \leq v, \alpha\beta\gamma)$ and $(Y - X \leq v, \neg\alpha\beta\gamma p)$, as depicted in Fig. 6-(b).

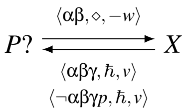
Proof. It is straightforward to prove the soundness of this rule, since it deals with the standard constraint between two ordered time-points. ☐

When there is a negative value on a constraint from Y to X, we have another case of label modification as shown in the following lemma.

Lemma 4.5 (Label-Modification Rule, R3)**.** Let σ be a dynamic execution strategy that satisfies the labeled



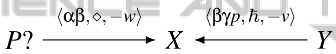
(a) Pre-existing edges, where $0 \leq w, v \geq w$; α, β and γ are labels that do not share any literals; p is a literal that does not appear in α, β or γ; and $\hbar$ is either $\diamond$ or an upper-case letter.



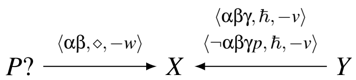
(b) New labels on the edge from Y to X.

Figure 6: The Label-Modification rule, R2 (cf. Lemma 4.4).

constraints shown in Fig. 7-(a). Then σ must also satisfy the labeled constraint, $(X - Y \leq -v, \alpha\beta\gamma)$. The original constraint, $(X - Y \leq -v, \beta\gamma p)$, is replaced by the pair of labeled constraints, $(X - Y \leq -v, \alpha\beta\gamma)$ and $(X - Y \leq -v, \neg\alpha\beta\gamma p)$, as shown in Fig. 7-(b).



(a) Pre-existing edges, where $0 \leq w$; $v \leq w$; α, β and γ are labels that do not share any literals; p is a literal that does not appear in α, β or γ; and $\hbar$ is either $\diamond$ or an upper-case letter.



(b) New labels on the edge from Y to X.

Figure 7: The Label-Modification rule, R3 (cf. Lemma 4.5).

Proof. Let σ be as in the statement of the lemma. Suppose that there is some drama, (s, ω), such that:

- the label $\alpha\beta\gamma$ is true in scenario s; but
- the schedule $\sigma(s, \omega)$, does *not* satisfy the constraint, $(X - Y \leq -v)$.

In that case, $X - Y > -v$, which implies that $Y < X + v \leq X + w \leq P?$ (Recall that $v \leq w$ and, given that $\alpha\beta$ is true, the constraint, $(X - P? \leq -w)$, must be satisfied by σ.) Note also that $X \leq P?$

Next, let $\tilde{s}$ be the same scenario as s except that the truth value of p is flipped. Let t be the first time at which the schedules, $\sigma(s, \omega)$ and $\sigma(\tilde{s}, \omega)$, differ. Thus, there must be some time-point T that is executed in one of the schedules at time t, and in the other at some time later than t. But in that case, the corresponding histories at time t must be different. But the only possible difference must involve the value of the proposition $P?$, since all other propositions and contingent durations are identical in the dramas, (s, ω) and $(\tilde{s}, \omega)$. Thus, $P?$ must be executed before time t. Now, in the schedule $\sigma(s, \omega)$, we have seen that both Y and X are executed before $P?$, and hence before t. Thus, $[\sigma(s, \omega)]_X = [\sigma(\tilde{s}, \omega)]_X$ and $[\sigma(s, \omega)]_Y = [\sigma(\tilde{s}, \omega)]_Y$. But then the value of $Y - X$ must be the same in both schedules. Thus, the constraint $X - Y \leq -v$ must be violated in both schedules. But this contradicts that the constraint $X - Y \leq -v$ is satisfied in scenarios where $\beta\gamma p$ is true.

Regarding the constraint $(X - Y \leq -v, \neg\alpha\beta\gamma p)$, it is straightforward to show that it is necessary to introduce it to maintain equivalence with the original constraint from Y to X.[5] Indeed, when α is false, the relation between $P?$ and X is not known. □

The application of rules R0, R1, R2 and R3 has to be considered for all pairs of time-points with respect to all suitable observation points.

4.2 A CSTNU DC-Checking Algorithm

This section presents an algorithm for determining whether arbitrary CSTNU instances are dynamically controllable.

This DC-checking algorithm works by applying the labeled constraint-generation rules of Table 2 and the label-modification rules of Table 3 to all relevant combinations of edges until:

- the associated AllMax matrix is found to be inconsistent; or
- the rules cannot generate any more new (stronger) edges; or
- a maximum number of rounds of rule applications has been reached.

The pseudocode for the algorithm is shown in Procedure : CSTNU-DC-Check, below.

The algorithm performs $p(n^2 + nk + k)$ rounds, where n is the number of time-points, k is the number of contingent links, and p is the number of propositional letters that appear in the network. In each round, all of the label-modification rules from Table 3 are first applied, followed by the edge-generation rules from Table 2. After those rounds have completed, if it is still possible to generate stronger constraints having the same labels, then the CSTNU is not DC. Proof of this is an easy extension of Morris and Muscettola's argument about the number of rounds in the MM5 algorithm.

Given the lemmas presented in this paper, it is straightforward to verify that the algorithm is sound.

[5] A similar check is done by Hunsberger et al. (2012).

Procedure : CSTNU-DC-Check(G).

Input: $G = \langle \mathcal{T}, \mathcal{C}, L, \mathcal{OT}, \mathcal{O}, P, \mathcal{L} \rangle$: a CSTNU instance
Output: the dynamic controllability of G.
$G' = G$;
for 1 *to* $|P|(|\mathcal{T}|^2 + |\mathcal{T}||\mathcal{L}| + |\mathcal{L}|)$ **do**
  if *(AllMax matrix of G is inconsistent)* **then**
  return *false*;
  // Label Modification Rules
  G =LabelModificationRuleR0(G);
  G =LabelModificationRuleR1(G);
  G =LabelModificationRuleR2(G);
  G =LabelModificationRuleR3(G);
  // Labeled Constraints Generation
  $G' = G' \cup$ needed LabeledNoCaseRule(G);
  $G' = G' \cup$ needed LabeledUpperCaseRule(G);
  $G' = G' \cup$ any LabeledCrossCaseRule(G);
  $G' = G' \cup$ any LabeledLowerCaseRule(G);
  $G' = G' \cup$ any LabeledLabelRemovalRule(G);
  $G' = G' \cup$ any ObservationCaseRule(G);
  if *(no rules were applied)* **then return** *true*;
  $G = G'$;
return *false*

Thus, whenever the algorithm is given a DC network, the algorithm invariably declares it to be DC. Stated differently, the algorithm never generates false negatives (i.e., if the algorithm declares a network to be non-DC, then the network must be non-DC).

We are continuing to study the question of completeness. (A DC-checking algorithm is complete if it never generates false positives—that is, it only says a network is DC if it really is DC.)

4.2.1 On the Management of PropLabels

The actual performance of the algorithm can also be affected by the strategy for managing the sets of labeled values on edges in the graph. To better introduce the issue, let us consider the application of the No-Case rule to a pair of constraints containing different labeled values, as in the example of Fig. 8. Even though, for a given edge, only the minimal value is stored for each possible label, it is still possible to have an exponential explosion in the number of labeled values, as suggested by Fig. 8-(a). However, exponential numbers of labeled values are not always necessary because it is possible that some subsets of labeled values might be represented by just one. For example, the pair of labeled values, $\langle \neg A, \diamond, 10 \rangle$ and $\langle A, \diamond, 10 \rangle$, can be represented by the single labeled value, $\langle \boxdot, \diamond, 10 \rangle$. In addition, since this labeled value is stronger than the labeled value, $\langle \boxdot, \diamond, 12 \rangle$, the latter labeled value is redundant. Similar reasoning shows that the six labeled values on the edge from Q to T in Fig. 8-(a) can be replaced by the two labeled values shown in

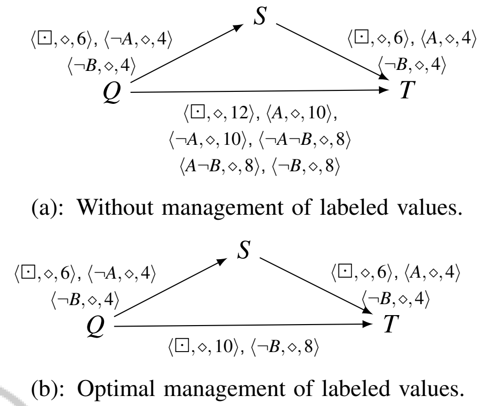


Figure 8: Two different strategies for managing the storage of labeled values in an application of the No-Case rule.

Fig. 8-(b). In the following we propose some labeled-value-management rules with the aim of minimizing the number of labeled values stored with each edge.

When there are two or more labeled values with labels that subsume the same “seed” label, and they have all the same numerical value, it is not necessary to explicitly represent all of them. Instead, it is sufficient to represent only the “seed” one. For example, the two labeled values of Fig. 8, $\langle A\neg B, \diamond, 8 \rangle$ and $\langle \neg B, \diamond, 8 \rangle$ can be replaced by the single labeled value, $\langle \neg B, \diamond, 8 \rangle$.

Rule 1 (Redundant Label Elimination 1, RLE 1)**.** If a set of labeled values contains two labels (ℓ_1, i) and (ℓ_2, i) where ℓ_1 subsumes ℓ_2, then the labeled value (ℓ_1, i) is redundant and it can be removed.

The previous rule can be simply extended to the case when two labels differ for only one proposition.

Rule 2 (Redundant Label Elimination 2, RLE 2)**.** If a set of labeled values contains two labeled values, $(\alpha p, i)$ and $(\alpha \neg p, j)$, where α contains neither p nor $\neg p$, there are two cases:

1. if $i = j$, then both labeled values can be represented by (α, i).
2. if $i \neq j$, then remove any labeled values of the form, (α, k), since the value of k would be greater than both i and j, making the labeled value redundant.

For example, in Fig. 8, $\langle A\neg B, \diamond, 8 \rangle$ and $\langle \neg A\neg B, \diamond, 8 \rangle$ can be replaced by $\langle \neg B, \diamond, 8 \rangle$.

Regarding the empty label ($\boxdot$), it effectively represents the disjunction of all possible labels. Thus, if there is an empty-labeled value, that value can be considered to be the default value. If there are other labeled values, their numerical values must be smaller than the numerical values associated with the default; otherwise, RLE 1 applies. It is possible to represent all possible combinations of labels not only with an empty label, but also with a suitable set of labels. For example, in the set $\{\langle A,\diamond,8\rangle,\langle\neg A,\diamond,8\rangle,\langle\boxdot,\diamond,10\rangle\}$, the pair of labels $\langle A,\diamond,8\rangle$ and $\langle\neg A,\diamond,8\rangle$ represent all possible labels of the Universe and their values are smaller than the empty-labeled one. Thus, the empty labeled value can be removed. These observations lead to rule RLE3, below.

Rule 3 (Empty Label Elimination, RLE 3)**.** If a set of labeled values contains a subset of labels that cover all possible combinations of a fixed set of propositions, then such a subset represents the base of all possible labels. Therefore, any empty-labeled value can be removed since, by construction, its numerical value must be greater than the numerical values associated with the labels of the base.

These rules explain how to maintain a set of labeled values for a given edge in order to rightly represent all the possible values, while maintaining the minimal number of such values represented explicitly.

In general, if we have to add the labeled values of a set S_1 to the labeled values of a set S_2 (e.g., as must be done when applying the No-Case rule to a pair of labeled edges), it is necessary to add each labeled value of the first set to each each labeled value of the second set.[6] The propositional label for the sum of the two labeled values is the conjunction of the two involved propositional labels. Assuming that those propositional labels are consistent, the new labeled value is put into a new set that represents the result of the overall operation. For example, given the two sets of labeled values seen in Fig. 8-(a), $S_1=\{\langle\boxdot,\diamond,6\rangle,\langle\neg A,\diamond,4\rangle,\langle\neg B,\diamond,4\rangle\}$ and $S_2=\{\langle\boxdot,\diamond,6\rangle,\langle A,\diamond,4\rangle,\langle\neg B,\diamond,4\rangle\}$, their sum is:

$$\begin{aligned}
S_1+S_2=\{&\langle\boxdot,\diamond,6\rangle+\langle\boxdot,\diamond,6\rangle=\langle\boxdot,\diamond,12\rangle, &(1)\\
&\langle\boxdot,\diamond,6\rangle+\langle A,\diamond,4\rangle=\langle A,\diamond,10\rangle, &(2)\\
&\langle\boxdot,\diamond,6\rangle+\langle\neg B,\diamond,4\rangle=\langle\neg B,\diamond,10\rangle, &(3)\\
&\langle\neg A,\diamond,4\rangle+\langle\boxdot,\diamond,6\rangle=\langle\neg A,\diamond,10\rangle, &(4)\\
&\langle\neg A,\diamond,4\rangle+\langle A,\diamond,4\rangle=\text{inconsistent}, &(5)\\
&\langle\neg A,\diamond,4\rangle+\langle\neg B,\diamond,4\rangle=\langle\neg A\neg B,\diamond,8\rangle, &(6)\\
&\langle\neg B,\diamond,4\rangle+\langle\boxdot,\diamond,6\rangle=\langle\neg B,\diamond,10\rangle, &(7)\\
&\langle\neg B,\diamond,4\rangle+\langle A,\diamond,4\rangle=\langle A\neg B,\diamond,8\rangle, &(8)\\
&\langle\neg B,\diamond,4\rangle+\langle\neg B,\diamond,4\rangle=\langle\neg B,\diamond,8\rangle\} &(9)
\end{aligned}$$

The sum of the labeled values in line (5) does not generate a new labeled value since the propositional labels, A and $\neg A$, are inconsistent. The rest of the newly generated labeled values can be represented by a small number of labeled values, as determined by the label-elimination rules. For example, $\langle\neg B,\diamond,8\rangle$ makes $\langle\neg B,\diamond,10\rangle$ redundant. Next, rule RLE2 says that $\langle A\neg B,\diamond,8\rangle$ and $\langle\neg A\neg B,\diamond,8\rangle$ can be replaced by $\langle\neg B,\diamond,8\rangle$, which is already present (Line 9). Finally, rule RLE1 says that $\langle\neg A,\diamond,10\rangle$ and $\langle A,\diamond,10\rangle$ can be replaced by $\langle\boxdot,\diamond,10\rangle$, which dominates the constraint, $\langle\boxdot,\diamond,12\rangle$, which is present in Line 1. Hence, the "reduced" set becomes:

$$S_1+S_2=\{\langle\boxdot,\diamond,10\rangle,\langle\neg B,\diamond,8\rangle\}$$

[6]Similar issues were discussed by Conrad et al. (2011).

5 DISCUSSION AND CONCLUSIONS

To verify and test the practical usability of the proposed algorithm, we have built a Java program, called CSTNU_EDITOR, that allows one to graphically design a CSTNU instance and to check its dynamic controllability. Fig. 9 depicts a screen shot of the program running on a sample CSTNU instance.

The program implements a variety of strategies for managing the sets of labeled values which enables the user to better monitor the propagation of labeled values and its impact on the convergence of the algorithm. Preliminary experiments show that the algorithm finds the solution in an average number of cycles one order of magnitude smaller than the theoretical estimated upper bound. Moreover, different policies in the management of labeled value sets have different consequences on the convergence of the algorithm: the number of cycles required to find a solution decreases when the management strategy minimizes (in any way) the number of stored labels, but the running time of each cycle of the algorithm increases. We are currently evaluating which management strategy provides the best trade-off between the sizes of the labeled value sets and execution time.

In summary, this paper presented a *DC-checking algorithm* for CSTNUs. The algorithm uses rules for generating labeled constraints/edges that extend the rules presented by Morris et al. (2005). It also uses new label-modification rules needed to manage different possible alternative executions. It is the first such algorithm in the literature. The algorithm is proven to be sound.

As for future work, we are going to formally analyze whether the algorithm is complete. Moreover, we will extensively test CSTNU_EDITOR with synthetic and real world complex CSTNU networks in order to evaluate its applicability in the area of temporal workflow systems.

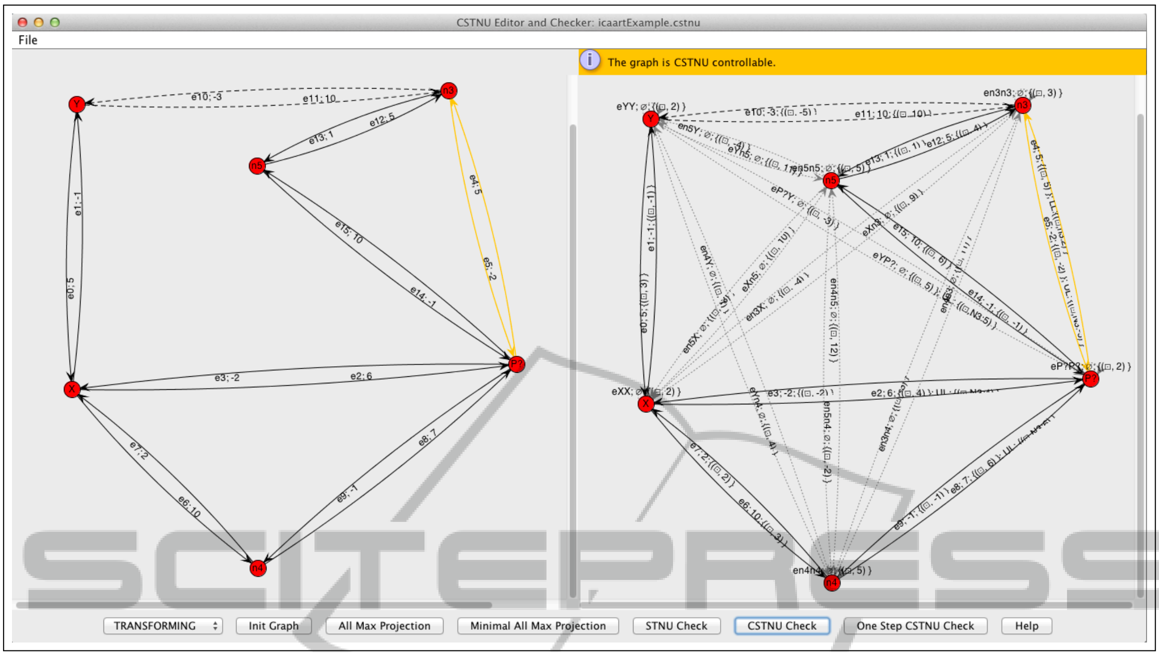


Figure 9: A screen shot of the CSTNU editor and checker.

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