# 4D Polygonal Approximation of the Skeleton for 3D Object Decomposition

Luca Serino, Carlo Arcelli and Gabriella Sanniti di Baja
*Institute of Cybernetics "E. Caianiello", CNR, Naples, Italy*

Keywords: 3D Object, Skeleton, Decomposition, Polygonal Approximation.

Abstract: We improve a method to decompose a 3D object into parts (called kernels, simple-regions and bumps) starting from the partition of the distance labeled skeleton into components (called complex-sets, simple-curves and single-points). In particular, each simple-curve of the partition is here interpreted as a curve in a 4D space, where the coordinates of each point are related to the three spatial coordinates of the corresponding voxel of the 3D simple-curve and to its associated distance label. Then, a split type polygonal approximation method is employed to subdivide, in the limits of the adopted tolerance, each curve in the 4D space into straight-line segments. Vertices found in the 4D curve are used to identify corresponding vertices in the 3D simple-curve. The skeleton partition is then used to recover the parts into which the object is decomposed. Finally, region merging is taken into account to obtain a decomposition of the object more in accordance with human intuition.

## 1 INTRODUCTION

Object decomposition is of interest to reduce the complexity of computer vision tasks such as description and recognition. The underlying theory is that human object understanding is based on recognition-by-component (Hoffman and Richards, 1984); (Singh et al., 1999).

Object decomposition can be achieved by deriving information from a representation of the object. If the surface delimiting a 3D object is used, curvature variations along the object boundary can be used to identify points through which surfaces, separating different object parts, should pass (Shamir, 2008); (Cheng et al., 2008). If the skeleton is used, its geometrical structure can lead to the identification of suitable skeleton subsets corresponding to different parts of the object (de Goes et al., 2008); (Macrini et al., 2008).

We favor the latter alternative and refer to the skeleton denoted as curve skeleton in (Arcelli et al., 2011). This is a subset of the 3D object, consists of the curves symmetrically placed within the object and has the same topology as the object. The skeletonization method (Arcelli et al., 2011) is related to the medial axis transform (Blum, 1973). According to this model, the skeleton is the locus of the symmetry points, i.e., the points of the object that can be seen as centers of balls bi-tangent to the object boundary and included in the object. Skeleton points are labeled with the radii of the associated balls, and the object can be recovered by the union of the balls associated with the symmetry points.

The skeleton can be computed in the distance transform of the object, where object voxels are labeled with their distance from the complement of the object. Each voxel can be interpreted as the center of a ball with radius equal to the distance label. A ball is maximal if is not included by any other single ball in the object and its center is called center of a maximal ball, CMB. The CMBs are equidistant from at least two parts of the object's boundary, hence they are symmetry points. Any ball can be built by applying to its center the reverse distance transformation (Borgefors, 1996). The object can be recovered by applying the reverse distance transformation to its CMBs.

Full object recovery from the skeleton is possible only if the skeleton includes all the CMBs. This happens only for objects consisting of parts with tubular shape, where the CMBs are almost all aligned along symmetry axes of the object. However, the CMBs are generally placed along symmetry planes and axes. Thus, to have a skeleton consisting exclusively of curves, only a subset of the CMBs can be included in the skeleton and full object



Serino L., Arcelli C. and Sanniti di Baja G. (2013).
4D Polygonal Approximation of the Skeleton for 3D Object Decomposition.
In *Proceedings of the 2nd International Conference on Pattern Recognition Applications and Methods*, pages 467-472
DOI: 10.5220/0004265204670472

recovery is not guaranteed. This notwithstanding, the skeleton has been profitably used for object decomposition, whichever is the shape of the object.

In this paper, we present a decomposition method based on skeleton partition and object reconstruction, which is the follow up of a method introduced in (Serino et al., 2010) and successively improved in (Serino et al., 2011).

## 2 DECOMPOSITION SCHEME

We achieve object decomposition via the partition of the distance labeled skeleton S. A key role is played by the regions recovered by applying the reverse distance transformation to the branch points of S, i.e., to the skeleton voxels with more than two neighboring skeleton voxels. For branch points sufficiently close to each other, a single region is obtain, which is called the *zone of influence* of the branch points it includes. The zones of influence of S allow us to group the branch points that for a human observer correspond to a single branch point configuration of an ideal skeleton representing the object. The zones of influence are also used to originate the partition of S. The components of the skeleton partition are used as seeds to recover the parts into which the object is decomposed.

### 2.1 Previous Work

The decomposition scheme (Serino et al., 2011) splits 3D objects in perceptually significant non overlapping parts by performing a partition of the skeleton into at most three kinds of subsets (called simple-curves, complex-sets, and single-points). See Figure 1 left and middle left, showing the 3D object horse and the partition of its skeleton into simple-curves, green voxels, and complex-sets, red voxels.

Simple-curves, complex-sets, and single-points were used to build respectively simple-regions, bumps and kernels. Kernels are a sort of main bodies of the object, from which simple-regions and bumps protrude. Object parts were built in two steps. The first step involves reverse distance transformation. The second step performs an expansion with the aim of assigning the object voxels not yet recovered by the reverse distance transformation to the regions to which they are closer. See Figure 1 middle right, where kernels and simple regions for the horse are shown in red and green, respectively.

A one-to-one correspondence exists between partition components and object parts. However, in some cases the number of parts may be not in accordance with human intuition. For example, some protrusions may be seen as negligible details that do not deserve to be represented by individual parts of the decomposition; similarly, two kernels linked to each other by a simple-region may be interpreted as constituting a unique main body, if the linking simple-region is scarcely elongated. Thus, it may be preferable to give up the one-to-one correspondence and favor a decomposition more in accordance with human perception. To this aim, criteria for merging bumps and simple-regions to their adjacent kernels were also suggested, so as to obtain a decomposition of the object into a smaller number of perceptually significant parts. See Figure 1 right, where the decomposition obtained after merging is shown. The two kernels and the simple-region in between them have been merged into a unique component, the torso of the horse.

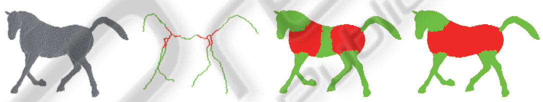

Figure 1: From left to right: the object horse; simple-curves (green) and complex-sets (red) of the skeleton; decomposition into kernels (red) and simple-regions (green); decomposition after merging.

### 2.2 New Ideas

To our opinion, kernels and bumps are regions whose description would not benefit of a further subdivision into simpler parts. In fact, kernels are almost convex bodies and bumps are elementary protrusions. In turn, a simple-region, though having the corresponding simple-curve as its unique symmetry axis, may still be interpreted as having an articulated structure. In fact, the surface separating a simple-region from the complement of the object may be characterized by curvature variations. In addition, also the thickness of a simple-region, measured in planes perpendicular to its associated simple-curve, may significantly change. Thus, in this paper we suggest an alternative decomposition scheme that allows us to subdivide simple-regions into smaller entities, called *basic-regions*, which are characterized by absence of significant curvature variations along the object boundary and by thickness that is either nearly constant or evolves in an almost monotonic manner.

We partition the skeleton as in (Serino et al., 2011). Then, we divide the simple-curves into segments, each of which consisting of voxels that are aligned along straight lines and whose distance values are either all equal or change in a monotonic way. In fact, curvature changes along the boundary of a simple-region are reflected by curvature changes along its associated simple-curve. In turn, thickness changes in a simple-region are reflected by changes in the distance values of the voxels in the associated simple-curve. Subdivision of each simple-curve is obtained by resorting to polygonal approximation in a 4D space, where any skeleton voxel is mapped into a point whose coordinates are related to the three Cartesian coordinates and the distance value of the skeleton voxel. After all simple-curves have been subdivided into straight-line segments, we build the regions into which the object is interpreted as decomposed. In particular, regions corresponding to simple-curves will result to be divided into a number of basic-regions, each of which characterized by constant or monotonically changing thickness and by absence of significant curvature changes along the boundary.

## 3 THE METHOD

We use binary voxel images in cubic grids. The 3×3×3 neighborhood of a voxel $p$ includes the six face- the twelve edge- and the eight vertex-neighbors of $p$.

The distance between two voxels $p$ and $q$ is the length of a minimal path from $p$ to $q$. If the weights 3, 4 and 5 suggested in (Borgefors, 1996) are used to measure moves from $p$ towards its face-, edge- and vertex-neighbors along the path, the <3,4,5>-distance is obtained.

The distance transform DT of an object P is a multi-valued replica of P, where voxels are labeled with their distance from the complement of P. We compute DT by using the <3,4,5>-distance.

The $k$-th layer of DT is the set of voxels having a distance value $d$ such that $(k\text{-}1)\times3<d\leq k\times3$ (Svensson and Sanniti di Baja, 2002). Except for the first layer that includes only voxels with distance label 3, any other layer in DT includes voxels that are characterized by up to three different values. The value of a voxel $p$ in the $k$-th layer depends on whether its closest neighbors in the ($k$-$1$)-th layer, i.e., the neighbors from which $p$ received distance information, are face-, edge- or vertex-neighbors of $p$. In Figure 2, a 3D object and a section of its distance transform are shown, where the voxels belonging to the same distance layer have been colored with the same color.

The polygonal approximation of the simple-curves in the partition of the skeleton S is computed by using a split type algorithm (Ramer, 1972). Given an open curve, the extremes of the curve are taken as vertices of the polygonal approximation. To identify the other vertices, we consider the straight line joining the extremes of the curve and compute the Euclidean distance from such a straight line of all points of the curve; the point with the largest distance is taken as a vertex if such a distance is greater than an a priori fixed threshold θ (to be set depending on the desired approximation quality). Any detected vertex divides the curve into two curves, to each of which the same splitting procedure is applied. The splitting process is repeated as far as points are detected having distance larger than the threshold from the straight lines joining the extremes of the curves to which the points belong.

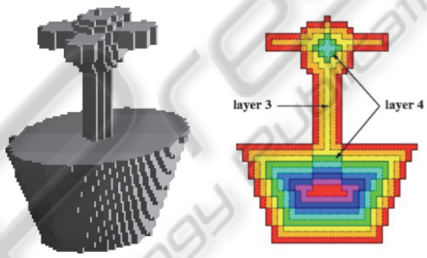


Figure 2: A 3D object, left, and a section of the <3,4,5>-distance transform, right.

To perform polygonal approximation by taking into account simultaneously changes in geometry along the simple-curves and changes in distance value of their voxels, we should represent the curves in a 4D discrete space, where the coordinates are the three Cartesian coordinates and the distance value of the voxels of the 3D simple-curves. A simple-curve in the 3D skeleton consists of voxels adjacent to each other (each voxel has exactly two neighbors in the curve, with the exception of the extremes of the curve having only one neighbor), but may result in a sparse set of points when passing to the 4D representation. To have a connected set also in the 4D discrete space, we exploit the fact that the algorithm (Arcelli et al., 2011) is based on the <3,4,5> distance transform, where layers are easy to detect, and adjacent skeleton voxels belong to the same layer or to layers whose indexes differ by one. Thus, we use the index of the layer to which a voxel belongs in place of its distance as the 4th coordinate of the corresponding point in the 4D space.

Given three points A, B and C in the 4D space, the square of the Euclidean distance $d$ of point C from the straight line AB can be computed as:

$$d^2 = ||AC||^2 - P_{ABC} * P_{ABC} / ||AB||^2$$

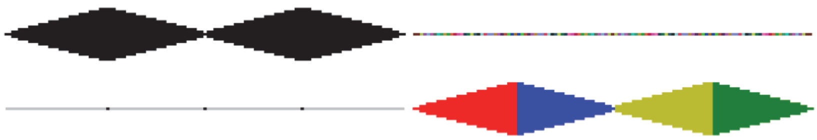

Figure 3: From top left to bottom right: an input object; its skeleton (different colors denote different distance labels); vertices (black voxels) found on the skeleton by the 4D polygonal approximation; object decomposition.

where ||AB|| is the norm of the vector AB, and $P_{ABC}$ is the scalar product between vectors AB and AC. Points with $d>\theta$ are taken as vertices of the polygonal approximation. Once vertices have been detected in the 4D space, we go back to the 3D skeleton representation and mark the skeleton voxels corresponding to the found vertices.

An example is given in Figure 3, showing an input object, top left, and its skeleton, top right. Different colors are used to show the different distance labels of the skeleton voxels. Though the skeleton is a straight-line segment in the 3D space, its voxels have different distance labels due to the fact that object thickness changes along the object. The 4D polygonal approximation is applied to the skeleton after the distance labels of the skeleton voxels have been replaced by the layer indexes. Vertices (black voxels in Figure 3 bottom left) are found in the skeleton, which divide the skeleton into four segments each of which corresponds to a portion of the object characterized by monotonically changing width (Figure 3 bottom right).

We have experimentally found that the threshold value $\theta=20$ is adequate to originate a polygonal approximation sufficiently faithful to the original curve and is, at the same time, able to prevent an excessive fragmentation of the curve. Such a threshold value has been used for the examples shown in this paper as well as for all other objects we have been working with. The vertices are shown in black in Figure 4 top left for the running example. The found vertices divide each simple-curve into a number of consecutive segments, each of which corresponding to a part of the simple-region associated with the whole simple-curve, which is characterized by absence of significant curvature variations along the object boundary and by thickness that is either nearly constant or evolves in an almost monotonic manner.

Consecutive segments share a common vertex, called *hinge*. We assign the same identity label to all voxels in a segment, except for the hinges to which we assign a unique identity label. The two extremes of any simple-curve are ascribed the identity labels of the two segments they belong to.

A process in two steps, following a strategy similar to that suggested in (Serino et al., 2011), is accomplished to build kernels, bumps and simple-regions into which the object is decomposed. Partition components are assigned identity labels and are interpreted as seeds for region growing. In the first step, the reverse distance transformation with identity label propagation is applied to the seeds. Care is taken to ascribe to the proper regions the voxels that, being at the same distance from different seeds, receive different identity labels and to guarantee that the surfaces separating adjacent regions are almost planar. The second step is done to achieve complete recovery of the various regions, since the skeleton of a 3D object generally does not include all CMBs. Thus, object voxels that have not been recovered by the reverse distance transformation are assigned the identity label of the region to which they result to be closer.

Since we aim at a decomposition where simple-regions are articulated into basic-regions, during the first step we propagate the labels assigned to the segments and hinges identified by polygonal approximation, rather than the labels ascribed to the simple-curves. Voxels that can be recovered by region growing applied to more than one segment/hinge receive the unique special identity label used for all hinges. Let us call *hinge-regions* the connected components of recovered voxels with the label of the hinges. Voxels of the hinge-regions have to be re-distributed between the adjacent regions. This is done during the second step by the same process used to complete recovery of all regions. Distance information is used to ascribe to the voxels not recovered by reverse distance transformation or belonging to hinge-regions the identity label of the regions to which they are closer. See Figure 4 top middle.

The final step of the process is devoted to merging to the adjacent kernels those bumps and simple regions that are perceived as not individually meaningful.

A simple-region is considered as a whole for merging, even if the region is articulated into basic-regions. In fact, in our opinion the articulation of a simple-region into basic-regions has to be taken into account only to distinguish objects in the same class.

The relevance of a region R, considered for merging to an adjacent kernel, is computed in terms

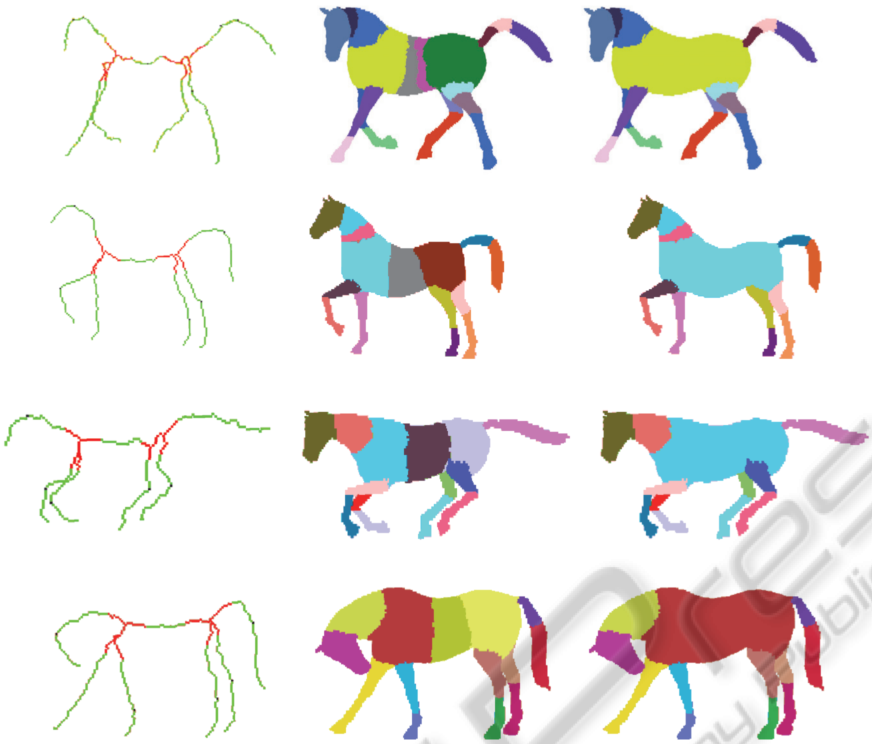

Figure 4: From left to right, skeleton partition (vertices of the polygonal approximation are in black), decomposition before merging and decomposition after merging.

of two measures, as suggested in (Serino et al., 2011). These are respectively the ratio between the volume of R and the volume of the compound region that would be obtained as result of merging, and the ratio between the visible portion of the surface of R (measured by the number of voxels in the surface of R having at least one face-neighbor in the background) and the non visible portion of the surface of R (measured by the number of voxels in the surface of R having at least one face-neighbor in the adjacent kernels). Two thresholds, $\tau$ and $\sigma$, are used for the above two ratios. In this work, the values $\tau$=1.2 and $\sigma$=2 have been used. For the running example, the final decomposition is shown in Figure 4 top right.

The decomposition after merging can be used to identify the class to which an object belongs, in terms of the number of kernels, bumps and simple-regions, and of their spatial relationships. Moreover, by taking into account the possibly existing basic-regions, the decomposition can be used to distinguish objects in the same class. If necessary, the decomposition before merging can also be used to derive information on the more or less articulated structure into basic-regions of those simple-regions that have been merged to the adjacent kernels. A few examples for the object horse in different poses are given in Figure 4.

The suggested method has been tested on several 3D images from publicly available shape repositories, e.g., (Shilane et al., 2004), producing generally satisfactory results. A few examples of the performance of our method can be appreciated in Figure 5.

## 4 CONCLUSIONS

A method to decompose a 3D object, starting from the partition of its skeleton into complex-sets, single-points and simple-curves, has been presented. Simple-curves are interpreted as curves in the 4D space, where the coordinates of each point are computed in terms of the Cartesian coordinates and the distance values associated to the skeleton voxels.

A polygonal approximation is done in the 4D space to subdivide simple-curves into straight-line segments. Straightness of segments regards both geometric curvature along the 3D simple-curves and distribution of distance values along the curves. The same threshold value has been used for polygonal approximation for all tested images. The elements of the skeleton partition are used as seeds to

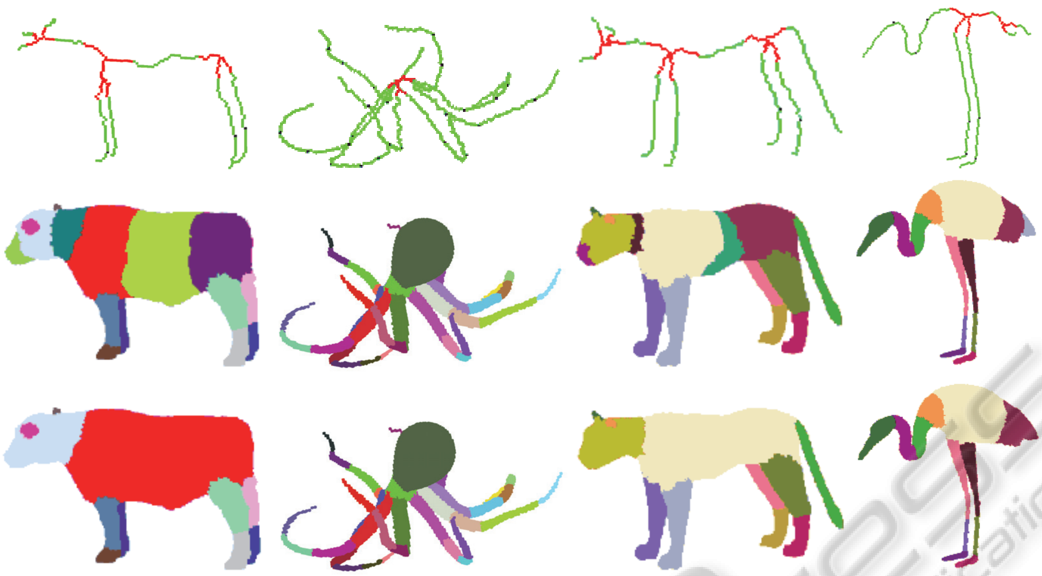

Figure 5: Skeleton partitions, where vertices found on the simple curves are shown in black, and corresponding decompositions before and after merging.

recover the parts (kernels, bumps and simple-regions) into which the object is decomposed. Simple regions may be articulated into basic-regions, due to the polygonal approximation done on the simple-curves. Merging is also accomplished to obtain a more stable final decomposition, whose parts agree with human perception.

## REFERENCES


C. Arcelli, G. Sanniti di Baja, L. Serino, 2011. Distance driven skeletonization in voxel images, *IEEE Trans. PAMI*, 33, 709-720.

H. Blum, 1973. Biological shape and visual science, *J. Theor. Biol.*, 38, 205-287.

G. Borgefors, 1996. On digital distance transform in three dimensions, *CVIU*, 64, 368-376.

Z-Q. Cheng, B. Li, G. Dang, S-Y. Jin, 2008. Meaningful Mesh Segmentation Guided by the 3D Short-Cut Rule, *Proc. AGMP 2008*, LNCS 4975, 244-257, Springer.

F.de Goes, S. Goldenstein, L. Velho, 2008. A Hierarchical Segmentation of Articulated Bodies, *Computer Graphics Forum*, 27, 1349-1356.

D. D. Hoffman, W. A. Richards, 1984. Parts of Recognition, *Cognition*, 18, 65-96.

D. Macrini, K. Siddiqi, S. Dickinson, 2008. From Skeletons to Bone Graphs: Medial Abstraction for Object Recognition, *Proc. CVPR 2008*, 1-8.

U. Ramer, 1972. An Iterative procedure for the polygonal approximation of plane curves, *CGIP*, 1, 244-256.

L. Serino, G. Sanniti di Baja, C. Arcelli, 2010. Object decomposition via curvilinear skeleton partition, *Proc. ICPR 2010*, 4081-4084, IEEE CS Press.

L. Serino, G. Sanniti di Baja, C. Arcelli, 2011. "Using the skeleton for 3D object decomposition", in A. Heyden and F. Kahl (Eds.): *SCIA 2011*, LNCS 6688, 447-456, Springer.

A. Shamir, 2008. A Survey on Mesh Segmentation Techniques, *Computer Graphics Forum*, 27, 1539-1556.

P. Shilane, P. Min, M. Kazhdan, T. Funkhouser, 2004. The Princeton Shape Benchmark, Shape Modeling International, Genova, Italy.

M. Singh, G. D. Seyranian, D. D. Hoffman, 1999. Parsing Silhouettes: the Short-Cut Rule, *Perception&Psychophysics*, 61, 636-660.

S. Svensson, G. Sanniti di Baja, 2002. Using distance transforms to decompose 3D discrete objects, *IMAVIS*, 20, 529-540.