Colour Processing in Tetrachromatic Spaces
Uses of Tetrachromatic Colour Spaces

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Abstract: We exploit the geometry of the 4D hypercube in order to visualize tetrachromatic images.

1 INTRODUCTION

Tetrachromatic images $i : N \times M \to [0,1]^4$ are images where each pixel has four spectral components, each component giving information regarding the energy contents of the pixel in a given spectral band. We assume that each component value of a pixel occurs in the interval $\mathbf{I} = [0,1]$ and the total gammut of the possible colours a pixel can take can be modeled with the hypercube $\mathbf{I}^4$, a *4D colour* being a point $[w,x,y,z]$ of the hypercube. Two points of the hypercube are the *black (or "schwartz") vertex* $\mathbf{s} := [0000]$, and the *white vertex* $\mathbf{w} := [1111]$; a subset of the hypercube is $\mathbf{A} := \{(t,t,t,t) : t \in [0,1]\}$, the *achromatic segment* between $\mathbf{s}$ and $\mathbf{w}$. See (Restrepo, 2012a) and (Restrepo, 2012b). Tetrachromatic images can be visualized by feeding the RGB channels of a projector or screen with 3 of the bands W, X, Y Z of the image, in one or several of the of the $3!\binom{4}{3} = 24$ possible ways of doing this.

2 GEOMETRY AND 4D COLOUR

The (tridimensional) boundary $\partial\mathbf{I}^4$ of the hypercube $\mathbf{I}^4 \subset \mathbf{R}^4$ has a rich geometrical structure; it consists of $\binom{4}{1}2^1 = 8$ solid cubes with 16 vertices, 32 edges, and 24 square faces. A colour $[w,x,y,z]$ is on $\mathbf{T} := \partial\mathbf{I}^4$ if at least one of its coordinates is 0 or 1; each solid cube consists of the points having a given coordinate at value either 0 or 1; for example, the cube $\{[w,x,y,z] \in \mathbf{I}^4 : w = 1\}$, which we denote as $\{w = 1\}$. Indeed, we write $\partial\mathbf{I}^4 = \{w=0\} \cup \{w=1\} \cup \{x=0\} \cup \{x=1\} \cup \{y=0\} \cup \{y=1\} \cup \{z=0\} \cup \{z=1\}$. $\partial\mathbf{I}^4$ is a *piecewise linear* (PL) tridimensional sphere that can be homeomorphed to a more standard, *round* $\mathbf{S}^3$.

In the 2-skeleton of the *complex* structure of $\mathbf{T} = \partial\mathbf{I}^4$, you find 24 PL 1-spheres (one per face), 8 PL 2-spheres and 3 PL Heegaard tori. Geometrically, these manifolds can be used to define an *orientation* of the points in the hypercube that, with corresponding coordinate systems, is used to define several types of *hue* for 4D colours.

2.1 Tint

To give *spherical coordinates* (d,Θ) to any point $\mathbf{p} \in \mathbf{I}^4$, denote the central point of the hypercube as $\mathbf{g} = [\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}]$, let d be a measure of the distance between $\mathbf{p}$ and $\mathbf{g}$ (e.g. the *max* of the absolute values of the components of $\mathbf{p} - \mathbf{g}$), and let $\Theta \in \mathbf{T}$ be the point where the ray from $\mathbf{g}$ through $\mathbf{p}$ leaves the hypercube. Call Θ the *tint*, or *generalized hue*, and call d the *colourfulness*, or *generalized saturation* of $\mathbf{p}$. In this sense, $\mathbf{T}$ is the set of tints. Note that the vertices $\mathbf{s}$ ("black") and $\mathbf{w}$ ("white") are fully colourful and are tints.

2.2 Chromatic Hue

A pair of vertices of the hypercube is said to be a pair of *opposing vertices* if the coordinates of one are the "negated" version of the coordiantes of the other, for example, [0000] and [1111], or [0101] and [1010]. Eight PL 2-spheres, that are dodecahedra of square faces, result by considering the faces that do not meet a given pair of opposing vertices. Each of these 2-spheres serves as an *equatorial* 2-sphere for $\partial\mathbf{I}^4$; for our purposes, the most relevant is the one having as opposing vertices $\mathbf{s}$ and $\mathbf{w}$. Call it the *chromatic dodecahedron* $\mathbf{D} = \{w=0, x=1\} \cup \{w=1, x=0\} \cup \{w=0, y=1\} \cup \{w=1, y=0\} \cup \{w=0, z=$

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$1\} \cup \{w = 1, z = 0\} \cup \{x = 0, y = 1\} \cup \{x = 1, y = 0\} \cup \{x = 0, z = 1\} \cup \{x = 1, z = 0\} \cup \{y = 0, z = 1\} \cup \{y = 1, z = 0\}$. Each of the faces of **D** has a "primary" w, x, y or z at its fullest value 1, and another at its minimum value 0. Each square face of **D** is subdivided into two triangles so that the points in each triangle obey the same ordering of their coordinates; e.g. the triangle with vertices $[0,1,0,1]$ $[0,1,0,0]$ $[1,1,0,1]$ of points $[w,x,y,z]$ with $y \leq w \leq z \leq x$, and the triangle $[1,1,0,0]$ $[0,1,0,0]$ $[1,1,0,1]$ of points $[w,x,y,z]$ with $y \leq z \leq z \leq w$, are the subdivision of the face $\{x = 1, y = 0\}$ of points $[w,x,y,z]$ with $min\{w,x,y,z\} = y$ and $max\{w,x,y,z\} = x$. There are 24 such *ordering triangles*; together, they give the subdivision of **D** called the *chromatic icositetrahedron* **IT**; on each ordering triangle, the relative contribution of the *primaries* is fixed; each ordering triangle represents a *family* of hue. To get the hue family corresponding to a colour not in **A**, find out the permutation that orders its coordinates. More precisely, the hue **h** of $[w,x,y,z]$ is the point **h** in **IT** that is obtained as $\mathbf{h} = \frac{1}{\rho}[w,x,y,z] - \frac{\nu}{\rho}[1,1,1,1]$ where ρ is the *chromatic saturation* given by the range of the primaries, and ν is the min. Each chromatic point $[wxyz]$ is in a unique *chromatic triangle* $\mathbf{w} - \mathbf{s} - \mathbf{h}$. Indeed $[wxyz] = (1 - \zeta)\mathbf{s} + \rho\mathbf{h} + \nu[1111]$ is an expresion in barycentric coordinates $[1 - \zeta, \nu, \rho]$ in the plane spanned by the points **s**, **w** and **h**.

2.3 Hue in a Rhombic Dodecahedron

When the points of $\mathbf{R}^4$ are projected along the direction [1111] onto the 3-subspace (through the origin)[1] the chromatic dodecahedron projects, without selfintersections, to a (2D) rhombic dodecahedron[2]. The achromatic segment projects to the central point of the rhombic dodecahedron and the cubes in **T** project to overlapping parallelepipeds in the (solid) rhombic dodecahedron. the orthonormal points $\mathbf{a} = [\sqrt{\frac{3}{4}}, -\sqrt{\frac{1}{12}}, -\sqrt{\frac{1}{12}}, -\sqrt{\frac{1}{12}}]$, $\mathbf{b} = [0, \sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{6}}, -\sqrt{\frac{1}{6}}]$ and $\mathbf{c} = [0, 0, \sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}]$ are a basis that gives 3D coordinates to the projection space. The coordinates of the projections of the vertices of **D** are shown in Table 1.

The **abc** coordinates of the intersection of the ray from the center of the rhombic dodecahedron trhough the projection of a chromatic point, and the boundary of the rhombic dodecahedron, gives an alternate *hue* η. The distance from the center of the rombic dodecahedron to the projection point is a measure of chromatic saturation σ; also, the projection [λ, λ, λ, λ] on **A** of $[w,x,y,z]$, $\lambda := \frac{w+x+y+z}{4}$, gives a measure of luminance. Thus $\sigma = \sqrt{w^2 + x^2 + y^2 + z^2 - 4\lambda^2}$. In this way an alternate colour space[3] to that with the $\rho\mu$ triangle results.

Table 1: The 14 vertices of the chromatic dodecahedron are projected onto the 3-subspace normal to [1,1,1,1]. Then, the projections are given 3-space coordinates in the third column.

vertex	projection	[a, b, c]
0111	$[-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$	[-0.8660, 0, 0]
0010	$[-\frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, -\frac{1}{4}]$	[-0.2887, -0.4082, 0.7071]
0011	$[-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$	[-0.5774, -0.8165, 0]
0001	$[-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{3}{4}]$	[-0.2887, -0.4082, -0.7071]
0101	$[-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}]$	[-0.5774, 0.4082, -0.7071]
0100	$[-\frac{1}{4}, \frac{3}{4}, -\frac{1}{4}, -\frac{1}{4}]$	[-0.2887, 0.8165, 0]
0110	$[-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}]$	[-0.5774, 0.4082, 0.7071]
1010	$[\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}]$	[0.5774, -0.4082, 0.7071]
1011	$[\frac{1}{4}, -\frac{3}{4}, \frac{1}{4}, \frac{1}{4}]$	[-0.2887, -0.8165, 0]
1001	$[\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}]$	[0.5774, -0.4082, -0.7071]
1101	$[\frac{1}{4}, \frac{1}{4}, -\frac{3}{4}, \frac{1}{4}]$	[0.2887, 0.4082, -0.7071]
1100	$[\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}]$	[0.5774, 0.8165, 0]
1110	$[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4}]$	[0.2887, 0.4082, 0.7071]
1000	$[\frac{3}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}]$	[0.8660, 0, 0]

2.4 Tori

The tint of a colour **p** different from **g** is given by $\Theta = \mathbf{g} + \chi(\mathbf{p} - \mathbf{g})$ where $\chi = \frac{1}{2max\{|w'|,|x'|,|y'|,|z'|\}}$ where $w' = w - 0.5$, $x' = x - 0.5$, $y' = y - 0.5$ and $z' = z - 0.5$. The indexes i of the coordinates Θ_i of $\Theta = [\Theta_0, \Theta_1, \Theta_2, \Theta_3]$ of value 0 or 1 indicate the cube Θ is at; for example, if $\Theta_1 = 0$, then $\Theta \in \{x = 0\}$.

A coordinate system for the points in an $\mathbf{S}^3$ results by considering the Heegaard splitting of genus 1. It uses two angles and a "signed radius" $r \in [-1, 1]$, rather than the better-known, spherical coordinates of three angles. A Heegaard torus splits the 3-sphere into two open solid tori and their common boundary. Out of the 24 square faces, 16 faces can be chosen that together are a Heegaard torus for $\mathbf{T} = \partial\mathbf{I}^4$; this can be done in three ways since the 8 cubes in **T** can be grouped in $\frac{1}{2}\binom{4}{2} = 3$ ways, into two groups of four cubes each, so that each group is a solid torus.

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[1] This is computed by subtracting the average of the coordinates from each coordinate.

[2] The rhombic dodecahedron is a Catalan solid, i.e. a polyhedron that is dual to an Archimedean solid; in this case, to the cuboctahedron, which has 12 vertices, 24 edges, 8 triangle faces and 6 square faces; two triangles and two squares meet at each vertex.

[3] To get a B&W image from a color image, in the trichromatic case, it gives better visual results to use the max (as in the HSV colour system) than to use the average.

Here, we consider the solid tori $V_{yz} := \{z=0\} \cup \{y=1\} \cup \{z=1\} \cup \{y=0\}$ and $V_{wx} := \{w=0\} \cup \{x=1\} \cup \{w=1\} \cup \{x=0\}$.

The boundaries of V_{wx} and V_{yz} are the torus H; H can be seen as the union of four *square pipe segments* in two ways; each pipe segment (topological cylinder or annulus) is a stack of 1-squares that are meridians for the solid torus in question and longitudes for the other solid torus. For the solid torus V_{yz} we have the pipes of square meridians with vertices

$P_0 := \{(0,0,s,0),(1,0,s,0),(1,1,s,0),(0,1,s,0) : s \in [0,1)\}$ (z=0),

$P_1 := \{(0,0,1,s),(1,0,1,s),(1,1,1,s),(0,1,1,s) : s \in [0,1)\}$ (y=1),

$P_2 := \{(0,0,1-s,1),(1,0,1-s,1),(1,1,1-s,1),(0,1,1-s,1) : s \in [0,1)\}$ (z=1) and

$P_3 := \{(0,0,0,1-s),(1,0,0,1-s),(1,1,0,1-s),(0,1,0,1-s) : s \in [0,1)\}$ (y=0),

similarly, the boundary of the V_{wx} is given by the pipes of square meridians with vertices

$Q_0 := \{(0,t,0,0),(0,t,1,0),(0,t,1,1),(0,t,0,1) : t \in [0,1)\}$ (w=0),

$Q_1 := \{(t,1,0,0),(t,1,1,0),(t,1,1,1),(t,1,0,1) : t \in [0,1)\}$ (x=1),

$Q_2 := \{(1,1-t,0,0),(1,1-t,1,0),(1,1-t,1,1),(1,1-t,0,1) : t \in [0,1)\}$ (w=1) and

$Q_3 := \{(1-t,0,0,0),(1-t,0,1,0),(1-t,0,1,1),(1-t,0,0,1) : t \in [0,1)\}$ (x=0).

As we remarked above, $H = \cup P_i = \cup Q_i$. Each point of $\mathbf{T}$ is either in the open solid torus T_{wx}, in the open solid torus T_{yz}, or in their common boundary H. The subindex n of the pipe segment together with the value of t or s, as in $n.t$, or $n.s$, give an angular measure that ranges from 0 to 4, mod-4.

For Θ in an open torus, there is a distance $r \neq 0$ from the boundary of the solid torus the tint is at; the distance from the boundary is measured with the product metric; that is, for example, for the piece of solid torus bounded by pipe P_0, a tint point $[w,x,t,0]$ is at distance $0.5 - \max\{|w-0.5|,|x-0.5|\}$ from its boundary. Also, there are two 1-squares in pipes say P_n and Q_m with corresponding parameters s and t such that one of them (a meridian) bounds a two-square the tint is in, and the other intersects the first 1-square at a point $\mathbf{u}$ on H that is closest to Θ [4]. Let $\mathbf{u} = (\phi,\psi) := (n.s, m.t)$ in H be the *toroidal hue* of $\mathbf{p}$. If Θ is on H, let $r = 0$. Denote Θ as (ϕ,ψ,r); with the understanding that if $r = \pm 0.5$ (i.e. if Θ is precisely on the axis or core of a solid torus), exactly one of the angles ϕ or ψ is left undefined and only the longitude of the corresponding solid torus that contains Θ is needed and a coordinate corresponding to the meridian is left undefined. For example, the tint of [0.9, 0.2, 0.3, 0.4] is [1, 1/8, 1/4, 3/8] = (3.625, 2.875, 0.25), corresponding to pipes P_3 and Q_3, with $s = 5/8$ and $t = 7/8$.

[4] On a disk, with the Euclidean metric, each point different from the center has a unique point on the circle boundary that is closest; in a square though, with the product metric, for each point on the diagonals, there are two points on the square perimeter that are closest to the point on the diagonals, 4 if it is the center of the square.

2.5 Spinning

We generalize Artin's concept of spinning is spinning with an S^1 to spinning with a sphere S^n. Given a subset E of $\mathbf{R}^2$ (such as the $\rho\mu$ triangle) with a closed subset F (such as the μ edge), form the topological space $(E \times S^n)/\approx$, where each set of the form $\{f\} \times S^n$, $f \in F$, is identified to a point. Artin's method provides a geometric embedding of subsets F of $\mathbf{R}^3$, in $\mathbf{R}^4$, as $\{(x,y,z\cos\theta, z\sin\theta) : f = (x,y,z) \in F, \theta \in [0,2\pi)\}$.

2.6 Runge Ball

A 4D round space is obtained by deforming the hypercube into the standard 4-ball $\{(w',x',y',z') \in \mathbf{R}^4 : w^2 + x^2 + y^2 + z^2 \leq 1\}$. This can be done in several ways; one is to spin the $\rho\mu$ triangle, deformed to a semicircle, around $\mathbf{S}^2$, with hinge the μ basis of the triangle, where $\mathbf{S}^2$ is derived from the chromatic dodecahedron; another is to spin the *midray* (that that originates at intermediate gray) with $\mathbf{S}^3$, with hinge the point of intermediate gray. In the first case we have a space with coordinates the luminance, the chromatic saturation and a 2D (the equatorial sphere derived from the chromatic dodecahedron) spherical hue; in the second case, we have a space with coordinates given by the generalized saturation r and a generalized 3D hue given vy the $\mathbf{S}^3$ that is derived from the boundary of the hypercube.

Let $[w,x,y,z]$ be a point in the hypercube, shift the hypercube so that intermediate gray ends up at the origin of 4-space R^4 and rescale so that the maximum values of the coordinates is 1 and the minimum is -1. Let $[w',x',y',z'] = 2[w-0.5, x-0.5, y-0.5, z-0.5]$ be the coordinates of the resulting hypercube $[-1,1]^4$.

The *lightness* in this space is given by the angle with the achromatic axis: $\lambda = arcos\frac{w'+x'+y'+z'}{2\sqrt{w'^2+x'^2+y'^2+z'^2}}$ $= arcos\frac{w+x+y+z-2}{2\sqrt{w^2+x^2+y^2+z^2+1-(w+x+y+z)}}$. Rather than using a chromatic saturation measure i.e. a distance measure to the achromatic line segment, we use a distance $\mathbf{g}$ obtaining a measure of colourfulness in the sense of "ungrayness". Let $\Lambda = max\{|w'|,|x'|,|y'|,|z'|\}$; if $\Lambda \neq 0$, the point on the boundary of the hypercube that is in the same direction is $\frac{1}{\Lambda}[w',x',y',z']$ (at least one of its coordinates has value of 1); let $d = \frac{1}{\Lambda}\sqrt{w'^2+x'^2+y'^2+z'^2}$ and normalize by this length (with the result that the hypercube is deformed into a 4-ball), getting the point $\mathbf{s} = [s_0,s_1,s_2,s_3] := \frac{1}{d}[w',x',y',z']$ whose distance from the center of the ball is $\kappa = \frac{\sqrt{w'^2+x'^2+y'^2+z'^2}}{\Lambda^{-1}\sqrt{w'^2+x'^2+y'^2+z'^2}} = \Lambda$. Thus

$\kappa = max\{2w-1, 2x-1, 2y-1, 2z-1\}$ is the *colourfulness* of the point $[w,x,y,z]$. $\chi = \frac{1}{2\Lambda}$.

3 PROCESSING

By colour processing a digital tetrachromatic image, we mean the application of a law to each pixel in the image, producing a new tetrachromatic image. The image is then to be visualized or fed to a computer vision algorithm. By appropriately modifying the hue, it is possible to visualize tetrachromatic images in such a way that certain aspects are made conspicuous.

The linear (i.e. noncircular, nonspherical) coordinates such as colourfulness, chromatic saturation and luminance, are transformed via exponential-law maps x^γ. The hue may be independently processed by automorphisms either of the 3-sphere, a hue sphere or of a hue torus. As the hue surfaces are rotated or otherwise *automorphed*, the colours of a tetrachromatic image may change in interesting ways when trichromatically visualized. The automorphisms respect the continuity; the rotations are isometries and respect the antipodicity or complementary colours as well. The simplest modification type of the hue of 4D colour is given by rotations of the 2-sphere, of the 3-sphere, or of the Heegaard torus. The rigid motions of $s \in \mathbf{S}^3$ or equivalently, the rotations of $\mathbf{R}^4$ are implemented by pre- and post-multiplying by unit quaternions p, q, as in psq, $s \in \mathbf{S}^3$. The rigid motions of $\mathbf{S}^2$ are implemented by pre and post multyplying a pure quaternion s times a unit quaternion q and its conjugate, as in qsq^*. The space of rigid motions of $\mathbf{S}^3$ has the group structure SO(4); it is the topoloical space $\mathbf{S}^3 \times \mathbf{RP}^3$ for which $\mathbf{S}^3 \times \mathbf{S}^3$ is a double cover.[5] The rigid motions of $\mathbf{S}^3$ can be coded as a pair $(\theta_1, \theta_2) \in \mathbf{S}^3 \times \mathbf{S}^3$ in the sense that a unit quaternion is being pre and post multiplied by unit quaternions. The space $\mathbf{H}$ of the quaternions can be seen as $\mathbf{R}^4$ or as $\mathbf{C}^2$. For $\mathbf{C}^2$, the analogous case of an orthogonal transformation is that of a unitary transformation that, rather than preserving the structrure of the inner product in $\mathbf{R}^2$, it preserves the standard hermitian form $(z_1, z_2).(w_1, w_2) = z_1\bar{w}_1 + z_2\bar{w}_2$. The set of unitary transformations has the group structure SU(2). A point of $\mathbf{S}^3$ can be denoted as a pair $(z_1, z_2) \subset \mathbf{C}^2$ with $z_1\bar{z}_1 + z_2\bar{z}_2 = 1$.

For toroidal hue, for PL rotations, the 1D squares with sides parallel to the axes w and x are meridians of the yz solid torus and longitudes of the wx solid torus; the 1D squares with sides parallel to the axes y and z are meridians of the wx solid torus and longitudes of the yz solid torus. Similarly for the other cases. Shifts around such squares implement modifications of hue.

[5] The set of rotations of the plane is the group SO(2) which has the topology of $\mathbf{S}^1$ while the set of rigid motions of $\mathbf{S}^2$ (of rotations of $\mathbf{R}^3$) is the group SO(3) which has the topology of $\mathbf{RP}^3$.

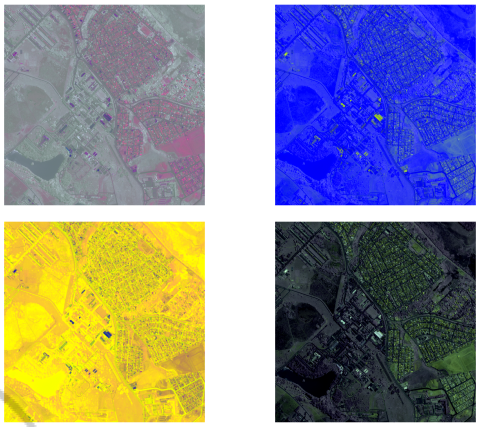

Figure 1: p=[1/2, 1/2, -1/2, -1/2], q=[-1/2, -1/2, 1/2, 1/2], $\gamma = 0.6$; bands 1 (in R), 3 (in G), 4 (in B). p=[1/2, 1/2, 1/2, -1/2], q=[1/2, -1/2, 1/2, 1/2], $\gamma = 1.0$; bands 1, 2 and 3. p=[1/2, 1/2, 1/2, -1/2], q=[-1/2, 1/2, 1/2, 1/2], $\gamma = 1.0$; bands 2, 3 and 4. p=[1/2, 1/2, 1/2, -1/2], q=[1/2, -1/2, 1/2, 1/2], $\gamma = 1.0$; bands 1, 2 and 4.

4 CONCLUSIONS

Tetrachromatic colour spaces find applications in the visualization of 4-spectral images. Its use in satellite imaginery (Landsat, 2012) is very likely providing alternate ways to the mere feeding of the visualizing RGB channels with permutations of the image wxyz channels. Also, as a technique for computational photography, the explotation of IR and UV bands is likely to be of use in different ways. Further work remains to be done in the exploration of automorphisms of spheres and tori different from isometries. Depending on the application diferent types of tetrachromatic colour processing will be needed.

REFERENCES

Landsat (2012). http://zulu.ssc.nasa.gov/mrsid/tutorial/landsat%20tutorial-v1.html.

Restrepo, A. (2012a). Tetrachromatic colour space. *SPIE Electronic Imaging, San Francisco*.

Restrepo, A. (2012b). Tetrachromatic colour spaces. *Visapp Rome*.