Image Pyramids as a New Approach for the Determination of Fractal Dimensions

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Abstract: The consideration of different scales and the application of fractal methods on digital images is of high importance if real world objects are investigated. In this context the fractal dimension is an important parameter to characterize structures and patterns. An accurate understanding of them is obligatory if significant and comparable results should be obtained. Recently a new method using an image pyramid approach was compared to the very popular Box Counting Method. The intriguing results showed that a trustable value for the fractal dimension could be obtained in much faster computational times compared to traditional Box Counting algorithms. In addition to these results of this new approach, which is only applicable to binary (black/white) images, we present developments toward the application to grey value/color images. Especially the determination of the grey value surface and the interpolation used to downscale the images seem to have major influence on the results achieved.

1 INTRODUCTION

The development of both image analysis and the available image processing techniques results in an increasing number of parameters, that can be extracted from investigated images. But discrepancies occurred in the results of different research groups which were studying images of the same object at different resolutions, especially if these objects were of natural origin.

In 1981 Paumgartner et al. (Paumgartner et al., 1981) showed in a pioneering study that the results obtained from image analysis of subcellular membrane systems seemed to depend on the resolution scale used. Their estimations for the surface density of inner mitochondrial membranes and endoplasmic reticulum increased significantly as the magnification of their images was increased. This behavior was explained with a *resolution effect*, using a concept proposed by Mandelbrot (Mandelbrot, 1977): the concept of *fractals*.

If properties of these objects called fractals are investigated, similar patterns and structures are gained at different scales because finer and finer shapes are revealed every time the resolution is increased. Together with a non-integer value for the dimension this is one of the main characteristics of fractals. The concept of fractals is illustrated in Figure 1, where successive steps of the construction process of an image with fractal dimension are shown. In his most popular publication titled *The Fractal Geometry of Nature*, Mandelbrot (Mandelbrot, 1983) showed that a lot of objects of natural origin have fractal characteristics.

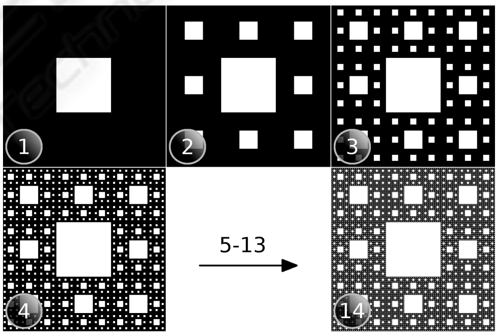

Figure 1: Image set of a fractal (”Sierpinski Carpet”). In every iterative construction step (in this example up to 14) the fractal dimension is increased.

The consideration of different scales and the application of fractal methods on these types of structures is not just possible, it is obligatory if significant and comparable results should be obtained. Conventional morphology based on a single scale measurement always implies a Euclidean geometry as its basis. In most studies dealing with images taken in a medical context a concept based on non-fractal geometry can-

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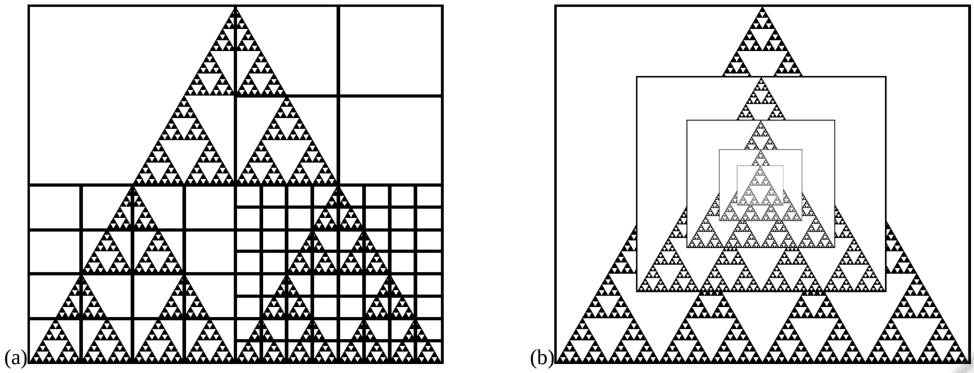

Figure 2: (a) Principle of Box Counting Method applied to a standard fractal (”Sierpinsky Gasket”) showing different iteration steps (box sizes) of the method in every quarter of the image. (b) Principle of Pyramid Method showing different iteration steps (image sizes).

not yield comparable results, as for example Losa et al. (Losa et al., 2011) stated in their discussion of a publication concerning pyramidal cells in the rat somatosensory cortex (Romand et al., 2011). The importance of scaling laws and network structures for the understanding of processes of health and disease in medical research was outlined by West very recently (West, 2012).

However, the application of fractal methods imposes certain requirements on the images under investigation. A sufficient resolution when calculating fractal properties as well as influences of edge detection algorithms were determined and discussed by our group some years ago (Ahammer et al., 2003; Ahammer and DeVaney, 2004). On the one hand images with higher and higher resolutions improve the gainable output of image analysis. On the other hand large amounts of data have to be handled by the implemented methods. Therefore, it can be stated that there is a demand for new approaches and methods in fractal analysis which are able to handle images with a high number of pixels in acceptable computational times.

2 METHODS

2.1 Box Counting Methods and Grey Value Surfaces

Based on the concepts of the Hausdorff definition of a dimension, the Box Counting Method (BCM, illustrated in Figure 2(a)) was defined by Russell et al. and is the most popular algorithm in contemporary fractal analyses (Hausdorff, 1918; Mandelbrot, 1983; Russell et al., 1980; Lopes and Betrouni, 2009).
The mathematical foundations and characteristics of the BCM are well investigated and its limitations (binarization, strictly valid only for statistically self-similar sets, box size sensitivity) are known (Keller et al., 1989; Normant and Tricot, 1991; Appleby, 1996; Pruess, 1995).

The approach for handling grey value images is called the Differential Box Counting Method (DBCM) and was proposed by Chaudhuri and Sarkar (Sarkar and Chaudhuri, 1992; Sarkar and Chaudhuri, 1995; Chaudhuri and Sarkar, 1995). Its development and improvements later on made it possible to overcome some of the limitations constraining the standard BCM (Jin et al., 1995; Biswas et al., 1998).

The BCM can easily be implemented and it is used throughout scientific communities dealing for example with images or signals originating in diverse medical investigations (e.g. photographs, histological images, optical coherence tomography, ...) (Higgs, 2011; Jelinek et al., 2011; Sullivan et al., 2011). Nevertheless, there exist several disadvantages of the Box Counting Methods, e.g. the high computational time needed for evaluating the fractal dimension of images with high pixel count. Furthermore, Box Counting Methods have limitations regarding the choice of the box sizes. As pointed out by Pickover and Khorasani there were studies to find upper and lower limits for the box sizes and it was shown that the Box Counting Methods underestimate the true values of the fractal dimensions. Disturbance by noise which also leads to lower fractal dimension values was investigated (Pickover and Khorasani, 1986; Bisoi and Mishra, 2001; Asvestas et al., 1998; Lee and Hsieh, 2010).

2.2 Pyramid Method

Recently the determination of the fractal dimension using a new method was developed by our group

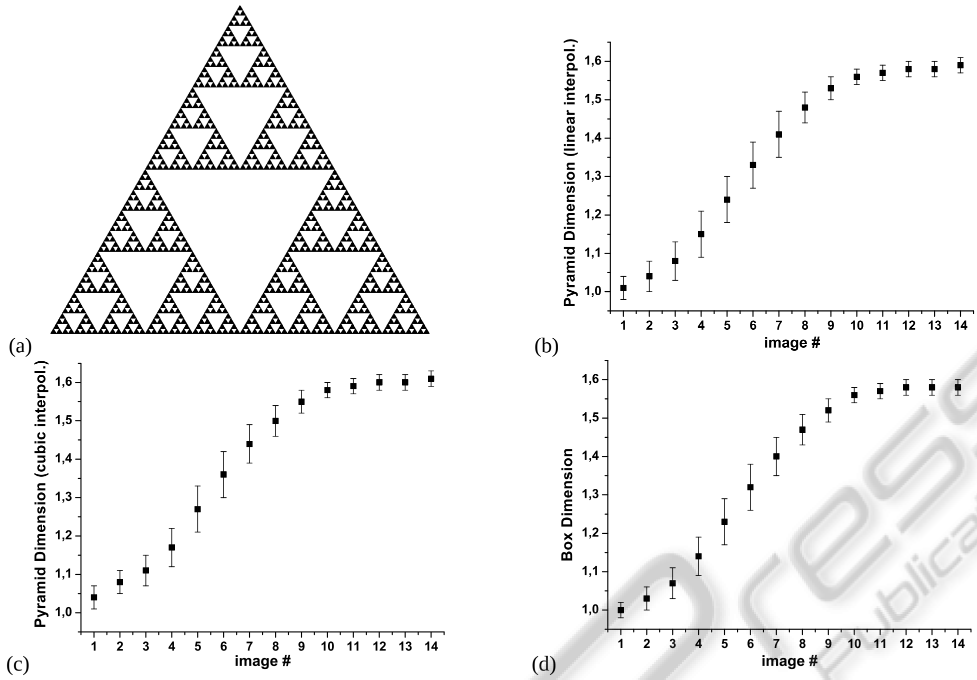

Figure 3: Calculated fractal dimensions of the Sierpinski Gasket ((a), 8192 x 8192 pixels, 1-14 iteration steps). (b) Values of the Pyramid Method (linear interpolation). (c) Values of the Pyramid Method (cubic interpolation). (d) Values of the Box Counting Method.

(Ahammer and Mayrhofer-R., 2012). The major conclusions of the analysis of the new method were that the obtained values for the fractal dimensions of binary images have the same quality as when determined with the BCM but within significantly shorter computational times. So far the method was only developed for binary (black/white) images.

The approach of the method is the origin of the introduced names *Pyramid Method* (PM) for the method itself and *Pyramid Dimension* for the fractal dimension value obtained, respectively. It uses image pyramids which are in fact sequences of identical images but at different sizes. The original image having the biggest size represents the bottom of the pyramid. The size of this bottom image is then reduced successively (using a suitable interpolation algorithm) until it is no more than one pixel, which can be thought of as the top of the pyramid (see Figure 2(b) for an illustration). For each image size the number N of object pixels is counted, with N_0 as the number of object pixels in the original image. A value for the pyramid dimension is obtained by applying the mentioned standard technique of fractal analysis. From the double logarithmic plot of N/N_0 versus the scaling variable used to scale down the images a slope can be extracted representing the value for the fractal dimension of the digital images.

The results obtained are quite identical to the data gained from using the BCM. Calculated values for the fractal dimensions of 14 subsequent images of a Sierpinski Gasket (Figure 3 (a)) are shown in Figure 3 (b)-(d). Results obtained by the Pyramid Method with linear interpolation for downscaling were equal to results obtained by the Box Counting Method. Differences to the results obtained by the Pyramid Method using cubic convolution interpolation were below one per mill and therefore negligible. Advantages of the PM over other comparable methods are its simple implementation and a significant reduction of calculation time. PM was shown to be approximately ten times faster than BCM. Nevertheless, it should be mentioned that the interpolation method used when downscaling the images affects the result of the calculation. With binary images linear and cubic convolution interpolation algorithms result in trustable results whereas nearest neighbor resampling should be avoided. Also resampling without interpolation does not yield acceptable results. Concluding it can be stated that for binary images PM is superior to BCM (Ahammer and Mayrhofer-R., 2012).

As the next step toward the applicability of the PM to high resolution images of various sources it has to be able to handle grey value images. For that purpose the two-dimensional grey value images are transformed to surfaces in three-dimensional space using the grey value as a third coordinate. The area of these surfaces should be a usable parameter for determining a value for the fractal dimension. The finding of an accurate and fast method for the calculation of these grey value surface areas is of high importance. So far, two known methods are already implemented: one method based on gradient analysis (Chinga et al., 2007) and another one known as Blanket Method (or Minkowski Dilation Method for surfaces) (Peleg et al., 1984; Tang et al., 2002). At the moment test results are evaluated. Further methods as the Isarithm Method (Shelberg et al., 1983) or the Triangular Prism Method (Clarke, 1986) and its improvements (Sun, 2006) will be tested. Also the significance of the gained parameters is expected to be highly influenced by the interpolation methods used when rescaling the images. Therefore, a detailed evaluation of different existing methods, an improvement of them together with a development of own algorithms both for the surface area calculation and for the interpolation are important tasks to tackle. The results of the developed method have to be compared to the results obtained by the well established Box Counting or other trustable methods. Especially significant improvements in computational time are expected as indicated by experiences with the Pyramid Method applied to binary images. This is of high importance considering the fact that the image size is expected to increase in the future, for instance because of steadily improving scanning devices, such as whole slide imaging.

3 CONCLUSIONS

The binary Pyramid Method applied with specific (linear and cubic convolution) interpolation methods yields results similar or at least comparable with results obtained from the popular Box Counting Method (Ahammer and Mayrhofer-R., 2012). Based on these results it is expected that an extension to grey value images, which is in development and evaluation at the moment, results in a reliable and fast method for determining fractal properties of structures in digital images. By applying the new Pyramid Method the analysis of grey value images having a high pixel count should be feasible in significantly faster computational times than compared to methods used previously by the scientific community.

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