A Study of Decision-making Model Considering Priorities based on Two Kinds of Evaluation

Decision Making Methodology Applying Risk Evaluation based on Prospect Theory

Rumiko Azuma[1] and Shinya Nozaki[2]

[1] *Department of Social Informatics, Aoyama Gakuin University, Sagamihara, Kanagawa, Japan*

[2] *Transdisciplinary Research Organization for Subtropics and Island Studies, University of the Ryukyus, Okinawa, Japan*

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Abstract: The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach aimed at reflecting a human's subjective judgment or vagueness. The conventional evaluation in AHP is considered to be a kind of utility. However, there are some cases where the traditional utility theory cannot explain risk aversion. This paper presents a new decision-making methodology for considering risk evaluation. We propose the hierarchy model that contains return and risk categories, and an AHP method that applies prospect theory, which is able to explain people's decisions when they face situations involving risks. Therefore, by proposing an AHP method that utilizes it, we enable the evaluation of alternatives under return and risk.

1 INTRODUCTION

In decision-making problems, it is necessary to simultaneously estimate benefits and risks. For example, in assessing supply chains, when companies find new suppliers for offshore sourcing decisions, they consider positive criteria, which may include low wages, lower-transportation costs, and higher reliability. These elements are generally expressed with a positive value as return. On the other hand, there are various types of risk such as poor quality, logistical failures, and natural disasters.

There are studies that solve the offshoring decision problem. Schoenherr's research (Schoenherr et al., 2008) proposed a method using Saaty's AHP (Analytic Hierarchy Process) (Saaty, 1980) to assess supply chain risks. The AHP is widely used for tackling multi-attribute decision-making problems in real situations. It uses a hierarchical model for the decision problem and is based on the use of pairwise comparisons, which lead to the elaboration of a ratio scale. In AHP, the degree of risks is also determined by a paired comparison. However, it is difficult to evaluate risk using a humans subjective judgments.

In our previous study, we extended AHP method for handling a satisfaction and a risk on the same structure, and proposed a decision-making model having pair criterion (Azuma and Miyagi, 2009). Because the conventional evaluation in AHP is considered to be a kind of utility, risk is represented by the utility of the probability of damage in the model. Furthermore, the expected utility is integrated, considering that satisfaction is a positive utility and damage by risk is a negative utility. Then, we applied the expected utility theory to the model by defining satisfaction as a positive utility and risk as a negative utility. However, studies have shown that an actual behavior of person is uncertain when choosing between risky alternatives (Barberis et al., 2003). In this kind of situation, it is considered inappropriate to use the utility theory for decision-making methods under risks.

In this study, we propose the introduction of the prospect theory (Kahneman and Tversky, 1979) concept to AHP for problem solving. The aim of our study is to develop a method that evaluates alternatives on the basis of return and risk standpoints.

2 PROSPECT THEORY

Prospect theory was developed as a psychologically more accurate description of preferences compared to expected utility theory. It is a theory of decision-making under conditions of risk. The theory says that

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preferences between positive and negative prospects are different.

The formula of prospect theory is given by

$$U = \sum_{i=1}^{n} \pi(p_i)v(x_i), \tag{1}$$

where U is the overall or expected utility of the outcomes to the individual making the decision, $x_1, x_2, \ldots$ are the potential outcomes and $p_1, p_2, \ldots$ their respective probabilities. The function π is a probability weighting. v is called value function that is defined on deviations from s-shaped the reference point. It expresses losses (= risk) have a significant influence more than gains feel good. A value function is displayed in Figure 1.

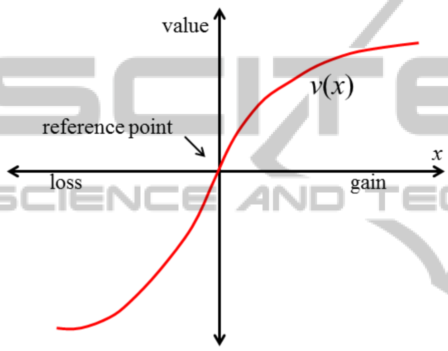

Figure 1: Example of a value function.

3 THE DEVELOPMENT OF THE AHP MODEL BASED ON PROSPECT THEORY

We propose a decision making model which evaluates considering a satisfaction and a risk. The feature of proposed model is that priorities of risk are evaluated based on prospect theory. Generally, risk is defined as the product of probability and resulting degree of damage. On the other hand, some scholars redefined the risk by using expected utility theory. We apply prospect theory as a non-linear expected utility theory for the evaluation of risks.

Specifically, flowchart of proposed method in Figure 2 is described as follows.

Step 1). Decision-maker is asked to extract the decision criteria and alternatives and to make up a hierarchy structure. This hierarchy model involves criteria of both satisfactions and risks, the structure of which can be seen in Figure 3.

Step 2). In this step, decision-maker makes up pairwise comparison matrices in criteria of satisfaction

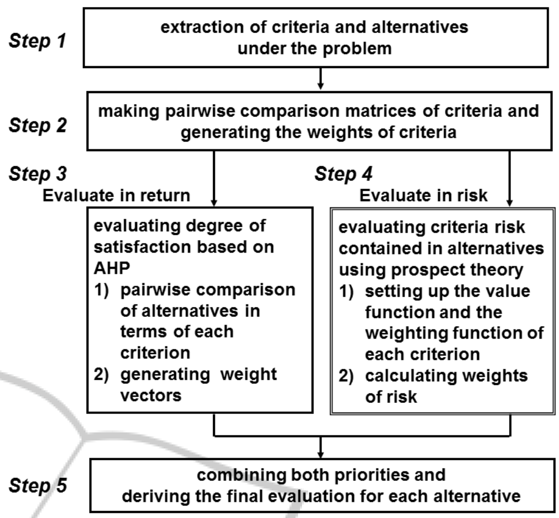

Figure 2: Flowchart in the proposed method based on AHP for risk.

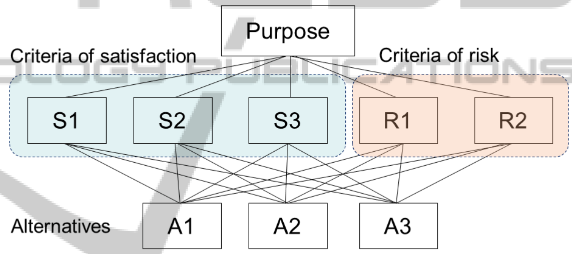

Figure 3: The hierarchy model for satisfaction and risk.

and risk. This procedure is same as AHP which is relative measurement approach. To calculate its eigenvector determines the weight of criteria.

The pairwise comparison matrix $\boldsymbol{S}$ in criteria of satisfaction is constructed:

$$\boldsymbol{S} = [s_{ii'}], \quad i, i' = 1, 2, \ldots, m, \tag{2}$$

and (i, i') is the number of criteria. The value of $s_{ii'}$ is given by linguistic scale of decision-maker as in Table 1. The scale is ratio-scale measure. In Table 1, the value is chosen under each criterion by answering question such as : How important is low-cost than high-quality when you determine a offshore company? Decision criteria are compared in pairs to assign weights. The relation between matrix $\boldsymbol{S}$, its eigenvector $\boldsymbol{w}_s$ and maximum eigenvalue λ_{max} is represented as

$$\boldsymbol{S}\boldsymbol{w}_s = \lambda_{max}\boldsymbol{w}_s, \tag{3}$$

where

$$\boldsymbol{w}_s^T = (\omega_{s_1}, \omega_{s_2}, \ldots, \omega_{s_m}),$$

and ω_{s_i} represents a weight of *ith* criterion.

Similarly, we define the pairwise comparison matrix $\boldsymbol{R}$ in criteria of risk as

$$\boldsymbol{R} = [r_{kk'}], \quad k, k' = 1, 2, \ldots, n. \tag{4}$$

By the same approach, we derive the vector of priorities for the matrix $\boldsymbol{R}$

$$\boldsymbol{R}\boldsymbol{w}_r = \lambda'_{max}\boldsymbol{w}_r, \tag{5}$$

$$where \quad \boldsymbol{w}_r^T = (\omega_{r_1}, \omega_{r_2}, \ldots, \omega_{r_n}).$$

and ω_{r_j} represents a weight of *jth* criterion in risk.

Table 1: Example of linguistic scale of paired comparison.

value	linguistic scale	
	satisfaction	risk
1	equally important	equally damage
3	moderate important	moderate damage
5	important	strong damage
7	very important	heavy damage
9	absolutely important	extreme damage

Step 3). The next step derives the weights of alternatives under each criterion. Then, we analyze weight of alternatives for satisfaction. The weights in satisfaction are derived by the same technique as Saaty's AHP. Decision-maker compares the alternatives based on each criterion S_i respectively, then constructs pairwise comparison matrices $\boldsymbol{A}_{s_i}(i = 1, \ldots, m)$ of alternative. The weights are acquired by calculating each eigenvector from matrices. The eigenvector of $\boldsymbol{A}_{s_i}$ regards as weight vector $\boldsymbol{w}_{s_i}$, then it can be represented by Eq.(6).

$$\boldsymbol{w}_{s_i}^T = (\omega_{s_i a_k}) = (\omega_{s_i a_1}, \omega_{s_i a_2}, \ldots, \omega_{s_i a_l}), \quad k = 1, 2, \ldots, l. \tag{6}$$

Here, $\omega_{s_i a_k}$ is a weight of alternative A_k based on criterion S_i. After calculating weights for each level of a class, the final weights of satisfaction are derived by these results as

$$\boldsymbol{u}_s = (u_{sa_k}) = [\boldsymbol{w}_{s_1}, \boldsymbol{w}_{s_2}, \ldots, \boldsymbol{w}_{s_m}] \cdot \boldsymbol{w}_s, \tag{7}$$

where u_{sa_k} is a priority weight of alternative A_k in satisfaction.

Step 4). In order to calculate the weights in risk, we utilize the utility of damage and its probability. In evaluation of risk, to calculate the weight of alternative A_k for each criterion R_j, we adopt prospect theory. p_{jk} represents a probability that risk R_j will occur under A_k. According to prospect theory, when the damage of R_j under A_k is represented as x_{jk}, the weight $\omega_{r_j a_k}$ of A_k based on R_j can be expressed as follows using Eq.(1).

$$\boldsymbol{w}_{r_j}^T = (\omega_{r_j a_k}) = (\omega_{r_j a_1}, \omega_{r_j a_2}, \ldots, \omega_{r_i a_l}), \quad k = 1, 2, \ldots, l \tag{8}$$

$$\omega_{r_j a_k} = \pi(p_{jk})v(x_{jk}) \tag{9}$$

where $\boldsymbol{w}_{r_j}$ is a weight vector of alternatives based on risk R_j. In a case that probability of a risk is not able to be expressed as a objective value, it is given as subjective value by linguistic scale, such as quite high, very high, high and so on (Takemura, 1996). All the weight vectors are generated after normalization.

From these results, weights of the whole hierarchy about risks are estimated. Assume weights of each alternative in risk are $u_{ra_1}, u_{ra_2}, \ldots, u_{ra_l}$, respectively. Then, a weight vector $\boldsymbol{u}_r$ can be formulated as

$$\boldsymbol{u}_r = (u_{ra_k}) = [\boldsymbol{w}_{r_1}, \boldsymbol{w}_{r_2}, \ldots, \boldsymbol{w}_{r_n}] \cdot \boldsymbol{w}_r \tag{10}$$

Step 5). Final evaluation is obtained in Step 5. The ultimate priority vector $\boldsymbol{U}$ is finally acquired by satisfaction evaluation and risk evaluation of alternatives. It is calculated from the ratio of each degree of satisfaction to a risk as

$$\begin{aligned} \boldsymbol{U} &= (u_{a_k}) = (u_{sa_k}/u_{ra_k}) \\ &= (u_{sa_1}/u_{ra_1}, u_{sa_2}/u_{ra_2}, \ldots, u_{sa_l}/u_{ra_l})^T. \end{aligned} \tag{11}$$

We reach the final weights after normalization such that

$$\sum_{k=1}^{l} u_{a_k} = 1. \tag{12}$$

4 CONCLUSIONS

This study suggested a new approach to construct a decision-making model for risk management. The proposed method is based on Saaty's AHP method and applied prospect theory for evaluating risk. By applying prospect theory to AHP it makes possible to quantify damage, which is derived by a human's judgments under risks. As a result, we think our approach enables people to make a decision about problems involving risk such as decision problems in supply chains.

In this study, the linear measurement of Saaty is used as the measurement of return in a paired comparison. On the other hand, since the measurement of a risk is calculated based on a value function which is nonlinear. In our future works, it needs to examine consistency of each measurement. One plan is to use the exponential measurement as relative value of return. We have to reconsider the method of comprehensive evaluation which is expressed in Eq.(11) in the case.

Moreover , in our previous model, we assumed that one criterion consists of the pair of return and risk. Then, we proposed a method for evaluating the weights of alternatives for criteria using expected utility. Therefore, one of the future areas of study involves applying a new method using prospect theory to our previous model instead of the expected utility. Furthermore, it is necessary to verify the effectiveness of our proposed method by comparing it with other methods.

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