Simulation Models for Grassland Ecosystem and Inter-species Plant Competition: Interation in NetLogo

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Abstract. In this article, we have first implemented El Hamidi, Garbey and Ali's nonlinear diffusion model of the competition of plants on the Netlogo platform. In parallel with this partial differential equation (PDE) model, an agent based diffusion model has been implemented to compare the structures of the two approaches. The multi-agent system (MAS) models how each individual grows up [1] and is spatially diffused thanks to reproduction. Furthermore, El Hamidi's nonlinear diffusion model has been extended to the case of n species in a system of inter-species plant competition. We have also studied how the inter/intra-specific competition parameters impact the space distribution by computing the surface ratios between species. Besides, terms representing resources have been added to measure the effects of environmental parameters. Finally, we propose a comparison between PDE and MAS approaches by identifying parameters of both models that correspond to each other.

1 Introduction

Plants evolve along time following a predefined life cycle: birth, growth, maintenance, sexual maturity, reproduction, death and decay. During these processes, plants alter the composition of the surrounding environment. When simulating plants' development two main phenomena must thus be taken into account: the growth of each individual and the diffusion of the population by reproduction. The growth and spread of plants are influenced by many environmental parameters such as the presence of light, the dispersion of resources in the earth and in the air [2], the wind direction or strength, etc. They are also influenced by parameters that depends on the plant properties such as the seed weight, shape, the height of the plant from which the seed is diffused, etc. Moreover, dissemination of asexual plants is different from that of bisexual plants because with the latter, the diffusion depends on the number of plants of different sexes which are present in a given radius. The models studied in this article are only focused on the case of asexual plants.

The first model we have implemented is a nonlinear diffusion model of the competition of two plants [3]. This model is based on a diffusive PDE (partial differential equation) implemented on the NetLogo platform with Neumann boundary conditions, and a discretization of both time and space. We have also extended El Hamidi's nonlinear diffusion model to the case of n species in a system of inter-species plant competition. Moreover, we have studied how the inter/intra-specific competition parameters impacts the space distribution by computing the surface ratios between species. In order to measure the effects on plants of environmental parameters such as the dispersion of

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resources in the earth and in the air, light and wind, terms representing resources have been added. Two conditions have been considered: nonlinear isotropic and anisotropic diffusion.

In parallel with the PDE model of two species, a diffusion agent based model has been implemented to compare the structures of the two approaches. The multi-agent system models how each individual grows up [1] and is spatially diffused thanks to reproduction. Finally, we propose a comparison between PDE and MAS approaches by identifying parameters of both models that correspond to each other. For instance, the inter-species competition of the PDE model is performed by the local plant strategy of space colonization.

2 Modeling Diffusion Species of a Prairie Ecosystem

2.1 Model of Nonlinear Diffusion of 2 Species with PDE

In recent years, the equation of diffusion-reaction in a competition system has been widely used in bioinformatics and mathematics [4–6]. Based on the model of Volterra-Lotka, [3] proposes this following model of diffusion:

$$\begin{cases} u_t - \varepsilon_1 div\left(\Phi\left(v\right)\nabla u\right) = u\left(\alpha_1 - \beta_1 u - \gamma_1 v\right), & (x,y) \in \Omega, t > 0, \\ v_t - \varepsilon_2 div\left(\Phi\left(u\right)\nabla v\right) = v\left(\alpha_2 - \beta_2 v - \gamma_2 u\right), & (x,y) \in \Omega, t > 0, \\ \nabla u \cdot n = \nabla v \cdot n = 0 & \text{on } \delta\Omega \times [0, +\infty[, \\ u(x,y,0) = u_0(x,y), v(x,y,0) = v_0(x,y) & (x,y) \in \Omega \end{cases} \tag{1}$$

where the density of the two species at time t and place (x,y) is denoted by u(x,y,t) and v(x,y,t) respectively. ε_i is the motility of a species, α_i the intrinsic growth rates, β_i the intra-specific competition rates and γ_i the inter-specific competition rates of u and v with i=(1, 2).

The nonlinear cross diffusion coefficient is the smooth function $0 \leq \Phi \leq 1$:

$$\Phi(s) = \begin{cases} exp(\frac{s^2}{s^2-\eta^2}) & \text{if } 0 \leq s < \eta \\ 0 & \text{if } s \geq \eta \end{cases}$$

Here are some existing classical results concerning the dynamical Lotka–Volterra system:

$$\begin{cases} u'(t) = u(t)(\alpha_1 - \beta_1 u(t) - \gamma_1 v(t)), & t > 0, \\ v'(t) = v(t)(\alpha_2 - \beta_2 v(t) - \gamma_2 u(t)), & t > 0, \\ u(0) = u_0 > 0, & v(0) = v_0 > 0 \end{cases} \tag{2}$$

where the solutions of interest have to be nonnegative. System (2) has 4 equilibrium points:

$(u_1, v_1) = (0, 0)$,
$(u_2, v_2) = (\frac{\alpha_1}{\beta_1}, 0)$,
$(u_3, v_3) = (0, \frac{\alpha_2}{\beta_2})$,
$(u_4, v_4) = (\frac{\alpha_2\gamma_1-\alpha_1\beta_2}{\gamma_1\gamma_2-\beta_1\beta_2}, \frac{\alpha_1\gamma_2-\alpha_2\beta_1}{\gamma_1\gamma_2-\beta_1\beta_2})$.

We can distinguish 4 situations:

Table 1. 4 situations of the solutions of interest have to be nonnegative.

	Cas 1		Cas 2		Cas 3		Cas 4	
Conditions	$\frac{\beta_1}{\gamma_2} > \frac{\alpha_1}{\alpha_2} > \frac{\gamma_1}{\beta_2}$		$\frac{\beta_1}{\gamma_2} < \frac{\alpha_1}{\alpha_2} < \frac{\gamma_1}{\beta_2}$		$\frac{\beta_1}{\gamma_2} < \frac{\alpha_1}{\alpha_2} > \frac{\gamma_1}{\beta_2}$		$\frac{\beta_1}{\gamma_2} > \frac{\alpha_1}{\alpha_2} < \frac{\gamma_1}{\beta_2}$	
	Saddle points	Stable points	Saddle points	Stable points	Saddle points	Stable points	Saddle points	Stable points
Results	(u_2, v_2), (u_3, v_3)	(u_4, v_4)	(u_4, v_4)	(u_2, v_2), (u_3, v_3)	(u_3, v_3)	(u_4, v_4), (u_2, v_2)	(u_2, v_2)	(u_4, v_4), (u_3, v_3)
	Steady point (u_1, v_1): linearly unstable							

The system is stable when the two species coexist (case 2) [4].
We show results by using two programming frameworks:

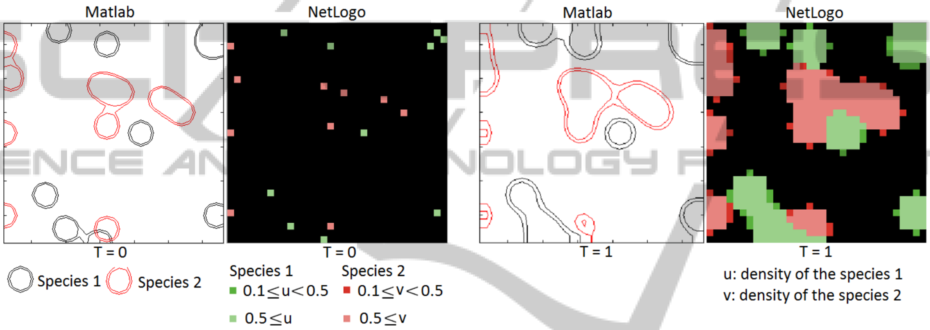

Fig. 1. Simulation with two species in case 2 with $\alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 1, \gamma_1 = 1.5, \gamma_2 = 2$.

Figure 1 stands for the diffusion of two species (green and red) with two programming frameworks Matlab and NetLogo. With the NetLogo framework, bright color presents the higher density than a threshold and the dark color presents the lower density than a threshold. With Matlab, the area inside the small circle presents the former, the area inside the big circle and outside small circle presents to the latter. We can see that two different programming frameworks give the same result.

As this model only represents the competition between to species, we have proposed to generalize this case to a larger number of species.

2.2 Model of Nonlinear Diffusion of n Species with EDP

We expand the model to n species:

$$\begin{cases} u_{it} - \varepsilon_i div\left(\prod_{k\neq i} \Phi\left(u_k\right) \nabla u_i\right) = u_i\left(\alpha_i - \sum_{j=1}^{n} \beta_{ij} u_j\right), & (x,y) \in \Omega, t > 0, \\ \nabla u_i \cdot n = 0 & \text{on } \delta\Omega \times [0, +\infty[, \\ u_i(x,y,0) = u_{i0}(x,y) & (x,y) \in \Omega \end{cases} \tag{3}$$

The function Φ presents the influence of the diffusion of the other species in the same area with the given species. This means that if the other species's development is too strong in the investigating area, the investigated species cannot diffuse (the case when$\Phi = 0$). We consider the function of n species is equal to the composition of n function of each species.

To better understand the behavior of this system of equations, we choose three levels of species behavior: dominant, average and lower levels.

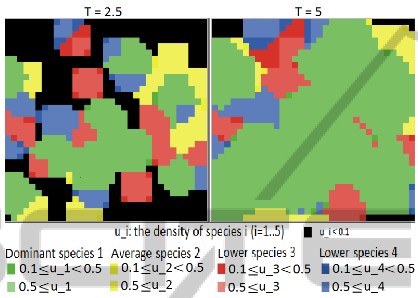

Fig. 2. Unequal competition between four species: one dominant species, one average species and two lower species.

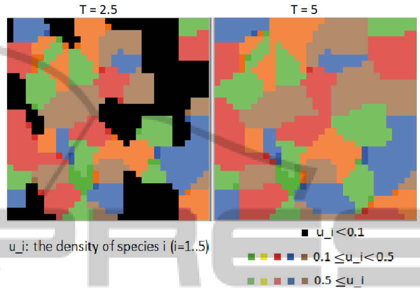

Fig. 3. Equal competition between five average species.

In the case of unequal competition (2), the dominant species grows faster than the other species. In the case of equal competition (3), the species almost develop at the same speed.

We also study the influence of inter-species and intra-species parameters on the spatial distribution by calculating the average of the surface of each species. For example, with five average species in case 2 of parameters α_i, β_i and γ_i. We change the number of initial points of each species. We run the program ten times and calculate the means of them. Each time, the positions of the initial points of each species are selected at random.

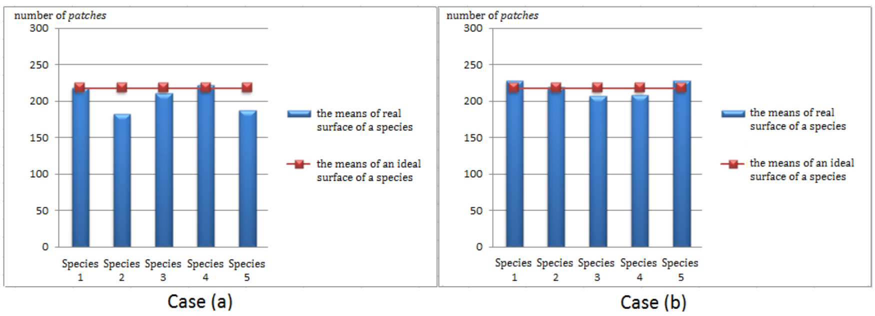

Fig. 4. The distribution of surfaces (number of patches) after averaging: Case (a): There are 5 initial points of each species. Case (b): There are 10 initial points of each species.

This figure represents the difference between the means of real surfaces and the means of ideal surfaces of each species after ten times running the program. Case (a) is the case in which there are five initial points of each species. Case (b) is the case with ten initial points of each species. We can see that the distribution of surfaces is dependent on the number of initial points.

2.3 Comparison between Isotropic Nonlinear Competition Model and Anisotropic Competition Model with EDP

The previous model has been conceived to represent the competition between the plants but the influence of the environment is not taken into account. We have thus proposed the following models to consider the distribution of resources.

In the case of the nonlinear isotropic diffusion, the diffusion is isotropic but depends on the gradient of resources. A resource function and a gradient of the resource function are added in the diffusion term:

$$\begin{cases} u_t - \varepsilon_1 div\left(\Phi\left(v\right)\left(\Phi_{1,s1}\left(R\right) + \Phi_{1,s2}\left(\nabla R \nabla u\right)\right)\nabla u\right) & \\ = u\left(\alpha_1 - \beta_1 u - \gamma_1 v\right), & (x,y) \in \Omega, t > 0, \\ v_t - \varepsilon_2 div\left(\Phi\left(u\right)\left(\Phi_{1,s1}\left(R\right) + \Phi_{1,s2}\left(\nabla R \nabla u\right)\right)\nabla v\right) & \\ = v\left(\alpha_2 - \beta_2 v - \gamma_2 u\right), & (x,y) \in \Omega, t > 0, \\ \nabla u \cdot n = \nabla v \cdot n = 0 & \text{on } \delta\Omega \times [0, +\infty[, \\ u(x,y,0) = u_0(x,y), v(x,y,0) = v_0(x,y) & (x,y) \in \Omega \end{cases} \tag{4}$$

In the case of the anisotropic diffusion, a tensor is used to represent the direction of resources:

$$\begin{cases} u_t - \varepsilon_1 \left(div\left(\Phi\left(v\right)\Phi_2\left(\nabla R \nabla u\right)\eta\eta^T \nabla u\right) + div\left(\Phi\left(v\right)\Phi_{1,s1}\left(R\right)\nabla u\right)\right) & \\ = u\left(\alpha_1 - \beta_1 u - \gamma_1 v\right), & (x,y) \in \Omega, t > 0, \\ v_t - \varepsilon_2 \left(div\left(\Phi\left(u\right)\Phi_2\left(\nabla R \nabla v\right)\eta\eta^T \nabla v\right) + div\left(\Phi\left(u\right)\Phi_{1,s1}\left(R\right)\nabla v\right)\right) & \\ = v\left(\alpha_2 - \beta_2 v - \gamma_2 u\right), & (x,y) \in \Omega, t > 0, \\ \nabla u \cdot n = \nabla v \cdot n = 0 & \text{on } \delta\Omega \times [0, +\infty[, \\ u(x,y,0) = u_0(x,y), v(x,y,0) = v_0(x,y) & (x,y) \in \Omega \end{cases} \tag{5}$$

where

$$\Phi_{1,s_*}\left(x\right) = \begin{cases} h_* & \text{if } thresoldMax_* \le x \\ \frac{h_* x}{thresoldMax_*} & \text{if } thresoldMin_* \le x < thresoldMax_* \\ 0 & \text{if } x < thresoldMin_* \end{cases}$$

$$s_* = \left(thresoldMax_*, thresoldMin_*, h_*\right)$$

$$\Phi_2\left(x\right) = \Phi_{1,s_*}\left(x\right) \text{ with } h_* = 1$$

$$\overrightarrow{\eta} = \frac{\overrightarrow{\nabla} R}{\|\nabla R\|}$$

We present the results of the two approaches. The difference in the selected context of resources is insignificant.

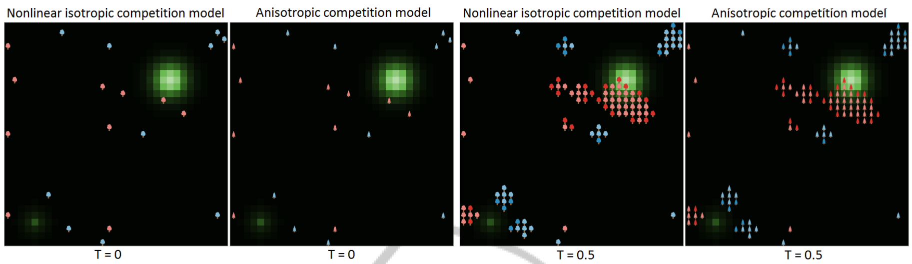

Fig. 5. The isotropic competition model and the anisotropic competition model.

2.4 Modeling Diffusion Species with MAS

In this model, we represent each individual in the community by an agent evolving on a NxM grid of patches. As mentioned above, the diffusion of the population depends on 2 processes: the plants' growth and their reproduction. The equation for the maintenance and growth metabolism was given by [1]:

$$\frac{dm}{dt} = am^{3/4}\left[1 - \left(\frac{m}{M}\right)^{1/4}\right] \tag{6}$$

where m is total body mass ($m = m_c N_c$); M is an asymptotic maximum body size, asymptotic mass ($M = \left(\frac{B_0 m_c}{B_c}\right)^4$); $a \equiv \frac{B_0 m_c}{E_c}$; B_0 is basic energy input; m_c is mass of a cell; E_c is metabolic energy required to create a cell; B_c is metabolic rate of a single cell; N_c is total number of cells. Practically, the spatial diffusion of the plant is then proportional to its mass (see figure 6 - right).

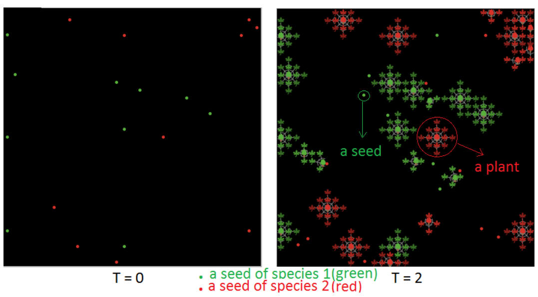

Fig. 6. The development of 2 species with MAS.

Considering the reproduction of the species, we have taken inspiration from the Neture model [7] developed on the NetLogo framework to simulate a terrestrial ecosystem. The model takes into account vegetal producers and their decomposers. Yet, it does not consider the competition between the vegetal producers. Considering the competition between producers and decomposers it is not symmetrical and we had to thus to propose a specific model.

In our model, the intra-species and inter-species competitions are simply modeled by the fact that only on plant at a time can be present on a given patch. When the biomass of a plant exceeds the necessary threshold to become a mature plant, the plant starts his reproduction. When a plant reaches its sexual maturity, it is considered that a seed of the plant can be dropped according to a Poisson distribution (with parameter $\lambda = 50$). The position where the seed falls is chosen randomly within a circle whose center represents the father plant and the radius parameter (dist) is defined by the user (see figure 6 - right). If the falling position is on an area outside the border or in an area where there is another plant, the seed dies. We consider this corresponding to the survivability of a seed in the wild.

Algorithm 1: Pseudocode of plant's growth.

biomass = mass-of-seed;
while biomass ¡ maximum-biomass {
    $\mathrm{a} = \frac{\mathrm{B}_0 * \mathrm{m}_c}{\mathrm{E}_c}$;
    delta-biomass =a $*$ mass$^{3/4} * \left[1 - \left(\frac{\text{mass}}{\text{maximum-mass}}\right)^{1/4}\right]$;
    biomass += delta-biomass;
}

Algorithm 2: Pseudocode of plant's distribution.

if (biomass ¿= biomass-of-mature-plant **and** biomass ¡= maximum-biomass)
    probability has a seed = Poisson distribution
if plant has a seed
    position of seed is random within a circle with radius = dist until selected position is empty

2.5 Comparison between Model-based EDP and Model-based Agents

Figure 7 shows the comparison between the final state obtained with PDE and MAS when the MAS model has different values of parameter "dist". Parameter "dist" determines the distance between the position of the parent plant and the new seed. The closer to the parent plant the new seeds are, the closer to the PDE the MAS model is.

To better understand the comparison between the two models, some key features must be enlightened:

- The PDE model represents a global representation of the system. We can measure the biomass on a given area (a patch) but we cannot know exactly how many individuals it corresponds to. This is a limitation of this model. In contrast, the MAS model represents individuals. Yet, the MAS computation time may be very important if the number of individuals and patches is high.
- In the MAS model, the spatial diffusion of a plant depends on 2 processes: the growth and the reproduction of the plants. The growth factor is modeled by the formula given in section 2.4. The reproduction depends on the maturity parameter, the probability of producing a seed and the probability that a seed can survive. Concerning the PDE model, the only parameter related to growth is α.

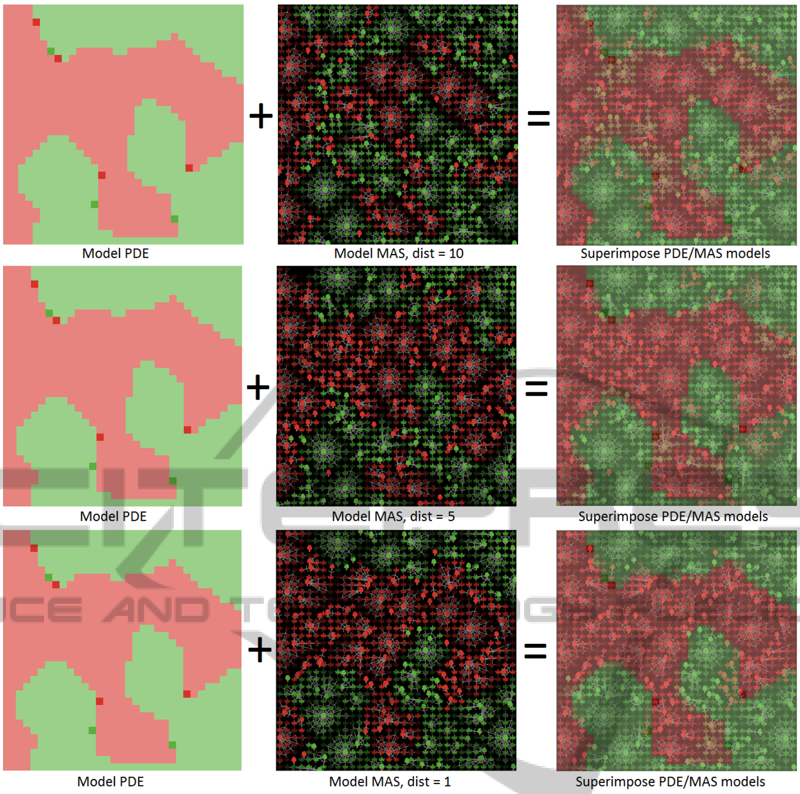

Fig. 7. The result of case 2 with model PDE, model MAS and the superimpose of two models.

– The rate of intra-species (β) and inter-species (γ) competition in the PDE model corresponds respectively, in the MAS model, to the competitive behavior between and individual and another individual of the same species and the competitive behavior between and individual and an individual of a different species. Yet, as in the MAS model we do not make the difference between those two kinds of competitions, it acts as if $\beta = \gamma$.

3 Conclusions

In this article, we have implemented [3]'s nonlinear diffusion of 2 species in competition. Besides, we have extended this model to a reaction-diffusion of one to n species in which the competition between species happens but has no effect on the species's living environment. Moreover, we have also developed two diffusion models of two species in which the diffusion of species is affected by environmental factors. In parallel, a simple MAS model was implemented in order to make the comparison with PDE model. It would be interesting to evaluate how time is included in each model. Though basic comparisons are made, they are to be discussed and studied so that we can combine these two models and enjoy their advantages.

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