The Algebraic and Descriptive Approaches and Techniques in Image Analysis

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Abstract. The main purpose of this review is to explain and discuss the opportunities and limitations of algebraic, linguistic and descriptive approaches in image analysis. During recent years there was accepted that algebraic techniques, in particular different kinds of image algebras, is the most prospective direction of construction of the mathematical theory of image analysis and of development an universal algebraic language for representing image analysis transforms and image models. So, the main goal of the Algebraic Approach is designing of a unified scheme for representation of objects under recognition and its transforms in the form of certain algebraic structures. It makes possible to develop corresponding regular structures ready for analysis by algebraic, geometrical and topological techniques. Development of this line of image analysis and pattern recognition is of crucial importance for automated image mining and application problems solving, in particular for diversification classes and types of solvable problems and for essential increasing of solution efficiency and quality.

1 Introduction

Automation of image processing, analysis, estimating and understanding is one of the crucial points of theoretical computer science having decisive importance for applications, in particular, for diversification of solvable application problem types and for increasing the efficiency of problem solving.

The specificity, complexity and difficulties of image analysis and estimation (IAE) problems stem from necessity to achieve some balance between such highly contradictory factors as goals and tasks of a problem solving, the nature of visual perception, ways and means of an image acquisition, formation, reproduction and rendering, and mathematical, computational and technological means allowable for the IAE.

The mathematical theory of image analysis is not finished and is passing through a developing stage. It is only recently came understanding of the fact that only intensive creating of comprehensive mathematical theory of image analysis and recognition (in addition to the mathematical theory of pattern recognition) could bring a real opportunity to solve efficiently application problems via extracting from images the information necessary for intellectual decision making. The transition to practical, reliable and efficient automation of image-mining is directly dependent on introducing and developing of new mathematical means for IAE.

B. Gurevich I., O. Trusova Y. and V. Yashina V..
The Algebraic and Descriptive Approaches and Techniques in Image Analysis.
DOI: 10.5220/0004394300820093
In *Proceedings of the 4th International Workshop on Image Mining. Theory and Applications* (IMTA-4-2013), pages 82-93
ISBN: 978-989-8565-50-1

During recent years there was accepted that algebraic techniques, in particular different kinds of image algebras, is the most prospective direction of construction of the mathematical theory of image analysis and of development of an universal algebraic language for representing image analysis transforms and image models.

Development of this line of image analysis and pattern recognition is of crucial importance for automatic image-mining and application problems solving, in particular for diversification classes and types of solvable problems and for essential increasing of solution efficiency and quality.

It is one of the breakthrough challenges for theoretical computer science to find automated ways to process, analyze, evaluate and understand information represented in the form of images. It is critical for computer science to develop this branch in terms of solving applied problems, in particular, increasing the diversity of classes of problems that can be solved and the efficiency of the process significantly.

Images are one of the main tools to represent and transfer information needed to automate the intellectual decision-making in many application areas. Increasing the efficiency, including automatization, of gathering information from images can help increase the efficiency of intellectual decision-making.

Recently, this part of image analysis called image mining in English publications has been often set off into a separate line of research.

We list the functions of particular aspects of image handling. Image processing and analysis provides for image mining, which is necessary for decision-making, while the very decision-making is done by methods of mathematical theory of pattern recognition. To link these two stages, the information gathered from the image after it is analyzed is transformed so that standard recognition algorithms could process it (we called this process "the reducing an image to a recognizable form" (RIRF)). Note that although this stage seems to have an "intermediate" character, it is the fundamental and necessary condition for the overall recognition to be feasible.

We need to develop and evolve a new approach to analyzing and evaluating information represented in the form of images. To do it the "Algebraic Approach" of Yu. I. Zhuravlev [43] was modified for the case when the initial information is represented in the form of images. The result is the descriptive approach to image analysis and understanding (DA) proposed and justified by I.B.Gurevich and developed by his pupils [2, 6, 14-16].

In this work, we give a brief review of the main algebraic methods and features.

2 State of the Art of Mathematical Theory of Image Analysis

"State of the art of mathematical theory of image analysis" is the section that describes modern trends in developing of mathematical tools for automation of image analysis, in particular in image-mining.

To automate image mining, we need an integrated approach to leverage the potential of mathematical apparatus of the main lines in transforming and analyzing information represented in the form of images, viz. image processing, analysis, recognition and understanding.

Done by pattern recognition methods, image mining now tends to multiplicity (multialgorithmic and multimodel) and fusion of the results, i.e., several different algorithms are applied in parallel to process the same model and several different models of the same initial data to solve the problem and then the results are fused to obtain the most accurate solution.

Multialgorithmic classifiers and multimodel and multiple-aspect image representations are the common tools to implement this multiplicity and fusion. Note that it was Yu. I. Zhuravlev who obtained the first and fundamental results in this area in 1970s [43].

From 1970s, the most part of image recognition applications and considerable part of research in artificial intelligence deal with images. As a result, new technical tools emerged to obtain information that allow representing recorded and accumulated data in the form of images and the image recognition itself became more popular as the powerful and efficient methodology to process and analyze data mathematically and detect hidden regularities. Various scientific and technical, economic and social factors make the application domain of image recognition experience grow constantly.

There are internal scientific problems that have arisen within image recognition. First of all, these imply algebraizing the image recognition theory, arranging image recognition algorithms, estimating the algorithmic complexity of the image recognition problem, automating the synthesis of the corresponding efficient procedures, formalizing the description of the image as the recognition object, making the choice of the system of representations of the image in the recognition process regular, and some others. It is these problems that form the basis of the mathematical agenda of the descriptive theory of image recognition developed using the ideas of the algebraic approach to recognition [43] to create a systematized set of methods and tools of data processing in image recognition and analysis problems.

There are three main issues one need to solve when dealing with images–describe (simulate) images; develop, study and optimize the selection of mathematical methods and tools of data processing in image recognition; and implement mathematical methods of image analysis on a software and hardware basis.

3 Algebraization of Pattern Recognition and Image Analysis (1970 – Till Now)

This section contains steps of the algebraization in image analysis, fundamentals and the basic theories of pattern recognition, different image algebras. Some words are concerned with contribution of the Russian mathematical school.

3.1 Fundamentals and the Basic Theories in Pattern Recognition

By now, image analysis and evaluation have a wide experience gained in applying mathematical methods from different sections of mathematics, computer science and physics, in particular algebra, geometry, discrete mathematics, mathematical logic, probability theory, mathematical statistics, mathematical analysis, mathematical theory of pattern recognition, digital signal processing, and optics.

On the other hand, with all this diversity of applied methods, we still need to have a regular basis to arrange and choose suitable methods of image analysis, represent, in an unified way, the processed data (images), meeting the requirements standard recognition algorithms impose on initial information, construct mathematical models of images designed for recognition problems, and, on the whole, establish the universal language for unified description of images and transformations over them.

In applied mathematics and computer science, constructing and applying mathematical and simulation models of objects and procedures used to transform them is the conventional method of standardization. It was largely the necessity to solve complex recognition problems and develop structural recognition methods and specialized image languages that generated the interest in formal descriptions–models of initial data and formalization of descriptions of procedures of their transformation in the area of pattern recognition (and especially in image recognition in 1960s).

As for the substantial achievements in this "descriptive" line of study, we mention publications by A. Rosenfeld [34], T. Evans [12], R. Narasimhan [29], R. Kirsh [21], A. Shaw [37], H. Barrow, A. Ambler, and R. Burstall[1], S. Kaneff [20].

In 1970s Yu. I. Zhuravlev proposed the so called "Algebraic Approach to Recognition and Classification Problems" [42], where he defined formalization methods for describing heuristic algorithms of pattern recognition and proposed the universal structure of recognition algorithms. In the same years, U. Grenander stated his "Pattern Theory" [18], where he considered methods of data representation and transformation in recognition problems in terms of regular combinatorial structures, leveraging algebraic and probabilistic apparatus. M.Pavel [31] was introduced "Theory of Categories Techniques in Pattern Recognition", that is formal describing of pattern recognition algorithms via transforms of initial data preserving its class membership.

Then, up to the middle of 1990s, there was a slight drop in the interest in descriptive and algebraic aspects in pattern recognition and image analysis.

Also we present a very brief description of the most important original results on algebraic tools for pattern recognition and image analysis in Russian mathematical school. There are algebras on algorithms, algebraic multiple classifiers, algebraic committees of algorithms, combinatorial algorithms for recognition of 2-D data[1], descriptive image models, 2-D formal grammars [34].

In the framework of scientific school of Yu.I.Zhuravlev several essential results were obtained in algebraic direction by V.L.Matrosov [26], by K.V.Rudakov [35] and V.D.Mazurov [27].

Algebraic method of analysis and estimation of information represented as signals. Apart from basic researches of Yu.I.Zhuravlev scientific school there are significant number of papers concerned with algebraic methods of analysis and estimation of information represented as signals, in partially V.G.Labunec [22], Ya.A.Furman [13], V.M.Chernov [4].

3.2 Steps of the Algebraization

The section presents leading approaches of mathematical theory for image analysis oriented tor automation of image analysis and understanding. First of all there is the history of developing algebraic construction for image analysis and processing –

formal grammars, cellular automata, mathematical morphology, image algebras, multiple algorithms, descriptive approach.

Algebraization of pattern recognition and image analysis has attracted and continues to attract the attention of many researchers. Appreciable attempts to create a formal apparatus ensuring a unified and compact representation for procedures of image processing and image analysis were inspired by practical requirements for effective implementation of algorithmic tools to process and analyze images on computers with specialized architectures, in particular, cellular and parallel.

The idea of constructing a unified language for concepts and operations used in image processing appeared for the first time in works by Unger [42], who suggested to parallelize algorithms for processing and image analysis on computers with cellular architecture.

Mathematical morphology, developed by G. Matheron [25] and Z. Serra [36], became a starting point for a new mathematical wave in handling and image analysis. Serra and Sternberg [39] were the first to succeed in constructing an integrated algebraic theory of processing and image analysis on the basis of mathematical morphology. It is believed [28] that it was precisely Sternberg who introduced the term “image algebra” in the current standard sense. (We note that U. Grenander used this concept in the 1970s; however, he was talking about another algebraic construction [18]). Within the limits of this direction, an array of works continues to be written, devoted to the development of specialized algebraic constructions implementing or improving upon methods of mathematical morphology.

From that time till 1990’s the interest to descriptive and algebraic aspects of image analysis is failing. The final view of idea of IA has become Standard Image Algebra by G.Ritter [32] (algebraic presentation of image analysis and processing operations).

DIA is created as a new IA provided possibility to operate with main image models and with basic models of procedure of transforms, which lead to effective synthesis and realization of basic procedures of formal image description, processing, analysis and recognition. DIA is introduced by I.B.Gurevich and developed by him and his pupils [14-16].

The history of algebraization: J.von Neumann [30], S.Unger [42] (studies of interactive image transformations in cellular space); M. Duff, D. Watson, T. Fountain, and G. Shaw [10] (a cellular logic array for image Processing); A. Rosenfeld [33] (digital topology); H. Minkowski and H.Hadwiger (pixel neighborhood arithmetic and mathematical morphology); G.Matheron, J.Serra, S.Sternberg [25, 36, 39] (a coherent algebraic theory specifically designed for image processing and image analysis - mathematical morphology); S. Sternberg [39] (the first to use the term “image algebra”); P. Maragos [24] (introduced a new theory unifying a large class of linear and nonlinear systems under the theory of mathematical morphology); L. Davidson [9] (completed the mathematical foundation of mathematical morphology by formulating its embedding into the lattice algebra known as Mini-Max algebra); G.Ritter [32] (Image Algebra); I.B.Gurebich [15] (Descriptive Image Algebra); T.R. Crimmins and W.M. Brown, R.M. Haralick, L. Shapiro, R.W. Schafer, J. Goutsias, L. Koskinen and Jaako Astola, E.R. Dougherty, P.D. Gader, M.A. Khabou, A. Koldobsky, B. Radunacu, M.Grana, F.X. Albizuri, P. Sussner [8, 7, 10, 11, 19, 40] (recent papers on mathematical morphology and image algebras).

4 Descriptive Approach to Image Analysis and Understanding

This section contains a brief description of the principal features of the DA needed to understand the meaning of the introduction of the conceptual apparatus and schemes of synthesis of image models proposed to formalize and systematize the methods and forms of image representation.

By the middle of 1990s, it became obvious that for the development of image analysis and recognition, it is critical to: 1) understand the nature of the initial information – images, 2) find methods of image representation and description that allow constructing image models designed for recognition problems, 3) establish the mathematical language designed for unified description of image models and their transformations that allow constructing image models and solving recognition problems, and 4) construct models to solve recognition problems in the form of standard algorithmic schemes that allow, in the general case, moving from the initial image to its model and from the model to the sought solution.

The DA gives a single conceptual structure that helps to develop and implement these models and the mathematical language [14-16]. The main DA purpose is to structure and standardize different methods, operations and representations used in image recognition and analysis. The DA provides the conceptual and mathematical basis for image mining, with its axiomatic and formal configurations giving the ways and tools to represent and describe images to be analyzed and evaluated.

The automated extraction of information from images includes (1) automating the development, testing, and adaptation of methods and algorithms for the analysis and evaluation of images; (2) the automation of the selection of methods and algorithms for analyzing and evaluating images; (3) the automation of the evaluation of quality and adequacy of the initial data for solving the problem of image recognition; and (4) the development of standard technological schemes for detecting, assessing, understanding, and retrieving images.

The automation of information extraction from images requires combined use of all features of the mathematical apparatus used or potentially suitable for use in determining transformations of information provided in the form of images, namely in problems of processing, analysis, recognition, and understanding of images.

Experience in the development of the mathematical theory of image analysis and its use to solve applied problems shows that, when working with images, it is necessary to solve problems that arise in connection with the three basic issues of image analysis, i.e., (1) the description (modeling) of images; (2) the development, exploration, and optimization of the selection of mathematical methods and tools for information processing in the analysis of images; and (3) the hardware and software implementation of the mathematical methods of image analysis.

The main purpose of the DA is to structure and standardize a variety of methods, processes, and concepts used in the analysis and recognition of images. The DA is proposed and developed as a conceptual and logical basis of the extraction of information from images. This includes the following basic tools of analysis and recognition of images: a set of methods of analysis and recognition of images, BFSR techniques, conceptual system of analysis and recognition image, DIM classes, the DIA language, statement of problems of analysis and recognition of images, and the basic model of image recognition.

The main areas of research within the DA are (1) the creation of axiomatics of analysis and recognition of images, (2) the development and implementation of a common language to describe the processes of analysis and recognition of images (the study of DIA), and (3) the introduction of formal systems based on some regular structures to determine the processes of analysis and recognition of images (see [14-17]).

Mathematical foundations of the DA are as follows: (1) the algebraization of the extraction of information from images, (2) the specialization of the Zhuravlev algebra to the case of representation of recognition source data in the form of images, (3) a standard language for describing the procedures of the analysis and recognition of images (DIA) [14-16], (4) the mathematical formulation of the problem of image recognition, (5) mathematical theories of image analysis and pattern recognition, and (6) a model of the process for solving a standard problem of image recognition. The main objects and means of the DA are as follows: (1) images; (2) a universal language (DIA); (3) two types of descriptive models, i.e., (a) an image model and (b) a model for solving procedures of problems of image recognition and their implementation; (4) descriptive algebraic schemes of image representation (DASIR); and (5) multimodel and multiaspect representations of images, which are based on generating descriptive trees (GDT) [14-16].

The basic methodological principles of the DA are as follows: (1) the algebraization of the image analysis, (2) the standardization of the representation of problems of analysis and recognition of images, (3) the conceptualization and formalization of phases through which the image passes during transformation while the recognition problem is solved, (4) the classification and specification of admissible models of images (descriptive image model - DIM), (5) RIRF, (6) the use of the standard algebraic language of DIA for describing models of images and procedures for their construction and transformation, (7) the combination of algorithms in the multialgorithmic schemes, (8) the use of multimodel and multiaspect representations of images, (9) the construction and use of a basic model of the solution process for the standard problem of image recognition, and (10) the definition and use of nonclassical mathematical theory for the recognition of new formulations of problems of analyzing and recognizing images.

Note that the construction and use of mathematical and simulation models of studied objects and procedures used for their transformation is the accepted method of standardization in the applied mathematics and computer science.

The creation of the DA was significantly influenced by the following basic theories of pattern recognition: (1) the algebraic approach to pattern recognition of Zhuravlev and their algorithmic algebra [43] and (2) the theory of images of Grenander [18], in particular algebraic methods for the representation of source data in image recognition problems developed in it.

As already noted, in the DA, it is proposed to carry out the algebraization of the analysis and recognition of images using DIA. DIA was developed from studies in the field of the algebraization of pattern recognition and image analysis carried out since the 1970s. The creation of a new algebra was directly influenced by algorithms of Zhuravlev [43] and the research of Sternberg [39] and Ritter [32], which identified classic versions of image algebras.

A more detailed description of methods and tools of the DA obtained in the development of its results can be found in [14-16].

5 Ontology-based Approach to Image Analysis

This section briefly describes the use of ontologies for representation of knowledge needed to support intellectual decision making in image analysis and understanding tasks.

The automation of image analysis assumes that researchers and users of different qualifications have at their disposal not only a standardized technology of automation, but also a system supporting this technology, which accumulates and uses knowledge on image processing, analysis and evaluation and provides adequate structural and functional possibilities for supporting the more intelligent choice and synthesis of methods and algorithms. The automated system (AS) for image analysis must combine the possibilities of the instrumental environment for image processing and analysis and a knowledge-based system. Therefore, one of its main components is a knowledge base. Knowledge bases usually contain modules of universal knowledge, which are not related to any subject domain (knowledge necessary for scheduling and control of the processing, result mappings, estimation of the processing quality, object recognition, and conflict resolution, as well as knowledge about methods of image processing and analysis) and knowledge modules related to a certain subject domain (segmentation strategies, object descriptions, and specialized strategies for feature extraction and object identification). The AS must provide software implementation of the hierarchies of classes of the main objects used in image analysis, have a specialized user interface, contain a library of algorithms that allow one to solve the main problems of image analysis and understanding with the help of efficient computational procedures, and provide accumulation and structuring of knowledge and experience in the domain of image analysis and understanding.

The need of efficient knowledge representation facilities can be fulfilled by using a suite of ontologies. Ontologies as an effective way for knowledge representation became very popular last years. Ontological knowledge representation has the following advantages: (1) it provides the opportunity to establish a common understanding of the considered field of knowledge, (2) it enables us to represent knowledge in a convenient form for processing by automated information processing and analysis systems, and (3) it provides the opportunity for acquisition and accumulation of new knowledge and for multiple use of knowledge.

Different works related to usage of ontologies for solving image-based tasks have been reported. For example, in [23], an approach devoted to semantic image interpretation for complex object classification purposes is proposed. The described framework is focused on the mapping between domain knowledge and image processing knowledge. In the proposed knowledge acquisition methodology, the mapping between the domain and the image is based on a visual concept ontology composed of three different types of visual concepts - texture concepts, color concepts and spatial concepts - associated with numerical descriptors useful for performing object recognition. In [3], an integrated knowledge infrastructure and annotation environment for multimedia description, analysis and reasoning is described. The framework comprises ontologies for the description of low-level audio-visual features and for linking these descriptions to concepts in domain ontologies based on a prototype approach. The key idea is to associate domain concepts with instances that serve as prototypes for these concepts. The proposed infrastructure includes a set of ontologies including the core ontology, Visual Descriptor Ontology (VDO), Multimedia Structure Ontology (MSO) and specific domain ontologies. The core ontology serves as a basis for all components of the knowledge infrastructure. The VDO represents the structure of the MPEG-7 visual part and models concepts and properties that describe visual characteristics of objects. The MSO models basic multimedia entities from the MPEG-7 Multimedia Description Scheme. Domain ontologies model the content layer of multimedia content with respect to specific real-world domains. The work described in [5] addresses the problem of explicit representation of objectives when developing image processing applications. Authors investigated kinds of information needed to design and evaluate image processing software programs and proposed an ontology-based model for formulation of user objectives. In [41], a cognitive architectural model for image and video interpretation is discussed. The proposed framework demonstrates that ontology-based content representation can be used as an effective way for hierarchical and goal-directed inference in high-level visual analysis tasks.

In [6], a novel knowledge-oriented approach to image analysis based on the use of thesauruses and ontologies as tools for representation of knowledge, which are necessary for making intelligent decisions on the basis of information extracted from images, is proposed. The main contribution of this work is the development of a sufficiently detailed and well-structured Image Analysis Ontology (IAO) needed for solving the following tasks: 1) construction of unified description and representation of image-based tasks and methods for solving these tasks; 2) automation of image analysis methods combination on the base of semantic integration; 3) automation of navigation and retrieval in knowledge bases on image analysis. As a main source of the information about concepts (including term definitions and basic relationships between terms) the Image Analysis Thesaurus (IAT) [2] has been used. The important feature of the IAT is a novel hierarchical classification of tasks and algorithms for image processing, analysis and recognition. The following IAO classes were defined: `Task`, `Method`, `Data`, `Context` and `Requirements`. The hierarchy of subclasses is based on term relations fixed in the IAT. By integrating the ontology with algorithms for image processing, analysis and understanding, high-level semantic information can be extracted from images.

6 Conclusions

Note that the idea to create a single theory that embraces different approaches and operations used in image and signal processing has a history of its own, with works of von Neumann continued by S. Unger, M. Duff, G. Matheron, G. Ritter, J. Serra, S. Sternberg and others [30, 10, 25, 32, 36, 39] playing an important role in it.

The main stages of algebraization are:

- Mathematical Morphology (G. Matheron, J. Serra [1970's])
- Algorithm Algebra by Yu.I.Zhuravlev (Yu. Zhuravlev [1970's]
- Pattern Theory (U. Grenander [1970's]
- Theory of Categories Techniques in Pattern Recognition (M.Pavel [1970's])

- Image Algebra (Serra, Sternberg [1980's]
- Standard Image Algebra (Ritter [1990's])
- Descriptive Image Algebra (DIA) (Gurevich [1990-2000])
- DIA with one king (Gurevich, Yashina [2001 to date])

Analyzing the existing algebraic apparatus, we came to the statement of the following requirements on the language designed for recording algorithms for solving problems of image processing and understanding: 1) the new algebra must make possible processing of images as objects of analysis and recognition; 2) the new algebra must make possible operations on image models, i.e., arbitrary formal representations of images, which are objects and, sometimes, a result of analysis and recognition; introduction of image models is a step in the formalization of the initial data of the algorithms; 3) the new algebra must make possible operations on main models of procedures for image transformations; 4) it is convenient to use the procedures for image modifications both as operations of the new algebra and as its operands for construction of compositions of basic models of procedures.

Acknowledgements

This work was supported in part by the Russian Foundation for Basic Research (projects nos. 11-01-00990, 12-07-31123) and by the Presidium of the Russian Academy of Sciences within the program "Fundamental Science to Medicine" as well as within the program "Information, Control, and Intelligent Technologies and Systems" (project no. 204) and the program of the Division of Computer Sciences, Russian Academy of Sciences "Algebraic and Combinatorial Methods of New_Generation Mathematical Cybernetics and Information Systems" (the project "Algorithmic Schemes of Descriptive Image Analysis").

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