Semi-centralized Reconstruction of Robot Swarm Topologies

The Largest Laplacian Eigenvalue and High Frequency Noise are used to Calculate the Adjacency Matrix of an Underwater Swarm from Time-series

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Keywords: Swarm Intelligence, Underwater Autonomous Vehicle, Topology Reconstruction, Eigenvalue Spectrum.

Abstract: An important task in underwater autonomous vehicle swarm management is the knowledge of the graph topology, to be obtained with the minimum possible communication exchanges and amid heavy interferences and background noises. Despite the importance of the task, this problem is still partially unsolved. Recently, the Fast Fourier Transform and the addition of white noise to consensus signals have been proposed independently to determine respectively the laplacian spectrum and the adjacency matrix of the graph of interacting agents from consensus time series, but both methodologies suffer technical difficulties. In this paper, we combine them in order to simplify calculations, save energy and avoid topological reconstruction errors using only the largest eigenvalue of the spectrum and instead of white noise, a high frequency, low amplitude noise. Numerical simulations of several swarms (random, small-world, pipeline, grid) show an exact reconstruction of the configuration topologies.

1 INTRODUCTION

Monitoring the marine environment is acquiring more and more importance because of scientific and economic reasons. Just to name some of them, we could indicate the search for natural resources, fishery, sea pollution mapping, maintenance activities.

In this paper we describe a methodology able to reconstruct exactly the graph from time series (the network inverse problem), using recently developed signal analysis and algebraic graph theory techniques.

Although our methodology depends on a semi-centralized data elaboration, there are particular situations when the noises, disturbances and interferences reach very high levels that may require such approach, as the last resort.

2 PROBLEM DESCRIPTION

Autonomous underwater vehicle systems (AUV) have moved from the prototype stage to scientific, and commercial uses (Nawaz et al., 2009).

An AUV must be considered (Dell'Erba, 2012) as a real cost alternative to other available technologies, such as manned submersibles, remotely operated vehicles and towed instruments led by ships. However, many problems are still to be solved to make AUV competitive especially for the issues relevant to power availability, information processing, navigation, and control. Communication channels are a major concern, as the acoustic underwater transmission is very slow and bandwidth limited, but, in the future, optical, high power transmission devices will be available for a number of different approaches integrating the acoustical data channel.

Nevertheless, a swarm could be advantageous compared to a single vessel, if high communication rate were available to reduce the dead reckoning errors. It can collect together all the data of all the vessels to minimize the errors in estimating position. Therefore, in some cases, a centralized formation analysis may be helpful, although economic costs rise (Pompili and Melodia, 2005). For example when the swarm size is large, since multi-hop paths are needed to reach every node causing delays in several ways.

This means that information about the configuration cannot be transmitted inside the swarm in the form of simple link look-up table

Fioriti V., Chiesa S. and Fratichini F..
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DOI: 10.5220/0004421800740081
In *Proceedings of the 10th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2013), pages 74-81
ISBN: 978-989-8565-70-9

because large delays are prone to cause instability. To be more precise, consider a large swarm and a consensus control protocol (see Paragraph 3). According to (Olfati-Saber and Murray, 2004) the stability of a fixed configuration is guaranteed iff:

$$\tau \leq \pi / 2\lambda_{max}$$

where τ is the uniform delay experienced by the consensus distributed computations. Similar constraints may be set for non uniform switching topologies.

The delay τ depends on a number of factors: CPU power, data transmission bandwidth, MAC protocols, the number of AUV N, the inter-symbol interference and finally the time necessary to acquire the largest eigenvalue of the Laplacian, T_λ.

If we calculate λ_{max} by means of a look-up table simply verifying the existence of an acoustical/optical link between two AUV and passing the information to other nodes, it is to be expected a relatively large amount of time T_λ . Then we have:

$$\tau = T_\lambda + \tau_1 + \tau_2 + \dots \tau_n \leq \pi / 2\lambda_{max}$$

Now, for a large swarm over a wide area it turns out that:

$$\tau \approx T_\lambda \leq \pi / 2\lambda_{max}$$

therefore T_λ is a sort of time-horizon beyond which the swarm configuration looses stability, i.e. the consensus solutions diverge.

Another situation that prevents from using multi-hop paths is the marine background noise (Traverso et al., 2012) together to the inter-symbol interference (ISI) (Dousse et al., 2005).

ISI is a signal in which one symbol interferes with subsequent symbols. The phenomenon is enhanced by multiple reflections of the signal and as has an effect similar to a non-gaussian noise.

The problem is produced by a number of emitting nodes (AUV) towards the receiver and by the background noise. Dousse has demonstrated that above a critical value of ISI the network of reliably communicating nodes splits into small isolated components, and as a consequence, the connectivity is lost (Dousse et al., 2005).

This result is true in general and even more so in the marine environment. Difficulties are exacerbate if a CDMA (Code Division Multiple Access), the transmission of multiple digital signals simultaneously over the same carrier frequency, is to be used (Appala Raju et al., 2012). As a consequence, data packets arrive deteriorated to the receiver node.

Nevertheless, there are many reasons to know the swarm topology. To name only two of them: the second largest laplacian eigenvalue (the Fiedler or algebraic eigenvalue) is relevant to the swarm connectivity and the maximum eigenvalue is relevant to the tolerable delay. Moreover, according to (Camperi et al., 2012); (Ballerini et al., 2008) in biological swarms maintaining a certain topology is preferred to maintain a metric graph, i.e. metric distances. It seems that topological interactions are more robust to predatory attacks, facilitating the group cohesion.

Then a solution may be to implement a semi-centralized scheme, taking advantage of the noise resilient procedure of (Ren et al., 2010) and of the spectrum distributed computation (Franceschelli et al., 2012), (Yang et al., 2008).

3.1 The proposed Scheme

We come to envision a large AUV swarm monitoring a very wide area. A Gausssian noise codifies the information about the topology and is transmitted on the consensus channel. Thus, provided the power of the added noise is large enough, the transmission inside the swarm is robust. The centralized data collection/elaboration task is carried out by one or more ships. A ship covers a part of the monitored area as shown in the pictorial representation of Figure 1. The radius r of the circumference covered by the ship depends on the inter vehicle distance, the number of AUV (Chiesa and Taraglio, 2012), the topology, and the maximum allowable delay. Ships exchange data each other by means of RF devices and with the swarm by optical transmitters. When the maximum extension D of the area is D >> r, it would be unfeasible to implement a multi-hop inter-swarm data transmission to control the configuration because delays would prevent any synchronization.

Clearly, this scheme is based on a centralized approach; instead the consensus control of position/velocity is completely distributed (Olfati-Saber and Murray, 2004). An appropriate position prediction algorithm may also alleviate consensus errors due to delays (Joordens and Ponds, 2010).

It should be noted that the consensus protocol is necessary to the swarm stabilization, thus no calculation encumbrance is the required to the system at least with respect to control tasks.

Another distributed procedure we use in this work is the determination of the laplacian spectrum of the swarm network. Since also the spectrum is determinate locally, we have two distributed calculations and a centralized one, i.e. a semi-centralized scheme. If the swarm configuration is fixed obviously the spectrum is known in advance.

Moreover, instead of a ship, the central elaborations could take place in one or more AUV, properly equipped and able to broadcast relevant data to the whole swarm by a gossip protocol.

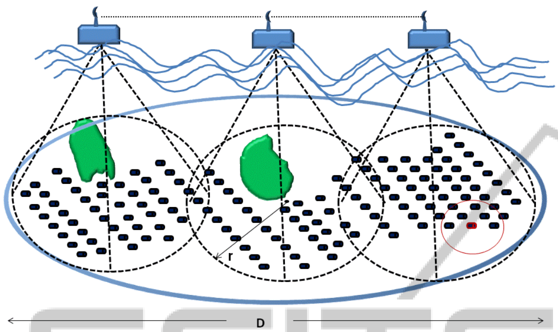

Figure 1: Ships receive information from the AUV inside the dotted circles of radius r and from the other ships. The small red circle is the transmission range of an AUV towards the closest neighbours. The same functions of ships could be fulfilled by one or more AUV properly equipped.

3 METHODS

Recently, a method to recover the Laplacian matrix of the a network of dynamical coupled systems has been given (Ren 2010). Starting from the general form of the i-th differential system:

$$\boldsymbol{x}_i' = \mathbf{F}_i(\, x_i \,)$$

$i = 1, \dots N$, and adding couplings and noise we have:

$$\boldsymbol{x}_i' = \mathbf{F}_i\,(\boldsymbol{x}_i) - c\,\Sigma_j\, L_{ij}\,\mathbf{H}(\boldsymbol{x}_j) + \eta_i \qquad (1)$$

$i, j = 1, \dots N$, where c is the coupling coefficient (here $c = 1$), $\mathbf{H}$ the coupling functions, $\boldsymbol{x}$ the state variables, η the white gaussian noise with strength σ^2, L_{ij} are the entries of the Laplacian matrix derived from the undirected graph of the systems. Vectors and matrices are in bold. The Laplacian matrix is:

$$\mathbf{L} = \mathbf{D} - \mathbf{A}$$

where **D** is a diagonal matrix formed by the node degrees and **A** is the adjacency matrix (1 if a link i-j exists, 0 otherwise) of the graph.

The very interesting point here is that the noise added enables the solution of the inverse problem: given the time series, reconstruct the graph. Because of the particular problem of the swarm control, in our paper we focus on the standard consensus form of (1):

$$x_i' = \Sigma_j\, a_{ij}(x_j - x_i) + \xi_i \quad , \quad j = 1, \dots N \qquad (2)$$

where a_{ij} are the entries of the adjacency matrix **A**, but here we consider a high frequency (HF) noise ξ instead of the white noise η, as it will be explained later.

It is known that for a connected network, the equilibrium point for (2) is globally exponentially stable. Moreover, the consensus value is equal to the average of the initial values; for small swarms the average is easy to calculate. In compact form (2) is written:

$$\boldsymbol{x}' = -\,\mathbf{L}\boldsymbol{x} + \boldsymbol{\eta}$$

Expression (2) and similar are utilized in the swarm control to coordinate the states of the robots on a common position/velocity agreement resilient to disturbs (Tanner et al., 2003); (Bullo et al., 2009); (Xi et al., 2012); (Olfati_Saber, 2007); (Cucker and Smale, 2007).

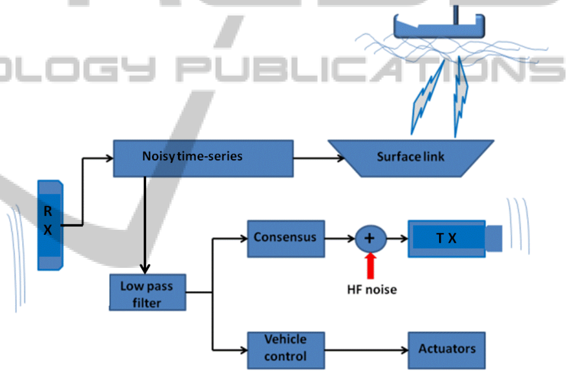

Figure 2: Scheme of data elaboration in an AUV.

After long enough time-series have been collected, it is demonstrated (Ren, 2010) that:

$$\mathbf{L} = \mathbf{C}^{+}\,(\sigma^2/2) \qquad (3)$$

where **C** is the correlation matrix among the time series between node i and node j, $\mathbf{C}^{+}$ is the Moore-Penrose pseudoinverse. Note that (2) requires the knowledge of all time series to calculate the pseudoinverse, hence the reconstruction is centralized. Authors of (Ren, 2010) find a one-to-one correspondence between the correlation matrix of time series from nodes and the Laplacian matrix; albeit no physical explanation of the phenomenon is clearly claimed, an analytical proof is sketched.

This remarkable, counterintuitive finding actually allows to set a threshold for the entries of $\mathbf{C}^{+}$: below it the entries are considered -1, above 0, thus the Laplacian and consequently the adjacency matrix, is reconstructed. The threshold procedure is not immediate to implement, anyway in (Ren, 2010) it is claimed a very good success rate. Nevertheless, some errors are reported to remain. Since the AUV has a non negligible economic value, any effort for eliminating the residual error is reasonable.

Moreover, considering the energy saving requirement of the signal transmission apparatus of the AUV, the average degree (i.e. the number of underwater communication links) should be kept as low as possible. At the same time, the consensus signals are needed also to control the swarm and in this respect, noise is a disturb to keep as small as possible. Therefore, bearing in mind these considerations, we suggest a node to transmit consensus signals added with HF noise and to low pass the noisy signals received, as in Figure 2.

3.1 The Spectral Estimation

To reduce or eliminate the residual error in the graph reconstruction we need extra information.

A relevant help could be the knowledge at least of some eigenvalues of the laplacian spectrum.

In some cases the graph is fixed and there is no need of topological variations, thus the desired spectrum is known and only a periodic verification is required, but usually the graph changes frequently and demands an on-line check.

The spectral reconstruction has been studied in (Franceschelli et al., 2012); (Yang et al., 2008).

Franceschelli calculates a distributed Fast Fourier Transform (FFT) of signals derived from a proper distributed protocol and received at a node i:

$$x_i' = z_i + \sum_j (z_i - z_j)$$
$$z_i' = - x_i - \sum_j (x_i - x_j)$$

with $j \in N_i$ (nodes at one hop of distance from node i). Thus, the state trajectory is a linear combination of sinusoids oscillating only at frequencies function of the eigenvalues of the Laplacian matrix λ_j, and the amplitude of the peaks in the spectrogram are related to the eigenvalues:

$$|\mathcal{F}(x_i(t))| = 1/2\sum_j a_{ij}\, \delta\, (f \pm (1+ \lambda_j)/2\pi)$$
$$|\mathcal{F}(z_i(t))| = 1/2\sum_j b_{ij}\, \delta\, (f \pm (1+ \lambda_j)/2\pi)$$

This method has some drawbacks (Kibangou and Commault, 2012): the multiplicities of the eigenvalues cannot be calculated and the FFT suffers from the presence of noise. Remember that independently from the Ren's procedure, underwater communications are polluted by several sources of noise.

On the other hand (Yang et al., 2008) provides an estimation of the laplacian spectrum based on matrix power iteration, but this way only an approximate solution can be obtained.

Finally, it is worth noting that even if an exact spectrum reconstruction was available, today is not clear if, at least theoretically, is possible to reconstruct univocally its adjacency matrix (Van Dam and Haemers, 2003). Alternative combinatorial optimization techniques such as the tabu search, simulated annealing or graph theory methods are not exact and some of them would anyway require a long computation time.

In spite of these limitations, having available the estimation of just a single eigenvalue, we show how to eliminate completely or at least reduce the estimation error in the graph reconstruction.

3.2 Error Reduction

Let us consider that only the largest lapalacian eigenvalue λ_N has been calculated by means of one of the previously described methods. It is intuitive to use it as a simple cost function, instead of the threshold procedure, to determine the non null entries of the adjacency matrix recovered by (3).

Therefore in our methodology the pseudoinverse C^+ is calculated from noisy consensus time-series and normalized. Then, starting from a convenient value, an initial adjacency matrix **A** is produced, its largest laplacian eigenvalue λ^*_N calculated and subtracted to the actual eigenvalue λ_N :

$$\min g(\lambda) = | \lambda_N - \lambda^*_N |$$

and when:

$$g(\lambda_N) = 0 \quad (4)$$

the actual matrix **A** is reconstructed (best results have been obtained with the largest eigenvalue, although other eigenvalus may be used). In Figure 3 it is shown how the zero estimation error of the eigenvalue is reached jointly with the complete reconstruction of the adjacency matrix.

If errors in the exact estimation of the maximum laplacian eigenvalue were present, the exact reconstruction as in Figure 2 is still possible for low – moderate amounts of the error. Moreover, accepting just a few errors in the link reconstruction, the acceptable error in the eigenvalue estimation increases quickly (see Table 2).

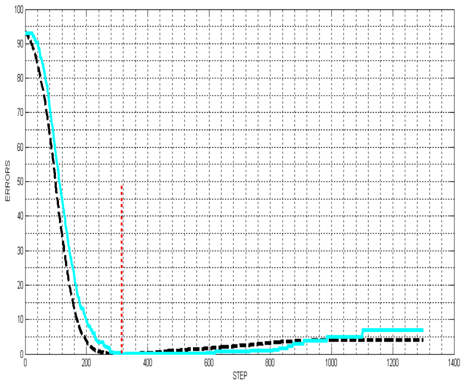

Figure 3a: Small World graph, 100 nodes (abscissa: time steps, ordinate: errors). Black dotted curve: actual error percentage of the adjacency matrix entries (not including diagonal and symmetric elements), continuous blue curve: the eigenvalue absolute error $|\lambda_N - \lambda^*_N|$. Vertical red and green (green line is not visible because is coincident with the red one) lines: exact reconstruction according to the (4). The minimum value of the continuous blue curve indicates the correct topology reconstruction, i.e. zero errors.

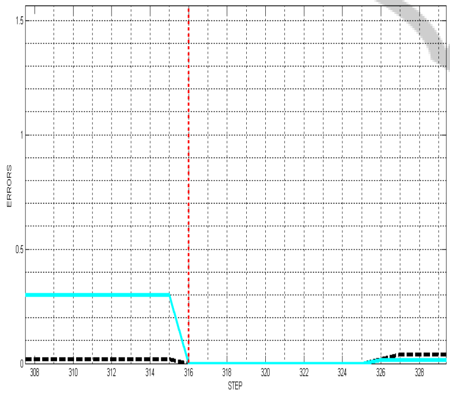

Figure 3b: Enlargement of the minimum area.

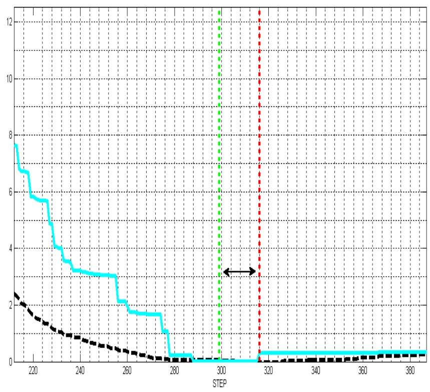

Figure 3c: In this case two entries are wrong and the minimum indicated by the largest eigenvalue (green dotted line), is no more coincident with the actual zero reconstruction error (dotted red line), see Table 2 also.

3.3 Noise Addition

For the methodology to work it is necessary the addition of noise to the consensus protocol. As pointed out in Paragraph 2, in a real environment it is already present a background of natural or artificial noise, then the previous noise level is increased. This does not undermine the methodology, provided the strength of the added Gaussian noise is large enough.

In order to save energy and allow the consensus signals to produce a proper control action on the AUV swarm, we add a high frequency (HF), low amplitude, zero mean, unitary variance Gaussian noise to (1).

Noise strength in simulations is $\sigma^2 = 0.01$, one order magnitude smaller with respect to (Ren, 2010). In Figure 4 is shown the HF noise and the signal power spectral density (psd) spectrum (frequencies are normalized).

In Figure 4 is shown a consensus signal, as it appears after the low-pass filtering, once the signal has been received in a node.

Aside the delay due to the low-pass filter, the original signal is recovered. Anyway, even without the low-pass filtering, the consensus solutions converge all (Figure 5), but the Erdos-Renyi random configuration that is not connected.

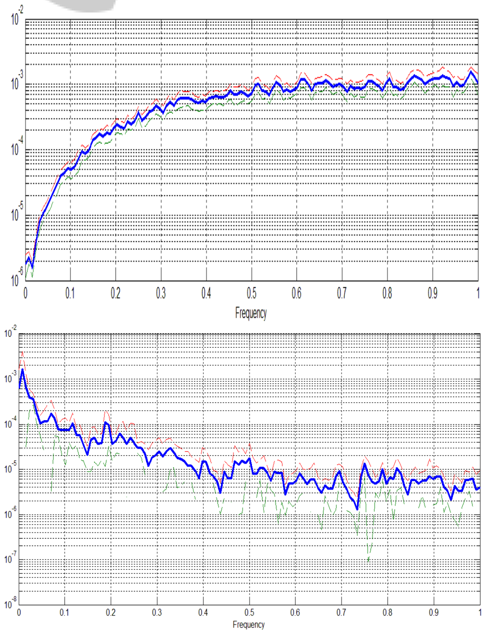

Figure 4: Top, psd of HF noise, middle, psd of the consensus signal with HF noise. Last image: HF noise in the time domain.

4 NUMERICAL SIMULATIONS

Numerical simulations have been conducted to validate the methodology, results are shown in Table 1. The task is to recovery exactly all of the significant ($N^2 - N$) / 2 entries of the adjacency matrix **A** of the swarm graph.

Four types of topologies have been considered, as relevant to underwater robot swarms: Erdos-Renyi (random, p = 0.01), small-world (average degree: 4, p = 0.1), pipeline (average degree: 4), grid (average degree: 4), N = 24, see Figure 6.

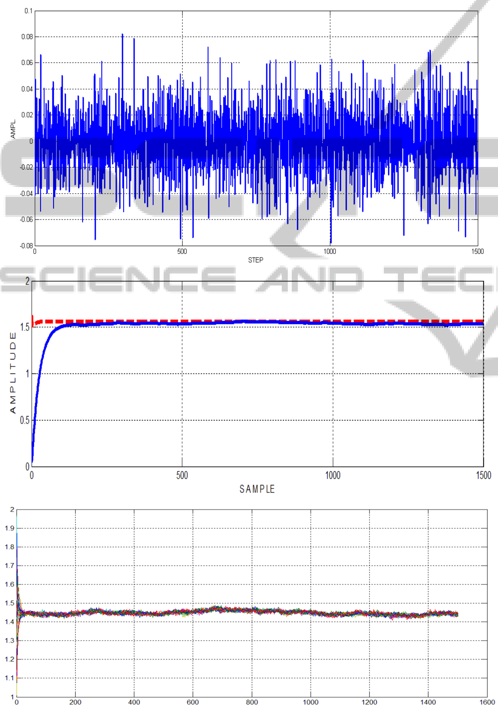

Figure 5: Top, the noisy time series. Middle: consensus time series after the low-pass filtering. The red dotted curve is the original consensus. Bottom: noisy consensus solutions for a SW (N = 24, average degree 4) before the low-pass filtering.

Table: 1 Simulation results.

Graph topology	Error	Nodes	Links	Integrati on steps
Erdos-Renyi	0	48	16	~150
Small-World	0	24	48	~150
Small-World	0	100	200	~370
Pipeline	0	24	43	~150
Grid	0	24	38	~150
Grid	0	100	180	~370

The average degree 4 has been chosen because biologically inspired investigations (Camperi et al., 2012) indicates in the range 5-10 the optimal number of neighbours in order to maintain the group cohesion while saving an evenly space distribution.

In the simulations the centralized elaborations are represented by the computation of the correlation matrix **C** among all the time-series received from the N nodes and by its pseudo inverse $\mathbf{C}^+$.

For each configuration a complete reconstruction (zero errors) has been achieved, see Table 1.

In particular, SW networks are very interesting for AUV configuration, as pointed out by (Olfati_Saber, 2005), because of the high consensus speed and connectedness properties.

The small-world consensus scheme seems to be the fastest also for low number of nodes. In fact, it is known (Olfati_Saber, 2005) that when a SW has a number of nodes N > 100 the convergence is very fast, but for N = 24, as in our case, there is no guarantee.

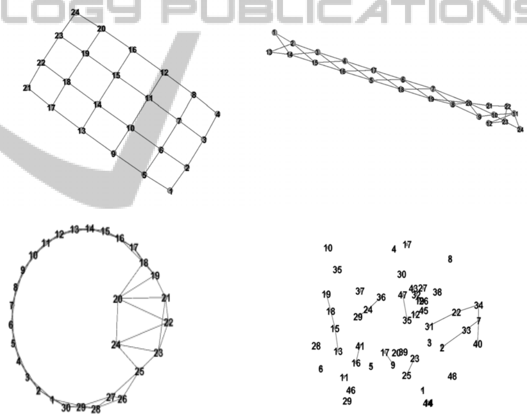

Figure 6: Left to right graph topologies: regular grid, pipe-line, small-world, random. Each node is an AUV, links are inter-swarm acoustical or optical communication channels. The grid topology is the most regular, the SW is half-way between regularity and randomness. Note the disconnected nodes of the ER topology.

All time series have a length of 150 simulation time-steps (1500 samples) for N = 24; the first 30 samples have been discarded because the transitory impair the calculations. As the size (in nodes) increases, longer time series are needed. As an example, when the node size of a SW graph is 100, about 370 time-steps are needed to recover the graph.

Note that an higher noise level reduces the time-step length, bur increases the energy dissipation. The trade-off should be analyzed on an *ad hoc* basis.

In Table 2 are shown the results for a large and a small SW graph in presence of errors on the estimation of the maximum laplacian eigenvalue, obtained by the methods of (Yang et al., 2008) or (Franceschelli et al., 2012).

The acceptable error on the maximum eigenvalue estimation (meaning that the number of mistaken entries of **A** is still zero) increases as N increases. For example for $N = 100$, the 3.22% estimation error means that the real value $\lambda_N = 4.0375$ is altered as much as: $\lambda_N \pm 0.13$, but the reconstruction of the matrix **A** remains exact.

Table 2: Stability of solutions.

Graph Topo-logy	Mista-ken entries	Nodes	Overall Entries	Acceptable error in the λ_N estimation
SW	0	100	4950	3.22%
SW	2	100	4950	7%
SW	0	24	276	0.22%
SW	2	24	276	15.2%

4 CONCLUSIONS

The control of an underwater robot swarm is a complex task because of the particular environment, especially when are present high levels of noises and interferences. To this end, new biologically inspired methodologies are currently under development.

One of the most important and unsolved control problems in this field is the reconstruction of the swarm topology. In fact, position sensors are often inaccurate or unable to work properly. At the cost of a semi- centralized elaboration of the consensus time series, we have shown how it is possible to achieve a complete topology reconstruction within the technological framework suited to the marine environment.

The methodology envisages the reconstruction of the graph of the swarm using the noisy signals of the consensus protocol. When received, signals are correlated and the resulting correlation matrix is elaborated according to a simple relation to obtain the Laplacian matrix. Since the largest eigenvalue of the Laplacian matrix can be estimated independently, although not exactly, it is possible to calculate the difference with the eigenvalue from the reconstructed one at every step of the algorithm. This information allows to decide the correct adjacency matrix with zero or minimum reconstruction error.

The original consensus signals necessary to control the AUV are recovered by low-pass filtering, as noise is allocated in the relatively high frequency band.

ACKNOWLEDGEMENTS

This work has been supported by the HARNESS Project, funded by the Italian Institute of Technology (IIT).

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