A New Modified Hough Transform Method for Circle Detection

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Keywords: Hough Transform, Incremental Hough Transform, Circle Hough Transform, Circle Detection.

Abstract: The Hough transform is a powerful tool in image analysis, e.g. circle detection is a fundamental issue in image processing applications of industrial parts or tools. Because of its drawbacks, various modifications of the basic circle Hough transform (CHT) method have been suggested. This paper presents a modified method based on the basic CHT algorithm and using no trigonometric calculations in order to improve the computational performance of the voting process for a good accuracy and robustness of circle detection in a binary image. This paper also provides the errors analysis of the proposed method against the basic CHT method to illustrate that it can replace the basic CHT method for small values of the resolution ε of the angle θ. It then compete the CORDIC algorithm when it is used in a hardware implementation.

1 INTRODUCTION

Shape recognition is one of the most important tasks in the image processing and pattern recognition. Many methods for detecting geometric primitives have been proposed. The Hough transform (HT) and its extensions constitute a popular and robust method for extracting analytic curves. It was first applied to the recognition of straight lines (Duda, 1972) and later extended to circles (Davies, 1987), ellipses (Yip, 1992) and arbitrarily shaped objects (Pao, 1992).

The principal concept of the HT is to define a mapping between an image space and a parameter space. Each feature point (or a set of feature points) in an image is mapped to the parameter space to vote for the parameters whose associated curves pass through the data point(s). The votes for each curve are accumulated, and after all the points in an image have been considered, local maxima in the parameter space correspond to the parameters of the detected curves. The curves detection in the image space therefore become a peak detection problem in the parameter space. The advantages of the HT include robustness to noise, robustness to shape distortions and to occlusions/missing parts of an object. Its main disadvantage is the fact that computational and storage requirements of the algorithm increase as a power of the dimensionality of the curve. This means that for straight lines the computational complexity and storage requirements are $O(n^2)$, for circles $O(n^3)$ and for ellipses $O(n^5)$ (Dimitrios, 1999).

In view of this disadvantage, in this paper, we introduce a new modified CHT method, called Incremental circle Hough transform (ICHT), that is aimed at improving the voting process. For each input point in the image, the new method computes incrementally the circle point coordinates passing through the input point using new formulation of the parametric representation of the circle. By using approximations on cosine and sine in the parametric representation of the circle, the new formulation is : easy to use because the point coordinates of the circle at the iteration n is computed from the coordinates point of the circle of the iteration $\mathrm{n}-1$ by using simple equations; provides a solution to the use of trigonometric functions that causes problems in digital device implementations such as FPGA; can be seen as a solution to the use of the CORDIC (Cordinate Rotation Digital Computer) algorithm, the equations used by our ICHT algorithm are simpler and very suitable for parallelization in the calculation of circles point coordinates that passing through the input point in the image.

In this paper, we show in detail the feasibility

Djekoune A., Messaoudi K. and Belhocine M..
A New Modified Hough Transform Method for Circle Detection.
DOI: 10.5220/0004424600050012
In *Proceedings of the 5th International Joint Conference on Computational Intelligence* (ECTA-2013), pages 5-12
ISBN: 978-989-8565-77-8

and the simplicity of the introduced method, and so, it can replace the basic CHT method not only in software applications but also in hardware applications.

The paper is organized as follow: In section 2, we positioning our work relative to some published CHT methods existing in the literature. In section 3, we present in more detail our new ICHT method: Starting with an overview of the basic CHT method followed by the ICHT algorithm development, the errors analysis and the implementation. We compare the ICHT method with the one often used to overcome the setback of the basic CHT method for hardware implementation in the section 4. The conclusion is given in the section 5.

2 BACKGROUND

Extracting circles from images has received more attention for several decades because an extracted circle can be used to yield the location of circular object in many industrial applications. Many variations on original CHT method have been proposed to increase its performance. One type of method address issues of efficiency to reduce significantly the amount of computation and storage required to implement the Hough transform (Lavin, 1986), Another type of method replaces the formal parameterization of the target object with a look-up table, the Generalized Hough Transform (GHT), allowing the Hough approach to be used to detect arbitrary shapes (Ballard, 1981). A third type of method uses the probabilistic interpretation of the Hough approach (Stephens, 1990), (Kälviäinen, 1995). Other type of method uses the randomized selection of edge points and geometrical properties of the circle (Bandera, 2006); (Xu, 1990); (Chun, 1995), and the edge orientation information of each edge pixel to reduce the computing time or the requirement of the accumulator (Kimme, 1975). And finally the type of method proposes a variety of voting scheme used in the Hough transform (Siyu, 2009). An excellent reviews of a number of circle detection methods based on variations of the Hough transform can be found in (Yuen, 1990).

Other than the software solution of the CHT drawbacks, we also find in the literature, the hardware solution which provides an attractive solution to computationally intensive applications in real-time whilst maintaining the flexibility of a software solution (Tagzout, 2001); (Djekoune, 2004). The Hough Transform has traditionally been implemented using complex processor architecture. These are either slow or complicated due to the transform's intensive calculations of trigonometric, multiplication and addition operations (Dixon, 2001). To overcome this major setback, the CORDIC algorithm is used (Dixon, 2001); (Ferhat-taleb, 2012). The CORDIC algorithm can be used to calculate elementary trigonometric functions such as sine, cosine, tangent, and arctangent as well as *ln* and *exp*.

In this context, we present in this work a new ICHT method aimed at improving the voting process. This method fully both exploits the software and the hardware solution advantages because it doesn't use any trigonometric calculations, simple to use, easily fitted into digital device, such as FPGA, without consuming too device resources, and very suitable for a parallel implementation.

3 THE PROPOSED ICHT METHOD

3.1 The Basic CHT Method: An Overview

A circle with center (*a*,*b*) and radius r, in a binary image, is specified by the parameters (*a*,*b*,*r*) in the equation:

$$(x-a)^2+(y-b)^2=r^2 \tag{1}$$

with *(x,y)* the set edge pixels that make up the circumference of this circle.

The parametric representation of the circle is:

$$\begin{cases} x = a + r\ \cos\theta \\ y = b + r\ \sin\theta \end{cases} \tag{2}$$

For each edge pixel, the basic Hough transform method constructs a circular cone, in the (*a,b,r*) parameter space (or Hough space), resulting from the voting process of the (*a,b,r*) parameters whose associated circles pass through the considered pixel by using a fourfold loop over *x*, *y*, *a* and *b* (Figure 1). This operation runs slowly because it is mainly due to the both use a large number of mathematical operations (1) and trigonometric calculations (2).This raises the computational cost of the transform, often to unacceptable levels.

For simplicity, some works in the litterature set the radius to a constant value (hard coded) or provide the user with the option of setting a range (maximum and minimum) prior to running the application, or use the edge direction information to limit voting to a section of the cone.

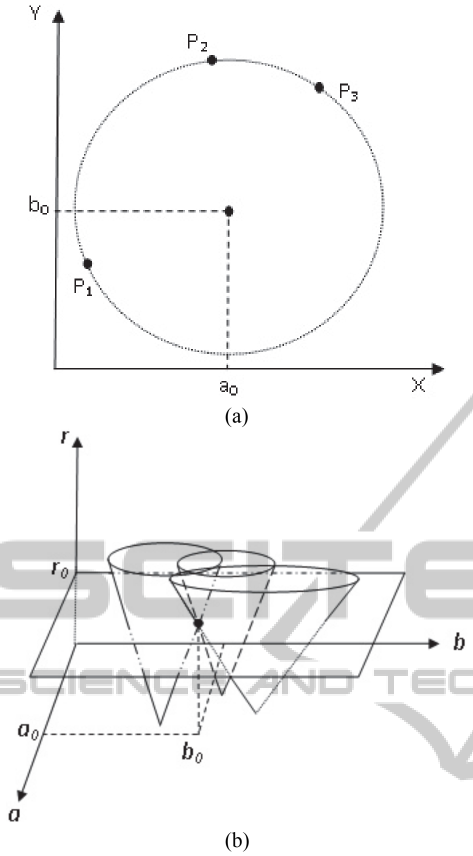


Figure 1: Relationship between a binary image plan and the Hough space. (a) P_1, P_2 and P_3 are edge pixels belonging to a same imaginary circle with r_0 the radius and (a_0, b_0) the coordinates of its center. (b) Each edge pixel from the binary image generates a circular cone in the Hough space. The cones in the Hough space intersect at (a_0, b_0,r_0) corresponding to the parameters of the circle formed by the edge pixels P_1, P_2 and P_3.

3.2 The New ICHT Method

a. Algorithm Development. The main goal of this work is to try to improve the basic CHT to make it simple to use, and easily adapted and fitted into the digital device without consuming too device resources. Thus combining both advantages of hard and soft solutions described above.

Our improvement mainly concerns the voting process, it uses new equations, or formulation, of the parametric representation of a circle. These equations compute incrementally the coordinates point of a circle, such that each coordinates point of a circle at the iteration n is computed from the coordinates point of the same circle of the iteration $n-1$. We proceeded as follows:

For discrete values of the angle, (2) is written as follows (we have used the same notation as in (Tagzout, 2001) and (Djekoune, 2004):

$$\begin{cases} x_n = a + r\ \cos\theta_n \\ y_n = b + r\ \sin\theta_n \\ x_0 = a + r \\ y_0 = b \\ \theta_n = n\,\varepsilon \\ n_\theta = \dfrac{2\pi}{\varepsilon} \\ 0 \le \theta_n < 2\pi \\ 0 \le n < n_\theta \end{cases} \tag{3}$$

with n, ε and n_θ are, respectively, the angle index, the angle resolution and the number of angle values in the θ interval.

To make (3) as incremental, it must be written in the following form:

$$\begin{cases} x_{n+1} = f(x_n, y_n) \\ y_{n+1} = f(x_n, y_n) \end{cases} \tag{4}$$

ie, the point coordinates of the circle (x_{n+1}, y_{n+1}) at the iteration $n+1$ is only computed from the point coordinates of the circle (x_n, y_n) of the iteration n.

By replacing n by $n + 1$ in (3), we will have:

$$\begin{cases} x_{n+1} = a + r\ \cos\theta_{n+1} \\ \qquad = a + r\ \cos[(n+1)\varepsilon] \\ \qquad = a + r\,[\cos n\varepsilon \cos\varepsilon - \sin n\varepsilon \sin\varepsilon] \\ y_{n+1} = b + r\ \sin\theta_n \\ \qquad = b + r\ \sin[(n+1)\varepsilon] \\ \qquad = b + r\,[\sin n\varepsilon \cos\varepsilon + \cos n\varepsilon \sin\varepsilon] \end{cases} \tag{5}$$

Making the approximation, in the expression above, on cosine and sine for the small values of the angle by assuming *cosε=1* and *sinε=ε*. The equation (5) becomes:

$$\begin{cases} x_{n+1} = a + r\,[\cos n\varepsilon - \varepsilon \sin n\varepsilon] \\ \qquad = x_n - r\,\varepsilon \sin n\varepsilon \\ y_n = b + r\,[\sin n\varepsilon + \varepsilon \cos n\varepsilon] \\ \qquad = y_n + r\,\varepsilon\ \cos n\varepsilon \end{cases} \tag{6}$$

Note that from (3), we can get:

$$\begin{cases} r\ \cos n\varepsilon = x_n - a \\ r\ \sin n\varepsilon = y_n - b \end{cases} \tag{7}$$

By replacing (7) in (6), then rearranging the obtained expression to get the following general expression of our new ICHT method:

$$\begin{cases} x_{n+1} = x_n - \varepsilon\, y_n + \varepsilon\, b \\ y_{n+1} = y_n + \varepsilon\, x_n - \varepsilon\, a \\ x_0 = a + r \\ y_0 = b \\ 0 \le n < n_\theta \\ n_\theta = \dfrac{2\pi}{\varepsilon} \end{cases} \tag{8}$$

We can note that (2) and (8) are almost similar except that (2) is highly dependent to the trigonometric functions, which is not the case with (8). We can therefore conclude that (8) is:

- Purely incremental,
- Doesn't use any trigonometric calculations,
- Simple to use,
- Can be seen as a solution to the use of the CORDIC algorithm (see section 4),
- Can be easily fitted into digital device such as FPGA,
- Can be very suitable for parallelization.

b. Error Analysis. In the following, we show the errors, if exist, caused by the above approximations when using (8) and (2) to draw circles.

When drawing two circles ($\text{Circle}_{\text{ICHT}}$ and $\text{Circle}_{\text{CHT}}$) with the same parameters using (8) and (2), we note that the points of the two circles overlap for small values of θ and diverge for larger values of θ (Figure 2).

There are many criteria which can be considered to evaluate this divergence, but in our study the most important point relates to errors analysis. The errors analysis is measured using: the average error ($\text{E}_{\text{average}}$) and the quadratic error ($\text{E}_{\text{quadratic}}$) between the radii resulting from the generated points using the two above equations; the difference area ($\text{Diff}_{\text{area}}$) and the *Jaccard* coefficient ($\text{Coef}_{\text{Jaccard}}$) to compare the similarity of the two generated circles. The *Jaccard* coefficient measures the ratio of the intersection area of two sets divided by the area of their union (Jaccard, 1912).

These errors are computed from different values of the radius R and the resolution ε of the θ angle. They are expressed as follows:

$$E_{average} = \frac{1}{n_\theta} \sum_{n=0}^{n_\theta - 1} (R_{ICHT}[n] - R_{CHT}[n]) \quad (9)$$

$$E_{quadratic} = \frac{1}{n_\theta} \sum_{n=0}^{n_\theta - 1} (R_{ICHT}[n] - R_{CHT}[n])^2 \quad (10)$$

with $\text{R}_{\text{ICHT}}[\text{n}]$ and $\text{R}_{\text{CHT}}[\text{n}]$ the radii computed from the coordinates point at the n^{th} value of θ using (2) and (8).

It is interesting to note that our new ICHT method achieves very small errors for small values of the resolution ε of the angle θ which remain within a narrow tolerance despite the high that can have the radius R. But these errors increase considerably when the resolution ε increases with high value of the radius R (Figures 3 to 6).

The figures 3 to 6 show that for values of the resolution ε less than 1°, the errors $E_{average}$, $E_{quadratic}$ and $Diff_{area}$ are very small, and consequently the $Coef_{Jaccard}$value reach the one value. The one value means that the two circles are substantially similar. Beyond the value 1° and for small values of the radius R, the $Coef_{Jaccard}$ value decreases giving rise to significant divergences.

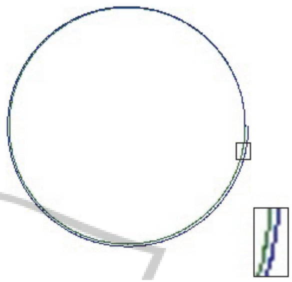


Figure 2: The circles $\text{Circle}_{\text{ICHT}}$ and $\text{Circle}_{\text{CHT}}$ drawn at a fixed position with radius=100 and ε=0.5°.

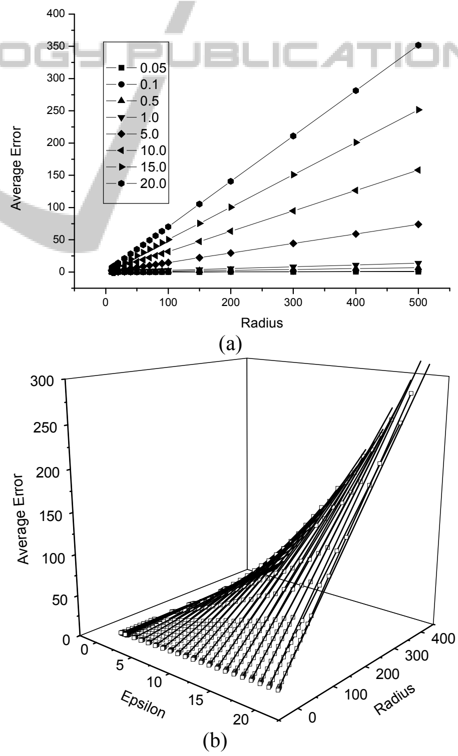


Figure 3: The average error in (a) 2D and (b) 3D version.

The figures 3 and 4 show that the average error and the difference area between $\text{Circle}_{\text{ICHT}}$ and $\text{Circle}_{\text{CHT}}$ circles increases linearly with a slope depending on the values of the resolution ε of the angle θ. This gives us an idea of how the computed points, from expression (8), diverge from those computed from (2).

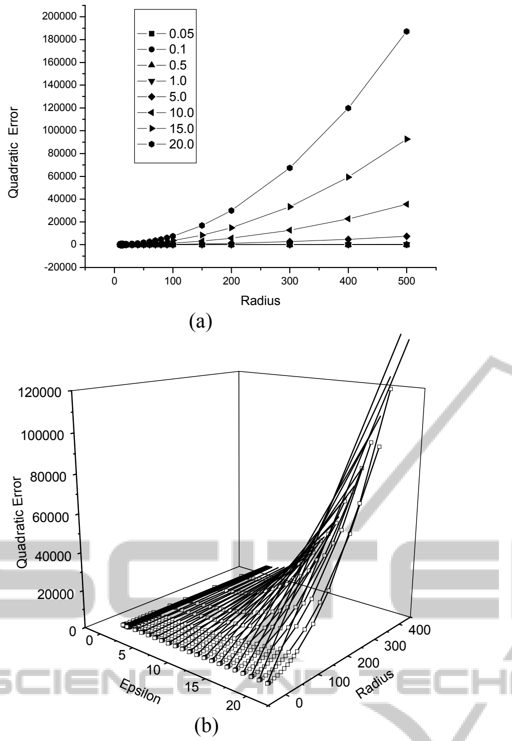


Figure 4: The quadratic error in (a) 2D and (b) 3D version.

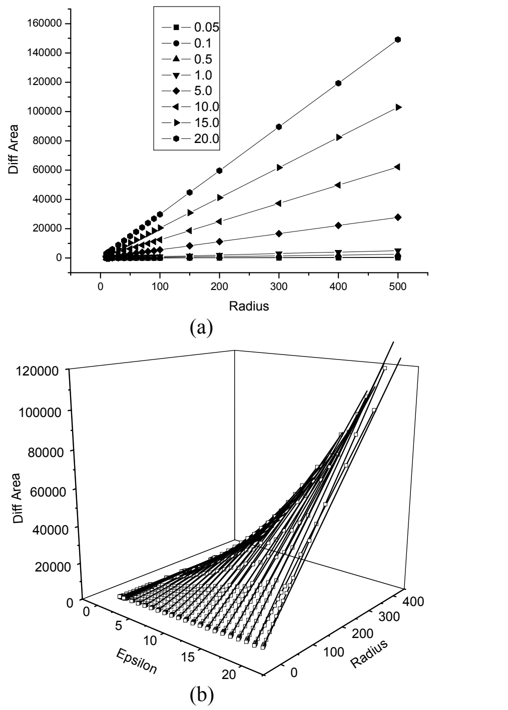


Figure 5: The difference area between $\mathrm{Circle}_{\mathrm{ICHT}}$ and $\mathrm{Circle}_{\mathrm{UCHT}}$ circles in (a) 2D and (b) 3D version.

In conclusion, our new ICHT method can replace the basic CHT method for small values of the resolution ε of the angle θ with the advantage of does'nt using any trigonometric calculations, It then compete the CORDIC algorithm when implemented into digital device.

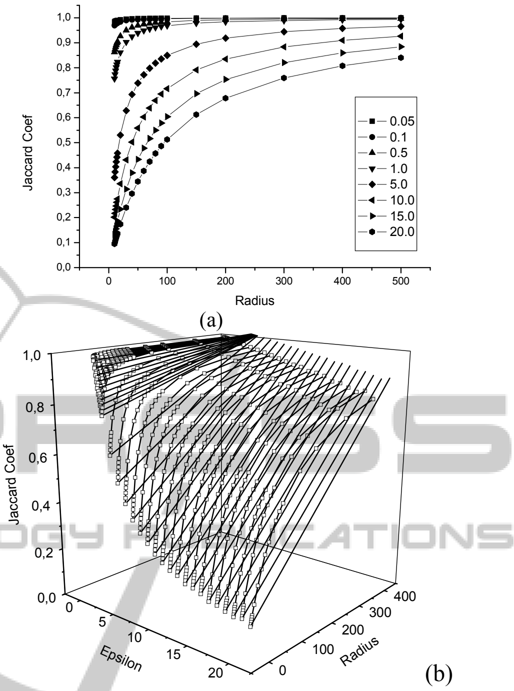


Figure 6: The *Jaccard* coefficient in (a) 2D and (b) 3D version.

Figure 7: The test image.

c. Implementation. The new ICHT method is tested against the basic CHT method. This will be done to illustrate the consequences of the used approximations in the parametric representation of the circle in the processing time of the voting process and to see the computational efficiency of the Hough space of the two methods. The new ICHT method and the basic CHT method were implemented in the programming language Matlab v.7. The implementation was performed using a labtop PC equipped with 2.6 GHz i5 processor and 6GB RAM. A real gray scale image, of size 225x220 pixel is used (Figure 7). The binary edge points shown in (Figure 8) are obtained by using the Matlab *Canny* operator.

The voting process algorithm of the new ICHT and the basic CHT methods, applied in a real binary edge image, are performed using $r_{min} = 10$ and $r_{Max} = \sqrt{225^2 + 220^2} \cong 315$. The ε resolution value of the θ angle is initially set by the user.

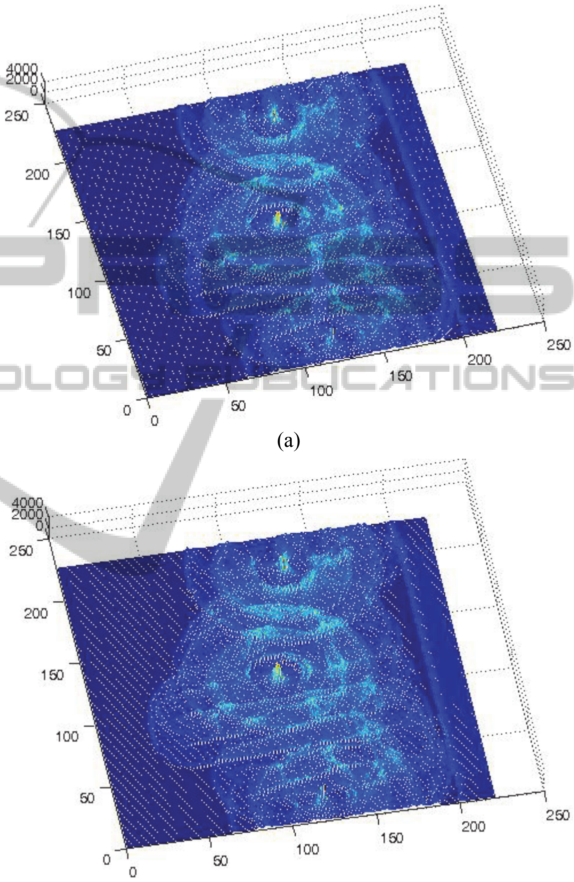
Figure 8: Binary edge image using *Deriche* operator.

The table 1 assess the time processing of the two methods, where the time of the voting process, the time required to process one binary edge pixel, and the time ratio are presented. The binary edge image of the figure 8 is used using different values of the resolution ε of the θ angle. The processing time per pixel, in milliseconds, is obtained by dividing the time of the voting process, expressed in second, by the number of the binary edge point contained in the figure 8, in our case this number is equal to 2508. The time ratio is obtained, in this case, by dividing the time of the voting process of the basic CHT method by the time of the voting process of the new ICHT method. The table 1 not only show that the new ICHT method is fast more than the basic CHT method but it is more than two time faster.

Table 1: Processing time of the voting process.

ε(°)	CHT (s)	ICHT (s)	Time per pixel(ms) CHT/ICHT	Time ratio
0.05	862.39	412.51	343.85/164.48	2.09
0.1	438	211.60	174.64/84.37	2.07
0.5	96.69	49.55	38.54/19.76	1.95
1.0	50.87	26.24	20.28/10.46	1.94
5.0	10.5	5.15	4.19/2.05	2.04
10.0	5.32	2.51	2.12/1.0	2.12
15.0	3.6	1.62	1.43/0.64	2.22
20.0	2.73	1.20	1.09/0.48	2.27

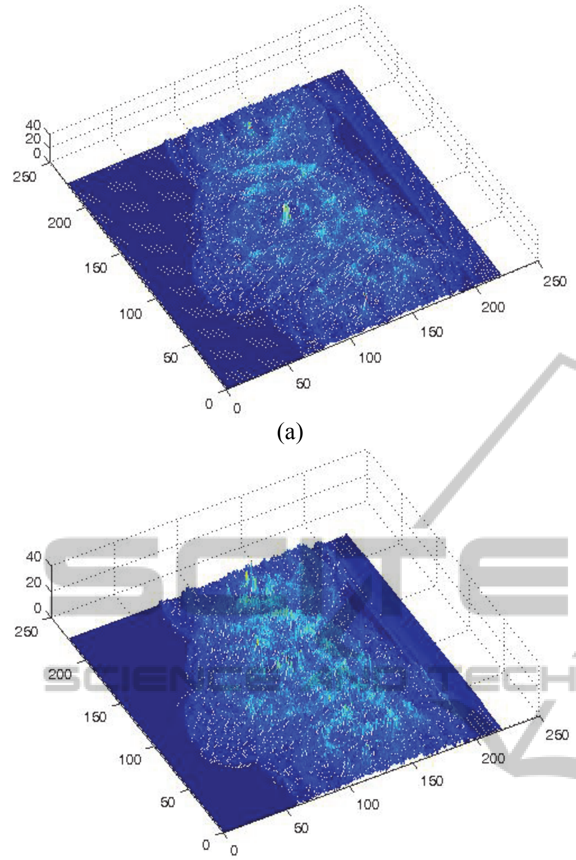
After evaluating the processing time of the two methods, now we try to show the Hough space obtained from these two methods. The figures 9 and 10 show the plans of the Hough space with a radius R = 10, and confirm the conclusions done in the section §3.2. The Hough spaces obtained from these two methods are the same for values of the resolution ε less than 1°, and diverge significantly beyond the value 1°.



(a)

(b)

Figure 9: Hough space plan with radius=20 and ε=0.05° obtained from both (a) the basic CHT method and (b) our new ICHT method.



(a)

(b)

Figure 10: Hough space plan with radius=20 and ε=5° obtained from both (a) the basic CHT method and (b) our new ICHT method.

4 THE NEW ICHT METHOD VS CORDIC ALGORITHM

The CORDIC algorithm, proposed by Volder in 1959 (Volder, 1959), is used to calculate elementary trigonometric functions such as sine, cosine, tangent, and arctangent as well as *ln* and *exp*. It provides an iterative method of performing vector rotations by arbitrary angles using only shifts and adds. The algorithm is based on the common rotation equations:

$$\begin{cases} x' = x\cos\varphi - y\sin\varphi \\ y' = y\cos\varphi + x\sin\varphi \end{cases} \tag{13}$$

which rotates a vector in a Cartesian plane by the angle φ. These can be rearranged so that:

$$\begin{cases} x' = \cos\varphi \cdot [x - y\tan\varphi] \\ y' = \cos\varphi \cdot [y + x\tan\varphi] \end{cases} \tag{14}$$

When restricting the rotation angles so that $\tan\varphi = \pm 2^{-i}$, the multiplication by the tangent term is reduced to simple shift operation. This allows the vector to be rotated by desired angle in a sequence of smaller rotations by angle $\varphi = \pm\tan^{-1}(2^{-i})$:

$$\begin{cases} x_{i+1} = K_i[x_i - y_i \cdot d_i \cdot 2^{-i}] \\ y_{i+1} = K_i[y_i + x_i \cdot d_i \cdot 2^{-i}] \\ \quad K_i = \cos(\tan^{-1} 2^{-i}) \\ \quad\quad = 1/\sqrt{1 + 2^{-2i}} \\ \quad\quad d_i = \pm 1 \end{cases} \tag{15}$$

The CORDIC algorithm is one of the existing hardware implementation solutions of the CHT (or HT) to overcome its intensive calculation of trigonometric, multiplication and addition operations. This solution results in a complication of the final architecture and a significant consumption of the device resources.

Unlike to the hardware implementation of the basic CHT where the CORDIC algorithm is used, our new ICHT method can be hardware implemented alone which leads better performance than the implementation of the basic CHT with the CORDIC algorithm minimizing the used device resources.

5 CONCLUSIONS

We presented a new modified CHT method with enhanced formulation for improving the computational performance and efficiency of the voting process of the basic CHT. Called Incremental circle Hough transform (ICHT), the method fully both exploits the software and the hardware solution advantages with no trigonometric calculations, it can be seen as a a solution to the use of the CORDIC algorithm, and consequently easily fitted into digital device, such as FPGA, without consuming too device resources, and very suitable for a parallel implementation.

We have presented theoretical and errors analysis of our method, and have shown experimentally that, for small values of angle, the new method has the same accuracy as the basic CHT method.

We are currently trying to further improve the time of the voting process of the proposed ICHT by changing if possible the expression (8).

ACKNOWLEDGEMENTS

The authors would like to thank all those who helped to achieve this modest work as well as their useful discussions and comments.

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