

# A Bayesian Approach to FDD Combining Two Different Bayesian Networks Modeling a Data-Driven Method and a Model-based Method

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**Abstract:** In this paper, we present an original FDD method. The interest of this method is her ability to coexist residuals and measures, under a same and a single tool. Indeed, our proposal is to combine two different Bayesian networks to FDD. A model-based method is associated to a data-driven method to enhance decision making on the system operating state. This method is evaluated on a simulation of a water heater system in some various circumstances.

## 1 INTRODUCTION

Nowadays, monitoring methods (also known as Fault Detection and Diagnosis methods (FDD)) are more and more used, due to the increasing systems complexity. They contribute to the reduction of faults or in the ideal case for their elimination by an entity (operator, engineer, automaton...) which will examine the state of malfunction caused by these past and makes a decision about the future of the system (adjusting settings, maintenance, closure...). These methods are used to describe and explain, at each instant, the situation in which the system is situated. They consist of two phases usually associated: detection and diagnosis phases.

The detection phase seeks to confirm if the system is still in normal operating state (In control) or is not (Out of control). The diagnosis phase is used in order to designate the faults responsible for the deviation of the system of his normal operating. This phase can be defined in different ways depending on the desired description level. According to (Chiang et al., 2001) we can distinguish three definitions: identification (determine the susceptible measures explaining the occurred fault on the system), isolation (distinguishes the measure responsible for system abnormal functioning), diagnosis (explains the faults occurred in the system by expressing their type, location, amplitude and duration).

In the last years, many monitoring methods have emerged (Chiang et al., 2001; Isermann, 2006; Qin, 2006; Ding, 2008). Among them, we can distin-

guish two classes of methods: model-based methods and data-driven methods. Model-based methods use *a priori* knowledge of the system for explaining its dynamic behavior. This knowledge corresponds to a specific set of mathematical equations representing the dependencies that exist between the variables of the system and contributing to the generation of residuals (differences between observed and estimated measurements when the system is supposed in normal operation). Once generated, their evaluation contributes to the understanding of the operating state. In contrast, data-driven methods are based only on measures taken at different times, and analyzed in relation to a historical of data regarding the system.

The ability of the data-driven methods to manage a significant number of data associated with the capacity of the model-based methods to describe accurately the dynamic behavior of the system and to provide a physical understanding, might improve the monitoring, increase the number of scenarios taken into account, benefit from the advantages of both methods and to struggle against the individual shortcomings of each one when they are used separately. However, despite many researchers (Chiang et al., 2001; Venkatasubramanian et al., 2003; Ding et al., 2009) suggesting that the creation of a common framework using both classes of methods, would allow a better monitoring system, these research fields remain unexplored. Nevertheless, we can find some recent works in the literature describing different association of the two methods.

In (Schubert et al., 2011) a unified scheme is pro-

posed. The authors combine subspace approaches and univariate and multivariate statistical control methods (data-driven methods) with inputs reconstruction method and banks of Unknown Input Observer (model-based methods). Luo et al. (Luo et al., 2010) for antilock braking system (ABS), propose a hybrid approach using parity equations and a nonlinear observer for residuals generation. These residuals are used by statistical tests with the aid of SVM (support vector machine) to detect and isolate different faults that may occur in the system. In (Ghosh et al., 2011), for monitor a laboratory distillation column a fusion of decisions of several monitoring methods is proposed. The authors use four monitoring methods: a model-based method: an extended Kalman filter, and three data-driven methods: SOM (Self Organized Map), artificial neural network and PCA (Principal Component Analysis). The output of each method corresponds to an assignment to one class of fault. A fusion strategy is then applied using them to make the right decision (Bayesian decision and other). Yew et al. (Yew and Rajagopalan, 2010) propose collaboration between different methods under a multi-agent framework using some decision fusion methods.

The proposals mentioned above, although interesting for the combination of data-driven and model-based methods, does not seem to cover or address a particular problem which is the lack of information or approximations of the system (decrease in performance of monitoring). We believe that the combination of the two methods is mainly interesting when the two methods are able to complete their information and to finally provide better oversight. For example, a combination of a model-based method, without an accurate model, and a data-driven method, with some data are missing or insufficiently represented. In this paper, we propose a new monitoring method based on Bayesian networks. This method uses the complementarities that may have a data-driven and a model-based method in a single and common tool. The major interest of this combination is their ability to improve the decision making when the two methods suffer from a information lack or an approximations of the system (decrease in performance of monitoring).

The paper is structured as follows: in section 2 we introduce Bayesian networks followed by a short description of data-driven and model-based methods in sections 4 and 3; section 5 describes the monitoring methodology proposed; finally, the results of the proposed method obtained in different conditions on a simulation of a water heater system are outlined in the last section.

## 2 BAYESIAN NETWORKS

A Bayesian network (Buntine, 1996; Jensen, 1996), is a probabilistic directed acyclic graph. Each node in the network represents a random variable that may be discrete with  $n$  modalities (multinomial) or continue (univariate or multivariate). Each node has a conditional probability table (marginal probability table for root nodes). The oriented arcs show the conditional dependencies/independencies that exist between different nodes of the graph. Each directed arc can link only two nodes: among these nodes, one is called the father and the other, the son. For updating the network and calculate the different *a posteriori* probabilities corresponding to each node, given the availability of new information on the network (evidence), calculations (eg: Bayes rule) named inference is required. A Bayesian network, in general, can be defined formally by:

- a directed acyclic graph  $G, G=(V, E)$ , where  $V$  the set of nodes of  $G$ , and  $E$  the set of arcs of  $G$ ,
- $E$  is a finite probabilistic space  $(\Omega, Z, p)$ , with  $\Omega$  a non-empty space,  $Z$  a set of subspace of  $\Omega$  and  $p$  a probability measure on  $Z$  with  $p(\Omega) = 1$ ,
- a set of random variables associated with to the nodes of the graph  $G$  and defined on  $(\Omega, Z, p)$ , such that:

$$P(V_1, V_2, \dots, V_n) = \prod_{i=1}^n p(V_i | C(V_i)) \quad (1)$$

where  $C(V_i)$  is the set of parent nodes of  $V_i$  in the graph  $G$ .

Nowadays, several variants of Bayesian networks exist. One of them is the Bayesian network classifier, who is based on a discrete root node modelling the fact of belonging to one class among others. Note that under the assumption of dependence/independence of variables  $X$  emitted, several types of structures are proposed (Friedman et al., 1997). Among them, we use two kind of Bayesian networks classifiers: one is the Naive Bayes network, it's making the strong assumption that the variables are class conditionally independent and the second network is the semi-naive condensed Bayesian network who provides a simple structure that take into account correlation that may exist under a group of variables.

## 3 BAYESIAN NETWORK AND MODEL-BASED METHODS

The model-based methods, in the presence of an analytical representation of the system, use resid-

uals generators (Isermann, 2006) which as their name suggests, contribute to the creation of residuals  $(r_1, \dots, r_n)^T$  (the difference between the existing measures on the system and their estimates). Once generated a consistency test (evaluation of residuals) is triggered to check if no residual is different from zero. Indeed, during normal operating, the residuals are assumed to be equal to zero. However, sometimes they are not only sensitive to the faults but also the noise measurements performed on the system, to the disturbances and the modeling errors, makes them different to zero even during normal operation, that's why generally each residual is considered to be statistically null with a given variation (eg: a residual follows a normal distribution, with a standard mean  $\mu = 0$  and variance  $\sigma^2 = 1$ ). Thus, generally binary statistical tests are used to making decision between  $H_0$  (corresponding to the distribution of residuals during normal operating) and  $H_1$  the alternative hypothesis (corresponding to faults). This is achieved by attempting to minimize the risk of first and second kind, respectively,  $\alpha$  and  $\beta$  (see Figure 1).

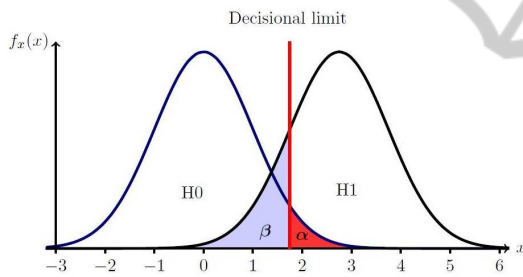


Figure 1: Statistical test.

The result of residuals evaluation  $(u_1^t, \dots, u_n^t)^T$ , in the case of the diagnostic methods based on structured residuals (constructed in order to be sensitive to certain faults and not to others), is compared (usually a logic test) to another vector representing the characteristics of each fault  $F_j \in \{F_1, F_2, \dots, F_k\}$ . These characteristics are generally assembled into a binary array called incidence matrix (an example is shown in Figure 1).

Table 1: Example of incidence matrix.

	IC	$F_1$	$F_2$	...	$F_k$
$u_1$	0	$b_{1,2}$	$b_{1,3}$	...	$b_{1,k}$
$u_2$	0	$b_{n,2}$	$b_{n,3}$	...	$b_{n,k}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$u_n$	0	$b_{n,2}$	$b_{n,3}$	...	$b_{n,k}$

Furthermore, in (Verron et al., 2009), Verron et al. model the last two phases of the model-based monitoring (knowing that monitoring methods

based model usually consists of three complementary phases: generation of residuals, residual evaluation (change detection) and decision making (diagnosis)). They propose a combination of two Bayesian networks works (modeling a control chart  $T^2$  (Verron et al., 2007) and the modeling of the incidence matrix (Weber et al., 2008)). To achieve this, a hybrid Bayesian network, representing  $k$  naive Bayesian network, is proposed. This network is made of discrete nodes (representing the  $k$  faults) with two modalities ( $\{\text{presence (yes) and not presence (No) of } F_j\}$ ) and continuous nodes (representing the  $n$  residuals) considered as a standard Gaussian variable (with a mean  $\mu$  and variance  $\sigma^2$ ).

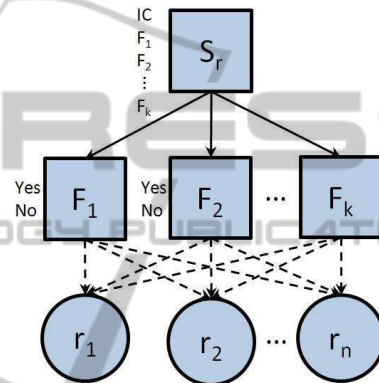


Figure 2: Bayesian network for model-based monitoring.

In order to combine explicitly the probabilities of belonging to one of the three faults (corresponding to the states of the system when it is out of control) and the probability that the system is always in normal operating state  $IC$ , we propose to add to the network a discrete parent node  $S_r$  with  $k + 1$  modalities (see 2) linked to all the other nodes  $F_j$ .

The conditional probability tables (CPT) of the node  $S_r$  and his son nodes  $F_{j \in \{1, \dots, k\}}$  are given in tables 2 and 3.

Table 2: CPT of the node  $S_r$ .

$S_r$			
IC	$F_1$	...	$F_k$
$\frac{1}{k+1}$	$\frac{1}{k+1}$	$\frac{1}{k+1}$	$\frac{1}{k+1}$

## 4 BAYESIAN NETWORK AND DATA-DRIVEN METHODS

Unlike methods that require an accurate model designed from first principles as a priori knowledge of the system, this methods try to detect and explain a change in the normal operating system, relying solely on measurements collected on the system

Table 3: CPT of the nodes  $F_1 \& F_2 \& \dots \& F_k$ .

$S_r$	$F_1 \& F_2 \& \dots \& F_k$	
	Yes	No
$IC$	0	1
$F_1$	$\frac{1}{2}$	$\frac{1}{2}$
$\vdots$	$\vdots$	$\vdots$
$F_k$	1	0
$\vdots$	$\vdots$	$\vdots$
$F_n$	$\frac{1}{2}$	$\frac{1}{2}$

(temporary or not). Several data-driven methods for monitoring purpose exist, they depends on the availability or unavailability of historical data on the system. Among this methods, we can firstly mention the subspace methods SMI (Subspace Model Identification) (Overschee and Moor, 1996), a linear identification algorithms, developed to address the problems of building an accurate model for complex systems. The principal component analysis (ACP) (Harkat et al., 2006), is a statistical method that can be used to modelise existing dependences between a set of system variables or as a method of data reduction (used when the number of variables is considerable). Mention may also the control charts methods (MacGregor and Kourti, 1995), statistics gathered over a period of time  $t$ , which are also widely used in industry. They (control chart  $T^2$  of Hotelling, MEWMA (Multivariate Exponentially Weighted Moving Average) are designed to monitor the normal operation of the system.

Otherwise, when a history of faults is available, supervised classification methods, adapted to the diagnostic can be used. Indeed, the diagnosis problem can be formulated for a given observation, as a problem of discrimination between several operating modes. Among this data-driven methods, we can mention neural networks (Duda et al., 2001), SVM (support vector machine) (Steinwart and Christmann, 2008), discriminant analysis (Fukunaga, 1990) (can be modelled under a Bayesian network, where the variables are assumed follow a multivariate normal distribution).

In our work, one assuming our knowledge about the faults covers almost all the space (assumed as closed space) out of control ( $H_1 \subseteq \{F_1, \dots, F_k\}$ ), we propose to use a Bayesian network classifier to discriminate between these faults and the state  $IC$  (where the system is considered in normal operating condition  $H_0$ ). To achieve this, we use a Bayesian Network (see figure 3) naive semi condensed (RBNSC) consisting of a discrete node representing  $k + 1$  modalities and continuous multivariate Gaussian node (with a mean  $\mu$  and variance  $\Sigma$  estimated on the fault database

by Maximum Likelihood Estimation (MLE) (Duda et al., 2001)) combining all the  $m$  variables of the system  $(x_1, x_2, \dots, x_m)$ .

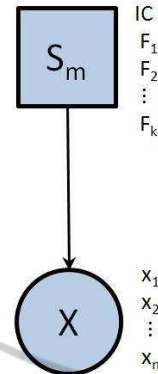


Figure 3: Bayesian network for data-driven monitoring.

By making assumption about normality of each modality, this network using a decision rule based on the Bayes formula (2):

$$P(Y/X) = \frac{P(Y)P(X/Y)}{P(X)} \quad (2)$$

corresponds to a quadratic discriminant analysis (5) :

$$\delta : x \in M_j^*, \text{ if } j^* = \operatorname{argmax}_{j=1, \dots, k+1} \{P(M_j/x)\} \quad (3)$$

$$= \operatorname{argmax}_{j=1, \dots, k+1} \left\{ \frac{P(M_j)P(x/M_j)}{P(x)} \right\} \quad (4)$$

$$= \operatorname{argmax}_{j=1, \dots, k+1} \{P(M_j)P(x/M_j)\} \quad (5)$$

Where  $P(M_j/x)$  is the probability *a posteriori* of  $Y$ ,  $P(x)$  is the density function of  $x$ ,  $P(x/M_j)$  is the likelihood and  $P(M_j)$  the prior probability of  $M_j$

Thus, it making us able to decide to a given instant, in which operating state, the system belongs among its various states separated quadratically ( $IC, F_1, F_2, \dots, F_k$ ). The probability tables for each node are shown in the tables 4, 5.

Table 4: CPT of the node  $S_m$ .

$S_m$			
$IC$	$F_1$	$\dots$	$F_k$
$\frac{1}{k+1}$	$\frac{1}{k+1}$	$\frac{1}{k+1}$	$\frac{1}{k+1}$

Table 5: CPT of the node  $X$ .

$S_m$	$X$
$IC$	$X \sim N(\mu_{IC}, \sigma_{IC}^2)$
$F_1$	$X \sim N(\mu_{F_1}, \sigma_{F_1}^2)$
$\dots$	$\dots$
$F_k$	$X \sim N(\mu_{F_k}, \sigma_{F_k}^2)$



### 5 DECISION FUSION

To improve the decision making, in this work, we propose to combine the two monitoring methodology in a same and a single tool. Indeed, our proposal consists to combine the two methods discussed above under a Bayesian network.

Thus to build our new network combining both methods, a new discrete node  $S_{r\&m}$  is added (see figure 4). This node represents like the root nodes  $S_r$  and  $S_m$  a variable with  $j + 1$  modalities. One of these corresponds to normal operating condition (IC) where the other modalities  $j$  corresponds to the known faults that may occur on the system. This new node  $S_{r\&m}$  is used to connect the two root nodes and so associate the both methods by fusing their decisions. Thus, a conjunctive combination (see (Xu et al., 1992; Chen et al., 1997) for others decision fusion methods) is made under the assumption that the two network are conditionally independent to the node added. Indeed, thanks to the use of the Bayes formula (2), the Bayesian network offers a naturally probabilistic fusion capacity.

Once the network is built, we enter the observations (evidences) in the network. They correspond to the residuals obtained and the measures taken on the system at a given instant. These evidences are then transmitted to other unobserved nodes in the network. Their marginal probabilities are then calculated using the inference method employed (we use junction tree). After having carried out this inference, the node  $S_{r\&m}$  indicates for each  $k + 1$  modalities, the probability of its occurrence.

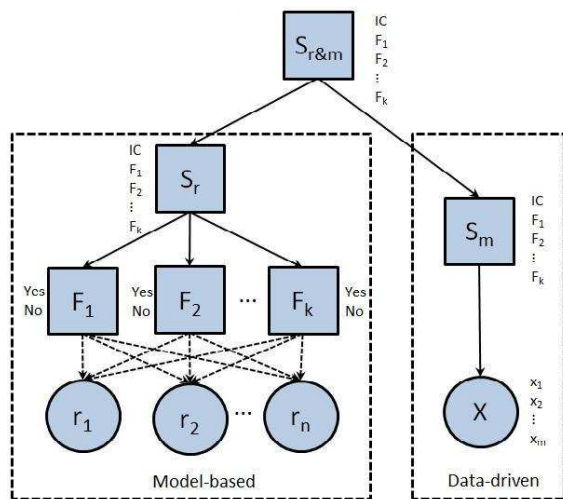


Figure 4: A two combination methods.

Regarding the decision, among others criterions, we chose to use the maximum *a posteriori* like we do for the others methods (model-based, data-driven),

Table 6: CPT of the node  $S_{r\&m}$ .

$S_{r\&m}$			
IC	$F_1$	...	$F_k$
$\frac{1}{k+1}$	$\frac{1}{k+1}$	$\frac{1}{k+1}$	$\frac{1}{k+1}$

where the modality with the higher probability *a posteriori* is choose. The table of conditional probabilities of node  $S_{r\&m}$  is shown in table 6.

### 6 APPLICATION

To illustrate our approach, we use a simulation of a water heater. It consists of a tank equipped with two resistors  $R_1$  and  $R_2$ . The inputs are the water flow rate  $Q_i$ , the water temperature  $T_i$  and the electric power for heating  $P$ . The outputs are the rate of water flow  $Q_0$  and the temperature  $T$  regulated around an operating point. The temperature of the incoming water  $T_i$  is assumed constant.

The objective of the system is to provide a water flow at a given temperature. Using hydraulic and thermal equations, in this analysis, only sensor faults are considered: water level sensor  $H$ , temperature sensor  $T$ , sensor flow of water from  $Q_0$ . The detailed mathematical model of the system is presented in (Weber et al., 2008).

A classic residuals generator is used: a Luenberger observer. The output vector is  $[H, T]^T$  and the input vector  $[Q_i, P]$ . Structured residuals  $[r_1, r_2, r_3]$  are generated and evaluated to detect faults of water level sensor  $H$  and temperature sensor  $T$ . According to the physical equations between the flow rate  $Q_0$  and the liquid level  $H$ , other residual can be established. The incidence matrix (the link between symptoms  $[u_1, u_2, u_3]$  and faults  $[T, H, Q_0]$ ) defined in table 7 will give the structure of our Bayesian network.

Table 7: Incidence Matrix of the heating water system.

	IC	T	H	$Q_0$
$u_1$	0	1	0	0
$u_2$	0	0	1	0
$u_3$	0	0	1	1

We simulated the system according to the scenarios described in table 8 in order to test the proposed method under different assumptions. Indeed, the value of combining the two methods is to be able to benefit of good results even when one or the other method is not very efficient (missing detection of abnormal operating state, faults misdiagnosis). Thus, we propose to test the Bayesian network, taking into account an accurate model ( $M+$ ) or a less accurate

( $M^-$ ), and a complete dataset of suitable size ( $D^+$ ) or incomplete dataset (a lack faults data, a few data) ( $D^-$ ). Thus, the scenarios previously presented will be tested on four assumptions described in 9.

Table 8: Simulated scenarios.

period	1-30	31-60	61-90	91-120
case	In Control	fault $T$	fault $H$	fault $Q_0$

Table 9: Hypothesis matrix.

hypothesis	M(Model)	D(Data)
$H_I$	$M^+$	$D^+$
$H_{II}$	$M^+$	$D^-$
$H_{III}$	$M^-$	$D^+$
$H_{VI}$	$M^-$	$D^-$

Every simulation was performed using Matlab/Simulink and BNT (BayesNet Toolbox). For each observation, we attribute the fault to the modality with the greater *a posteriori* probability. The different results of the simulation obtained by testing our methods under four different assumptions are presented in figure 5.

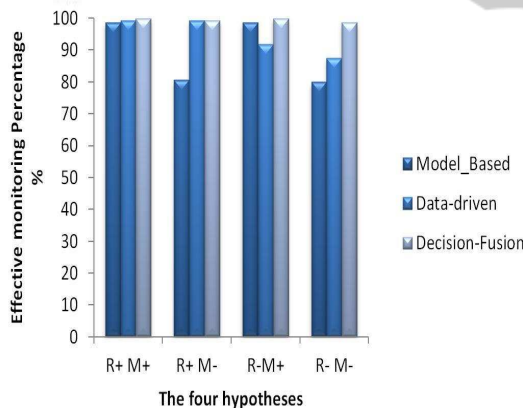


Figure 5: Simulations results under the four hypothesis.

One can notice that for each hypothesis, the proposed method can usually equalize the performance of each method and even to improve the monitoring. In hypothesis  $I$ , the three methods are good, they give right answers. The proposed method is slightly more efficient. For the hypothesis  $II$ , we suppose that the data of the fault  $H$  misses us. In this condition, we see that the method proposed can have good results. The same thing is happen, in the case of the third hypothesis, where we have degraded the model. Finally, in the hypothesis  $VI$ , the proposed approach allows a better decision than the two other methods. This shows that the proposition made is efficient and can take advantage of the two basic methods under a Bayesian network.

## 7 CONCLUSIONS

The interest of this paper is to present a new method for monitoring industrial systems. We have presented a particular structure of Bayesian network which consists of discrete and Gaussian nodes allowing to models and combines two Bayesian networks dedicated to monitoring: one for data-driven monitoring and one representing the incidence matrix and the evaluation of residuals for the model-based monitoring. This original structure can enhance decision making during monitoring using simultaneously data and residuals. This method has been tested on a water heater system, where an improvement of the decision is made and this in the most cases (specific model, model degraded and more or less data).

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